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A
NEW and COMPENDIOUS
SYSTEM
OF
OPTICS.

In THREE PARTS, *viz.*

PART I. CATOPTRICS, or the Doctrine of Vision by Rays reflected from *Mirrours*; or polished Surfaces.

PART II. DIOPTRICS, or the Theory of Vision by Rays refracted through *Lenfes*, or transparent Substances.

PART III. A Practical Description of a great Number of the most useful Optical *Instruments* and *Machines*, and their Construction shewn from the Theory; *viz.* The EYE, CAMERA OBSCURA, *single* and *double* MICROSCOPES, *Refracting* and *Reflecting* TELESCOPES, PERSPECTIVE GLASSES, the MAGIC LANTHORN, &c. The Manner of adapting MICROMETERS to MICROSCOPES and TELESCOPES of the reflecting Sort.

The whole explained, exemplified, and illustrated by a great Variety of Copper-Plate Figures, as big as the Life.

By BENJAMIN MARTIN.

Author of the *Philological Library of Literary Arts and Sciences*; and *A Treatise of Logarithms, Common and Logistical, in Theory and Practice.*

Oculi, pars corporis pretiosissima, & qui lucis usu vitam distinguant à morte. PLIN. Nat. Hist. Lib. II. Cap. 37.

Nam sic possunt figurari perspicua & specula, ut unum appareat multa — ut longissimè posita, appareant propinquissima — ut maxima appareant minima, & alta appareant infima, & occulta videantur manifesta, — & stellas faceremus apparere quo vellemus, ROG. BACON. Epist.

L O N D O N :

Printed for JAMES HODGES, at the Looking-Glass on London-Bridge. MDCCXL.



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TO
MARTIN FOLKES, *Esq*;
VICE-PRESIDENT
OF THE
ROYAL SOCIETY, &c.

S I R,

THE Reputation You have acquired of a general and profound Knowledge in the Arts and Sciences, as well Literary as Mathematical, and Your particular Taste for *Optics*, and the more delicate

a 3

vi *DEDICATION.*

cate Pleasures arising from the various Effects and Inventions of that noble Science, point You out as the most proper Patron for my *System of Optics*. And I presume the more freely to put my Book out under the Patronage of Your Name, as You have been pleased to let me know You thought my former Pieces worth Your Notice and Perusal. And as You have honoured me with Your Approbation of my new Pocket Microscopes, I shall take the Freedom, e'er long, of presenting You with something new in the Reflecting-Telescope Way, if Success attends my Designs. I should think myself in nothing happier, than in doing something that You shall approve and accept. And if the following Book can be thought to have so much Merit,

DEDICATION. vii

Merit, my highest Ambition will be satisfied : And I take this Opportunity of letting You know how much I value Your Judgment and Friendship, and am, with all Gratitude and Respect,

S I R,

Your most obliged, and

Most obedient Servant,

B E N J. M A R T I N.

1871
[Faint, illegible text follows, appearing to be a list or account of items and their values, possibly in a ledger or inventory format.]

THE
PREFACE.

A*S the Eye among the Organs of the Body, so Optics among the Sciences, is the most delicate, curious, and useful; as the Parts and Structure of the Eye surpass that of most other organical Parts, in Point of Mechanism and wondrous Contrivance, so the Principles and Theorems of Optics are of a peculiar Nature, wonderful in their harmonious Origin, and express a whole Science in a Line. Lastly, As the Eye is that Organ by which*

which we have all our Ideas of the Magnitude, Order, Number, Disposition, Colours, &c. of Things about us ; so Optics is that Science which alone accounts for the Reason and Manner of such Sensations : And a Man not versed in the Visual Science, can no more properly be said to see Things rationally, than a Quadruped ; and has little better Notions of apparent Magnitudes and Distances of Things, than a blind Man has of Colours.

THUS Optics is in itself of the last Importance, and to be well understood to discriminate the Man from the Brute, in Regard of the noble Sense of Seeing. But this is not all ; it is also the Ground-Work, or Fundamental Science, to many others, as Perspective, Painting, Architecture, Astronomy,

my, Dialling, Surveying, &c. *How lame and imperfect must any of those Arts and Artists be, were they not assisted and succoured by the Principles and Rules of this noble Science?*

AGAIN, If we regard Curiosity, what Science can compare with Optics, in the whole Encyclopædia? What Gratifications of Sense so exquisite as those of Sight? By Optics the Heavens have been revealed to us; the Spots and Feculæ on the Face of the Sun, the Horns and waning Phases of Venus, the Mountains and Vales in the Moon, the Satellites and Belts of Jupiter, and Saturn with his wondrous Ring; besides innumerable Stars not otherwise to be seen but by the Telescope! What Pleasure, yea, how useful is it, to have Objects thirty or forty Miles

Miles off, brought within the Distance of one Mile, or half a Mile? Yet this the Telescope effects. Again, What Discoveries have been made in the World of Miniature, where Objects so very small as otherwise must ever have been unseen and unknown by us, are made conspicuous, and rendered visible in their minuter Parts? Who could ever have thought of the Animalculæ in Water, in Semine, &c. The Eels in Vinegar and Water; the Pores and Air-Vessels in Wood; the pearly Drops on Leaves of various Plants; the vesicular Substance of Beans, Pease, and all Kind of Pulse; the curious Forms, the particular Structure of Parts, and the rich Colours that adorn most of the invisible Tribes of Animals! Yet all this, and ten thousand times more, is performable by the Microscope; an Instrument

The P R E F A C E. xiii

strument which no reasonable Man should want, inasmuch as it serves him instead of Microscopic Eyes. As to the Benefit those receive from this Science, whose Eyes require the Aid of Spectacles, it is so great and so general, that it would be meer Impertinence to pretend to expatiate upon it; and many other invaluable Blessings result from this Art of improving Sight, which the Experience of Mankind has long since evinced, we can't be without, and not be miserable.

YET notwithstanding what has been said of the exceeding Usefulness of this Science, I am too well convinced but very few understand any Thing of it. If you ask why they do not study Optics, they re-interrogate, what Books should we read? If you refer them to
Mr

Mr Molyneaux, that is too large, and too much perplex'd with algebraical Solutions, and is therefore only fit for Scholars. If you recommend Dr Gregory's Elements, the Geometrical Demonstrations of every Proposition deter the Reader, and Mr Browne's Supplement thereto involves him in a Labyrinth of analytical Investigations and Solutions, with little Order and Perspicuity, and great want of Schemes. If, lastly, you advise them to read Dr Smith's Treatise of Optics, they tell you it is too expensive, and so voluminous, that they cannot pretend to have Time for reading so much upon the Subject, besides that by far the greatest is above their Understanding. These, and such like, are the Objections to the Books extant, and therefore it was judged necessary to draw up a new System

System of Optics, which might, if possible, obviate those Objections, and remove the Difficulties that have hitherto discouraged Persons from the Study of so excellent a Science.

*I*N Order to this I judged it necessary to dispatch the Theory in as short, yet plain a Manner as possible; this in Catoptrics I have done from the admirable universal Theorem, invented by the late Mr Humphrey Ditton; and in Dioptrics I was supplied with that wondrous Theorem, which expresses the whole Science in half a Line, and is one of the many noble Inventions of the justly renowned Dr Halley. These two general Theorems I have explained and branched out into all the particular Cases that can arise from different Rays, Mirrours, and Lenses, Distances, and

and ^aPositions of Objects ; and after that, lest any should not understand the Theory in Species, I have carefully explained, or rather expressed, each Theorem (in Catoptrics and Dioptrics) in Words at Length, and so reduced them to Rules, by which any Case may be solved truly, by those who know nothing of Algebra or Geometry. I have also given Examples (to every Case that required it) in Numbers, and illustrated them by Schemes and Figures of Rays, Mirrours, and Lenses, as large as the Life, which has never been done before that I know of.

AFTER the Theory, in the Third Part, you have an Account of all the useful Optical Instruments and Machines, whose Nature and Construction, are fully explained from the Theory ;
and

The P R E F A C E. xvii

and their Uses exemplified by divers and familiar Examples. In short, whatever I judged curious, new, and worth the Reader's Notice, I have inserted it all along; but studiously avoided all nugatory Remarks, and the Minutiæ of the Art, on one Hand; and all ostentatious Subtilties, and useless Disquisitions, that may puzzle but not profit the Reader. My sole Design being to render the Study of this Science as easy, delightful, and as general, as possible; to effect which, I have done all that is in my Power; and I can neither do nor say any more.

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C H A P.

C H A P. I.

D E F I N I T I O N S.

I. **O**PTICS is a Science which teaches the Nature, Properties, and Laws of *Vision*, arising from the Rays of Light, either *reflected* from the Surfaces of Bodies, or *refracted* in passing thro' them, and painting the Images of Objects on the *Retina* on the Bottom of the Eye. Also this Science, in it's most extensive Acceptation, comprehends the whole Doctrine of *Light* and *Colours*, and all the *Phænomena* or Appearances of *visible Objects*. Optics, therefore, consisteth of three Parts, *viz.* *Catoptrics*, *Dioptrics*, and *Chromatics*. 27-

II. CATOPTRICS is that Part which treats of *Reflek Vision*, or all that relates to the viewing of Objects by Light *reflected* from the Surfaces of Bodies, whether *plain*, *convex*, *concave*, or otherwise; and in Rays *diverging*, *converging*, or *parallel* to each other. 28-

A

III. DIOP-

III. DIOPTRICS treats of the Properties of Light and Vision, arising from Rays passing thro' transparent *Media* or Bodies, as *Air, Water, Glass, Crystal, Diamond, &c.*

IV. CHROMATICS treats of the *Colours* of Light and natural Bodies. Of this Part Sir *Isaac Newton's* Optics does almost entirely consist.

V. LIGHT is that Property of some Bodies by which Objects^a are rendered *visible*, or capable of being *seen* by the Eye. It consists of very small Particles which issue from the luminous Body in strait Lines.

VI. RAYS or BEAMS of Light are those Streams or Emanations of Light, which proceed from the luminous Body, and enlighten or illuminate all Objects so that they may be *seen*.

VII. The RADIANT is that Body or Object which emits, or from which proceed, the Rays of Light under Consideration at any Time.

VIII. The SPECIES of an Object is the Image or Representation thereof, made by the Rays of Light in the *Focus*, or Place where they unite.

IX. PARALLEL

A
A
DEFINITIONS.

3

IX. PARALLEL Rays, are such as proceed *equally distant* from each other thro' all their Course ; as those from the *Sun*, and other vastly distant Objects. See Fig. 1. Plate I.

X. CONVERGING Rays are such as, proceeding from a Body, approach nearer and nearer together in their Progress, tending to one certain Point, where they all unite ; thus the Rays proceeding from the Object A B to the Point F, are said to *converge* towards that Point. Fig. 2.

XI. DIVERGING Rays are those, which, proceeding from any Point, as A; do continually recede from each other as they pass along in their Course towards B C. Fig. 3.

XII. The Focus of Rays is that Point to which all Converging Rays tend, and in which they unite and intersect each other ; as the Point F, Fig. 2. And this is called the *Real Focus* ; but

XIII. The VIRTUAL or *imaginary Focus*, is a Point, as F, to which the Rays A B tend, and where they would unite, were they not intercepted by the Obstacle (suppose a Mirrour) C D ; by which Means they are turn'd aside, and made to converge in their *Real Focus* F. Fig. 4.

A 2

XIV. RE-

XIV. REFLECTION of Rays is their *Regress* or *Returning* from the Surface of such Bodies on which they fall, and cannot penetrate or enter. Thus the Ray BC falling on the Surface AD, is reflected or turned back or up again in the Direction CE. 765

XV. The PLANE of *Reflection* is that in which the reflecting Point, or Surface, is situated, as AD in Fig. 5. and *ad* in Fig. 6 and 7.

XVI. MIRRORS or *Speculums*^a are those Bodies whose Surfaces are so very smooth, and fine polish'd, as to be impervious to the Rays of Light which fall on them, and which therefore they reflect so entirely, as to represent the *Images of Objects* opposed to them. These are generally made of Glass polish'd on one Side, and quicksilver'd on the other; and are either *plain*, *convex*, or *concave*.

XVII. PLAIN MIRRORS are those whose Surfaces are perfect *Planes*, and whose Section is a *strait Line*, as AD, Fig. 5. Note, these are vulgarly called *Looking-Glasses*.

XVIII. CONVEX MIRRORS are such whose Surfaces do every Way equally and uniformly rise above the Plane of their Bases or lowest Parts; the Section of which Sort of Mirror is a *Curve*, either *Circular*, *Elliptical*,
Parabolical,

DEFINITIONS. 5

Parabolical, or Hyperbolical. See Fig. 6. where AD is a *Circular* Section, and the *Mirroure* is the Segment of a *Globe*, or *Spherical* Surface, which are of most common Use. As

XIX. CONCAVE MIRROURS are those whose Surfaces sink down with an uniform ~~Hollowness~~ or Curvity, below the upper Parts AD, and whose Section also is a *Curve*, as various as the Convex above; but AD in Fig. 7. is circular, and it's Surface the internal Part of an *hollow Sphere*, as being most in Use.

XX. The INCIDENT Ray is that which comes from any Object, and falls on the reflecting Surface as BC; and CE is the *reflected Ray*.

XXI. The ANGLE of INCIDENCE is that which is contained between the *incident Ray* BC, and a perpendicular to the reflecting Surface in the Point of Reflection FC, viz. the Angle BCF. Fig. 5, 6, 7.

XXII. The ANGLE of REFLECTION is that contained between the said perpendicular FC, and the reflected Ray CE; viz. the Angle FCE. Fig. 5, 6, 7.

XXIII. REFRACTION of Rays is their being bent or turned out of their first Course,

A 3

in

6 DEFINITIONS.

in passing out of one *Medium* into another. Let ADHI be a Body of *Water*, AD it's Surface, C a Point in which a Ray of Light BC (in the Air) begins to enter the same; this Ray, by the greater Density of the Water, will be resisted, and instead of passing strait forwards in it's first Direction to K, it will be bent therefrom, and made to describe the Tract CE, which is called the *refracted Ray*. Let FG be drawn perpendicular to the Surface of the Medium in C, then it is plain the Ray BC, in passing out of a *rarer Medium* (*viz.* of *Air*) into a *denser Medium*, (*viz.* of *Water*) is refracted into a Ray CE, which is nearer to the perpendicular CG, than the *incident Ray*; and, on the contrary, the Ray EC passing out of a *denser* into a *rarer Medium*, will be refracted into CB, which is farther from the perpendicular.

XXIV. The ANGLE BCF is the Angle of *Incidence*, as before; and ECG is the *Angle of Refraction*, as being contained between the refracted Ray CE, and the perpendicular CG.

XXV. A LENS is a Medium, generally of *Glass*, of a proper Form to *collect* or *disperse* the Rays of Light which pass through it. Of these there be various Forms, and which, from thence, receive divers Names. As

XXVI. A

XXVI. A PLANO-CONVEX, which hath one Side plain, the other Spherical or Convex; as (Fig. 9.) A.

XXVII. A PLANO-CONCAVE, *plain* on one Side, and *concave* on the other; as B, Fig. 9.

XXVIII. A DOUBLE-CONVEX, is one *Convex* on both Sides; as C, Fig. 9.

XXIX. A DOUBLE-CONCAVE, is one *Concave* on both Sides; as D, Fig. 9.

XXX. A MENISCUS, is one *Convex* on one Side, and *Concave* on the other; as E, Fig. 9.

XXXI. The VERTEX of a MIRROR or Lens, as A B, is the middle Point V, every Way equally distant from it's Base. Fig. 10.

XXXII. The AXIS of a *Mirror* or *Lens* is the Right Line E D, drawn thro' the Vertex V, and the Center C, on which it was described.

XXXIII. The VISUAL or OPTIC *Angle*, is that which is contained under the two Right Lines drawn from the extreme Points of an Object to the Eye; thus A E B or C E D

A 4

is

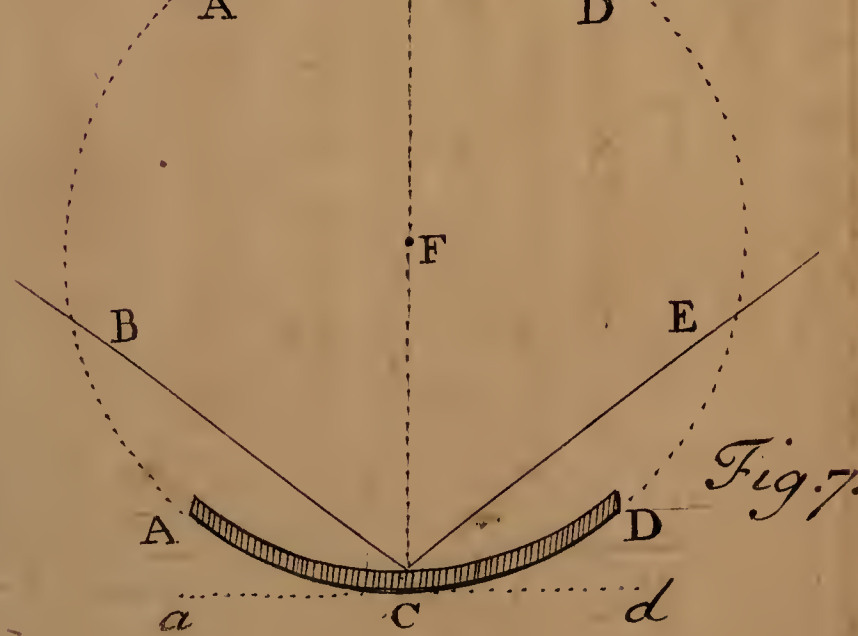
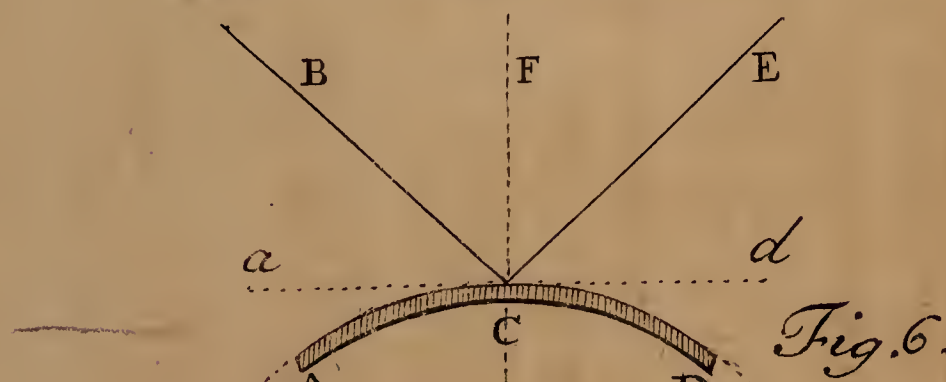
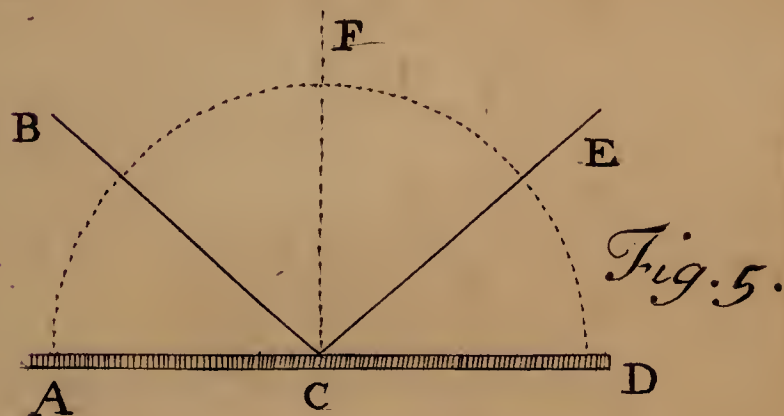
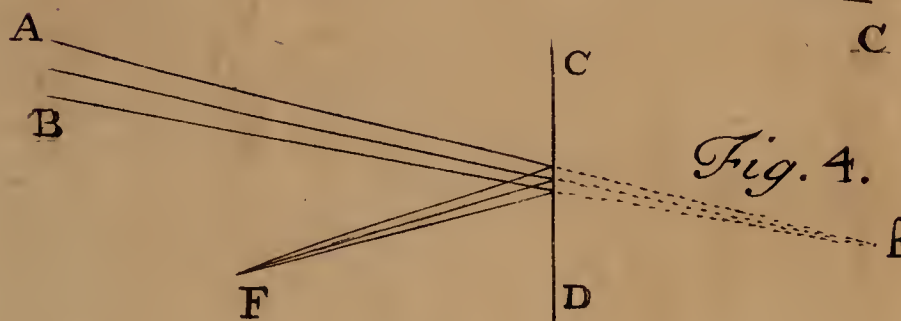
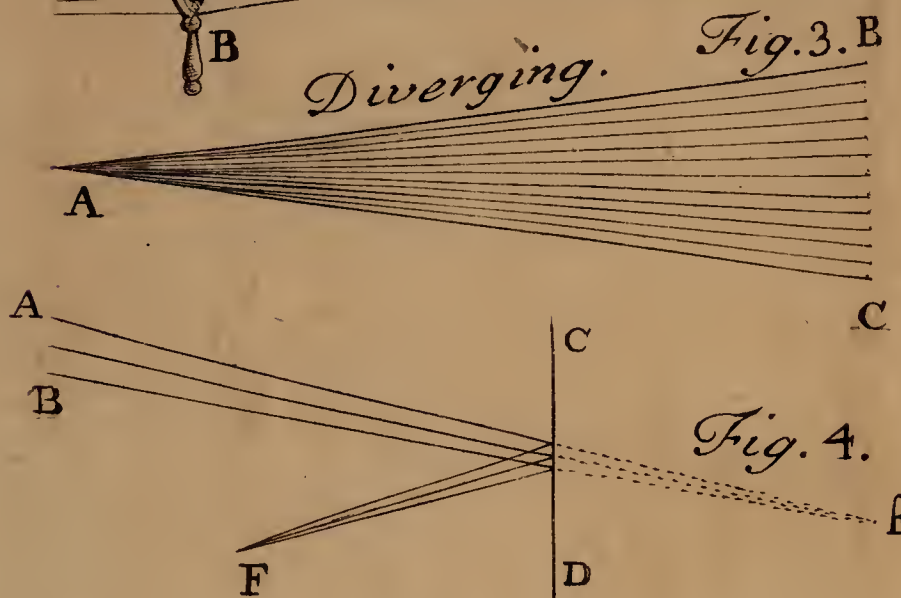
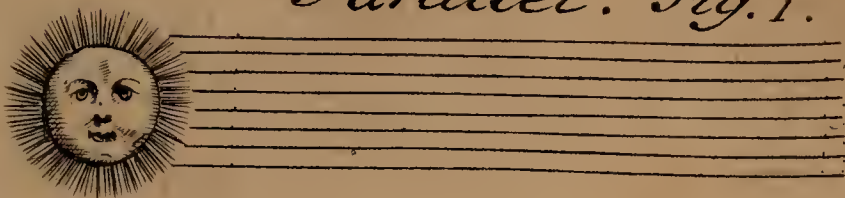
8 DEFINITIONS.

is the *Optic Angle*, or that under which the Object AB or CD appears to the Eye at E .
Fig. 11.

XXXIV. A PENCIL of Rays is a double Cone of Rays, as $LONFL$, joined together at the Base in the *Lens* LN ; of which one Cone LON has it's Vertex in some Point of an Object, as at O ; and the other Cone LFN has it's Vertex in the Point of Convergence, or *Focus* F , Fig. 12. The middle Line OF is called the *Axis* of that Pencil.

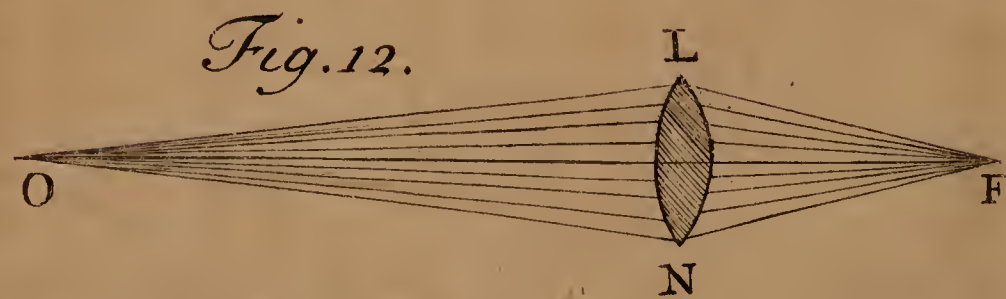
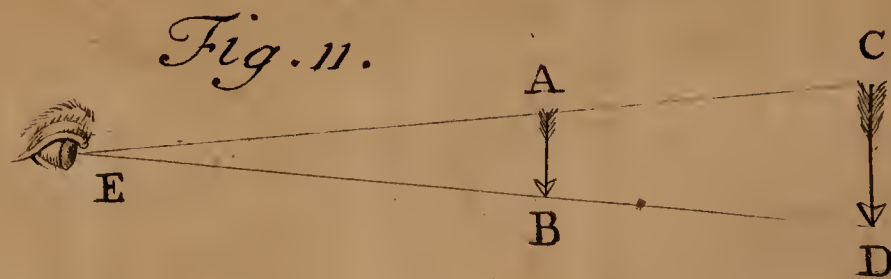
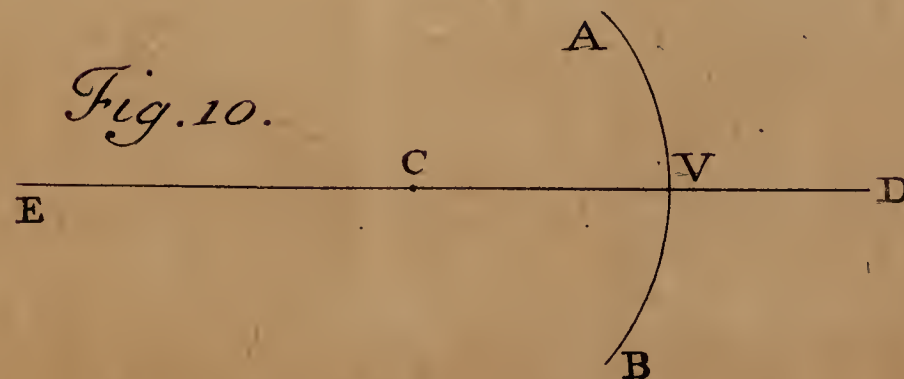
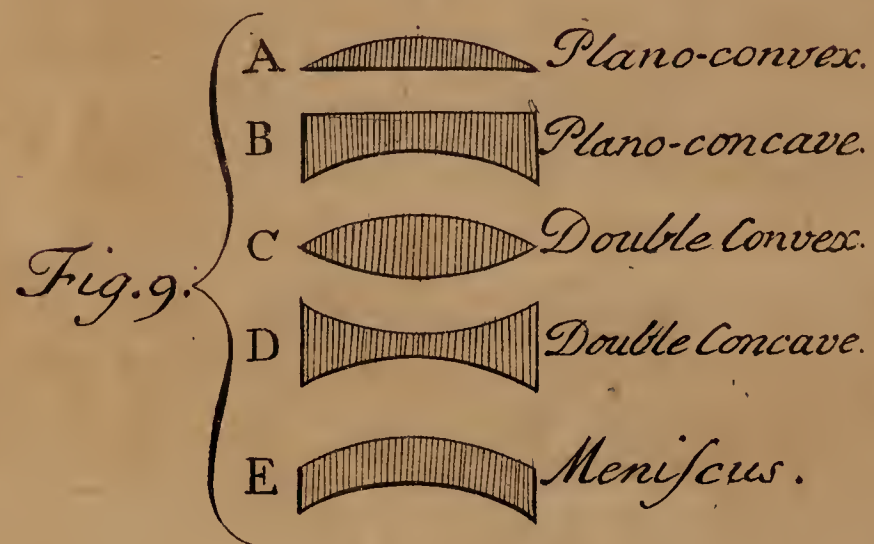
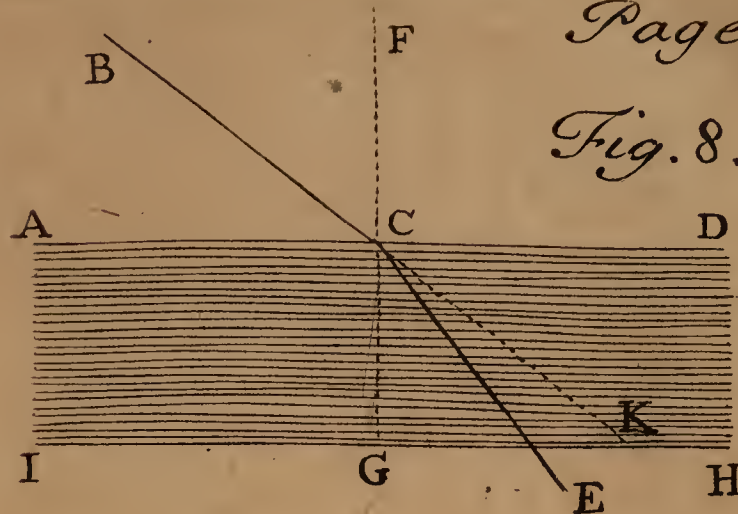


Parallel. Fig.1.



~ 1.

Fig.8.



C H A P. II.

The Principles of CATOPTRICS and DIOPTRICS.

WHAT I shall here propose and lay down as the *Principles of Optics*, are those things on which the following *New Theory*, both of *Catoptrics* and *Dioptrics*, does chiefly depend; and which are in themselves very evident, or have been demonstrated by Mathematicians, or confirm'd by *Experiments*; or lastly, are such as tho' not *strictly or geometrically true*, may nevertheless be *assumed as such*, without any sensible Error.

PRINCIPLE I.

In very small Angles BCE, the Sine of the Angle AD, the Arch DB, and the Tangent EB, are very nearly equal to each other.
Plate 2. Fig. 1.

For suppose the Radius CB divided into 100000 equal Parts, the Sine, the Arch, and the Tangent, will each of them be but 29 of these Parts, when the Angle or Arch is but one Minute of a Degree. Or if the Arch be
of

of one Degree, it's Length will be $1745\frac{3}{10}$, that of the Sine $1745\frac{2}{10}$, and that of the Tangent $1745\frac{5}{10}$, of such Parts. The Difference being so extremely small, the Proposition is evident.

PRINCIPLE II.

In every Triangle, the Sides are proportional to the Sines of their opposite Angles.

This is demonstrated by all the Writers of *Trigonometry*. And therefore, when these Angles are very small, *they have the same Proportion with their opposite Sides*, by Princip. I.

PRINCIPLE III.

In every Plain Triangle BCD, (Fig. 2.) if the Side CB be produced, the outer Angle ABD shall be equal to the Sum of the Angles at C and D. By Euclid's 32d Prop. of Book I.

PRINCIPLE IV.

Let AB (Fig. 3.) be part of a Circle described with the Radius CB; and EB an Arch described with the Radius DB, let CD join the Centers C and D, and draw DE; this shall intersect the Arches in G and E, and GE will be the Difference between DE and DB. Now it is evident the greater DC is, the less will be GE, the Angle BDE remain-
ing

CATOPTRICS *and* DIOPTRICS. II

ing the same ; also the nearer the Point E is to B, the less is the Difference GE ; and therefore, when E is very near to B, the said Difference will be very small, and inconsiderable ; and consequently, the Lines DB and DE may, in such a Case, be assumed equal to each other.

PRINCIPLE V.

The incident and reflected Ray are both in the same Plane.

PRINCIPLE VI.

The Angle of Incidence BCF is ever equal to the Angle of Reflection FCE. (Plate 1. Fig. 5.)

This is evident from Experiment. See the *Philos. Grammar*, Book I. Chap. VI, Note *, and is demonstrated by Dr Gregory in his *Elem. of Catop.* Prop. 1.

PRINCIPLE VII.

The incident and refracted Ray are both in the same Plane.

PRINCIPLE VIII.

Refraction out of a rarer Medium into a denser, is made towards the perpendicular ; and vice versa into a rarer Medium. See Defn. 23.

PRINCIPLE

PRINCIPLE IX.

The Sine AD of the Angle of Incidence BCF, is either accurately or very nearly in a given or constant Ratio, or Proportion to the Sine HR of the Angle of Refraction ECG.
Fig. 4.

This Ratio of the Sines, when the Refraction is made out of *Air* into *Water*, is as 4 to 3; that is, $AD : HR :: 4 : 3$, in *Water*.

When the Refraction is out of *Air* into *Glass*, the Proportion is as about 17 to 11, or more nearly, as 31 to 20, or as 77 to 50; but for common Use, as 3 to 2; that is, $AD : HR :: 3 : 2$, in *Glass*.

If the Refraction be out of *Air* into *Diamond*, it is as 5 to 2; and $AD : HR :: 5 : 2$. For the denser the Medium is, the less will be the Angle and Sine of Refraction.

PRINCIPLE X.

X *Wherever the Rays of Light, which come from all the Points of any Object, meet again in so many Points after they have been made to converge by Reflection or Refraction, there they will make a Picture of the Object upon any white Body on which they fall.*

It

It was necessary to premise this as a *Principle of Optics*, tho' the Grounds and Truth of it cannot, till after a few Pages, be made to appear.

PRINCIPLE XI.

An Object seen by reflected or refracted Rays appears in that Place, from whence, after their last Reflection or Refraction, they diverge in falling on the Spectator's Eye.

PRINCIPLE XII.

The apparent Magnitude of an Object is determined or estimated by the Magnitude of the Optic Angle which it subtends, or under which it appears at the Eye of a Spectator.

Thus (in Fig. 11. Plate I.) the Magnitude of the Object AB is estimated or measured by the Quantity of the Angle AEB; so is also the Object CD; but CD is greater than AB; and since they appear under the same Angle, it is evident, the *apparent Magnitudes* of Objects may be equal, when their *true or real Magnitudes* may be unequal in any given Proportion.

C H A P. III.

The Theory of CATOPTRICS.

1. **L** E T G E represent a Portion of a Convex *Speculum* Mirrour ; C it's Center ; C A or C E the Semidiameter or Radius ; let D be a radiant Point in the Axis of the *Speculum*, from whence D A, a Ray of Light proceeding, falls on the Point A, and is reflected into the Direction A f, tending to a Point F, it's *virtual Focus*, in the Axis of the *Speculum* behind the Vertex E. Fig. 5.

2. Let $DE = d$; $CA = CE = r$; $CF = z$; and $FE = f = r - z$; and consequently, $f + z = r = CE$.

3. Suppose the Point A very near to E ; then will the Angles at D and C be very small, and consequently will have the same Proportion to each other as their opposite Sides A C and A D have ; by Princip. 1, and 2. But $AC = AE$, and DA may be esteemed equal to DE ; by Princip. 4. Therefore it will be $ADC : ACD :: CE : DE :: r : d$.

4. Produce CA to I, so shall I A be perpendicular to the Mirrour in the Point of Reflection A ; and therefore the Angle D A I = I A f, by Princip. 6. But $DAI = dAC$,
and

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and $\angle I A f = \angle C A F$, as being Angles at the Vertex of two intersecting Lines, by *Euclid*, Book I. Prop. XV. Therefore $\angle A C = \angle C A F$. Again, $\angle A C = \angle A D C + \angle A C D = r + d$, by Princip. 3. and consequently the Angle $\angle C A F = r + d$.

5. In the Triangle CFA (the Point A being very near E) the Angles at A and C will be very small, and will have the same Proportion as their opposite Sides FC and FA , by Princip. 1, and 2; that is, the Angle $\angle F A C : \angle F C A :: FC : FA$: But in this Case FA may be esteemed equal to FE , by Princip. 4, and therefore the Angle $\angle F A C : \angle F C A :: FC : FE :: z : f$.

6. But the Angle at C is as DA or DE , that is, as d , by the 3d Step of this Theory; and the Angle $\angle F A C$ as $r + d$, by the 4th Step. Therefore $f : z :: d : d + r$. And by Composition of Ratio's, $f + z : f :: 2d + r : d$; but $f + z = r$, by Step 2. hereof; and therefore $r : f :: 2d + r : d$; then multiplying the Extremes and Means together, we have $dr = 2df + fr$; dividing this Equation by $2d + r$, there results this Equation $\frac{dr}{2d + r} = f = EF$.

7. Therefore in any *Speculum*, when r , or the Radius of it's Curvity, and d , or the Distance of any Object in the Axis thereof, are known, then f , or the Distance of the Focus F from the

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Vertex E, will also be known, or given, for all Rays proceeding from the Point D, and falling on the Mirrour A E on either Side the Axis.

8. This Theorem is applicable to the Cases of all Kind of Rays, reflected from all Sorts of Mirrours, whether *Convex*, *Concave*, or *Plain*; and is therefore the primary and fundamental Theorem of *Catoptrics*; from which I shall now deduce such *particular Theorems* as are subservient to the Solution of all the different Cases that may arise from the threefold Variety of Mirrours and Rays incident upon them. And first for

CONVEX MIRROURS.

9. CASE I. Of DIVERGING RAYS. The Ray D A proceeding from a radiant Point D in the Axis D C, at an indefinite Distance from the Vertex of the Mirrour E, gives the Distance of the Focus F from the Vertex E, according to the fundamental Equation, which therefore I shall call the first Theorem, *viz.*

$$\frac{dr}{2d+r} = f. \quad \text{Theorem 1.}$$

10. If $d = r$, we have $\frac{dr}{2d+r} = \frac{rr}{3r} = \frac{r}{3} = f.$ Theor. 2.

If $d = \frac{1}{2}r$, then $\frac{dr}{2d+r} = \frac{\frac{1}{2}rr}{2r} = \frac{r}{4} = f.$ Theor. 3.

If

If $d = \frac{1}{3} r$, then $\frac{dr}{2d + r} = \frac{\frac{1}{3} r r}{\frac{2}{3} r + r} = \frac{r}{5}$
 $= f$. Theor. 4.

If $d = \frac{1}{4} r$, then $\frac{dr}{2d + r} = \frac{\frac{1}{4} r r}{\frac{1}{2} r + r} = \frac{r}{6}$
 $= f$. Theor. 5.

From hence it is evident, that the Point D and F do both approach the Mirrour in a regular Manner, till at last they coincide at it's Vertex.

II. CASE II. Of PARALLEL RAYS. Fig. 6.
 If the radiant Point D be supposed to recede from the Mirrour E, to a vast or infinite Distance, the Rays DA, which come from it to the Mirrour, will be *parallel* to the Axis DC, or very nearly so; as is represented Fig. 6. In this Case, therefore, d or DA being *infinite*, the Quantity r as being *finite*, will vanish out of the Equation: For no infinite Quantity can be made *greater or less*, by the Addition or Substraction of any finite Quantity how great soever. Consequently the Equation

$$\frac{dr}{2d + r} = \frac{dr}{2d} = \frac{r}{2} = f. \text{ Theor. 6.}$$

12. CASE III. Of CONVERGING Rays.
 As in *Diverging Rays* the Radiant D was always posited in the Axis of the *Speculum*, directly before it's Vertex E, so in *Converging*
 B Rays,

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Rays, the incident Ray D A will tend towards a Point on the Axis behind, or on the other Side of the Vertex, as at d , in Fig. 7, 8, 9, 10. And as the Distance D E or d , in the former Case, had an *affirmative* Sign, viz. $+d$; so in this present Case, $E d$ being on the contrary Part will have a contrary, that is, a *negative* Sign, thus $-d$; so that the fundamental Theorem for Converging Rays will stand

$$\text{thus } \frac{-dr}{-2d+r} = f.$$

13. In this Theorem, the Dividend being a negative Quantity, viz. $-dr$; and the Divisor $-2d+r$, consisting of a negative Part $-2d$, and an affirmative one $+r$; it is evident, when the negative Part exceeds the affirmative, the Quotient f will be affected with an affirmative Sign, viz. $+f$, as before; but if $-2d$ be less than $+r$, the Quotient will be negative, viz. $-f$; that is, the incident Ray D A will be so reflected as to converge towards a Point f , in the Axis before the Mirrour, as it did to a Point F, behind it in the other Cases.

14. This being premis'd, it is plain the Theorem for Converging Rays will admit of four considerable Varieties, viz.

$$\text{If } 2d \text{ exceed } r, \text{ then } \frac{-dr}{-2d+r} = +f.$$

Theor. 7. Fig. 7.

If

If $d = r$, then $\frac{-dr}{-2d+r} = \frac{-rr}{-2r+r}$
 $= \frac{-rr}{-r} = r = +f$. Theor. 8. Fig. 8.

If $2d = r$, then $\frac{-dr}{-2d+r} = \frac{-\frac{1}{2}rr}{0}$
 $= \infty f$. Theor. 9. Fig. 9.

If $2d$ be less than r , $\frac{-dr}{-2d+r} = -f$.
 Theor. 10. Fig. 10.

Note, $\left\{ \begin{array}{c} +f \\ \infty f \\ -f \end{array} \right\}$ signifies an $\left\{ \begin{array}{c} \text{Affirmative} \\ \text{Infinite} \\ \text{Negative} \end{array} \right\}$ Focus.

CONCAVE MIRROURS.

15. As in *Convex Mirrours* the Radius CE lay on the *Right Hand* of the Vertex E, and had an *affirmative* Sign, as $+r$; so in *Concave Mirrours*, the Radius CE lying on the other Side the Vertex E, or to the *Left*, must have a *negative* Sign of Course, or be represented by $-r$; and therefore the fundamental Theorem for *Concave Mirrours* will become

$$\frac{-dr}{2d-r} = f.$$

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16. CASE I. Of DIVERGING RAYS. The Theorem is as above, and hath also four Varieties, *viz.*

If $2d$ be less than r , then $\frac{-dr}{2d-r} = +f$.

Theor. 11. Fig. 11.

If $2d = r$, then $\frac{-dr}{2d-r} = \frac{-\frac{1}{2}rr}{0} = \infty f$.

Theor. 12. Fig. 12.

If $d = r$, then $\frac{-dr}{2d-r} = \frac{-rr}{2r-r} = -r = -f$. Theor. 13. Plate III. Fig. 1.

If $2d$ exceed r , then $\frac{-dr}{2d-r} = -f$.

Theor. 14. Fig. 2.

17. CASE II. Of PARALLEL RAYS. In this Case, DA or d being infinite, the finite Quantity r will vanish where it is found

by it self; and the Theorem $\frac{-dr}{2d-r}$ will

become $\frac{-dr}{2d} = \frac{-r}{2} = -f$. Theor. 15.

Fig. 3.

18. CASE III. Of CONVERGING RAYS. Here again the incident Ray DA will have the

Fig. 1.



Fig. 2.

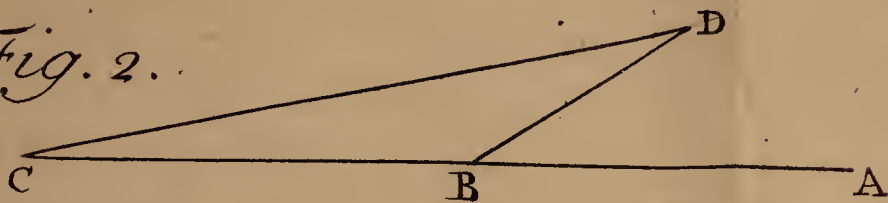


Fig. 3.

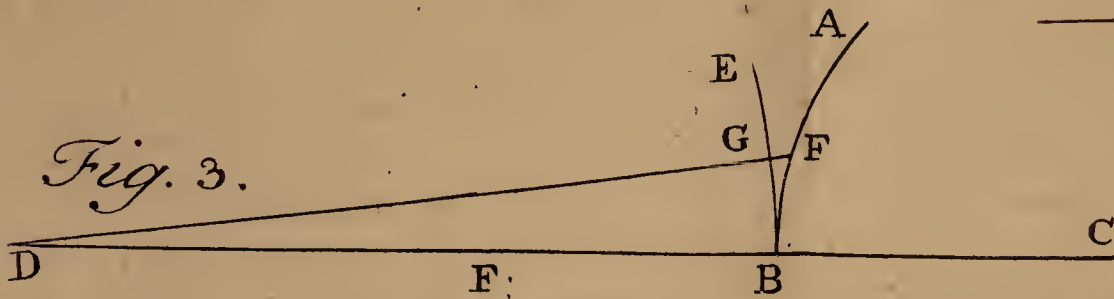


Fig. 4.

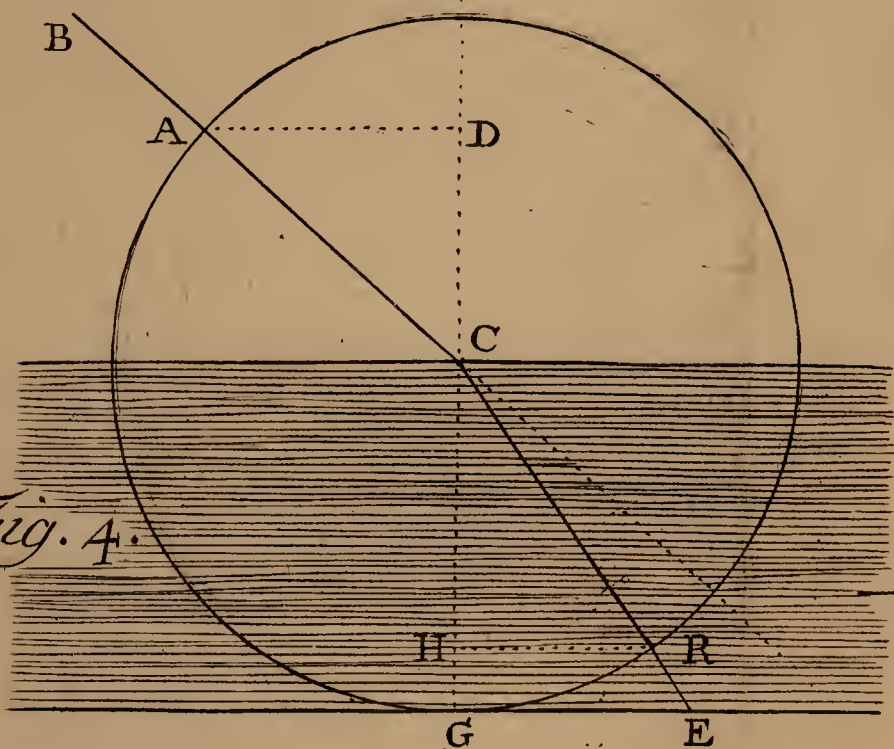


Fig. 5.

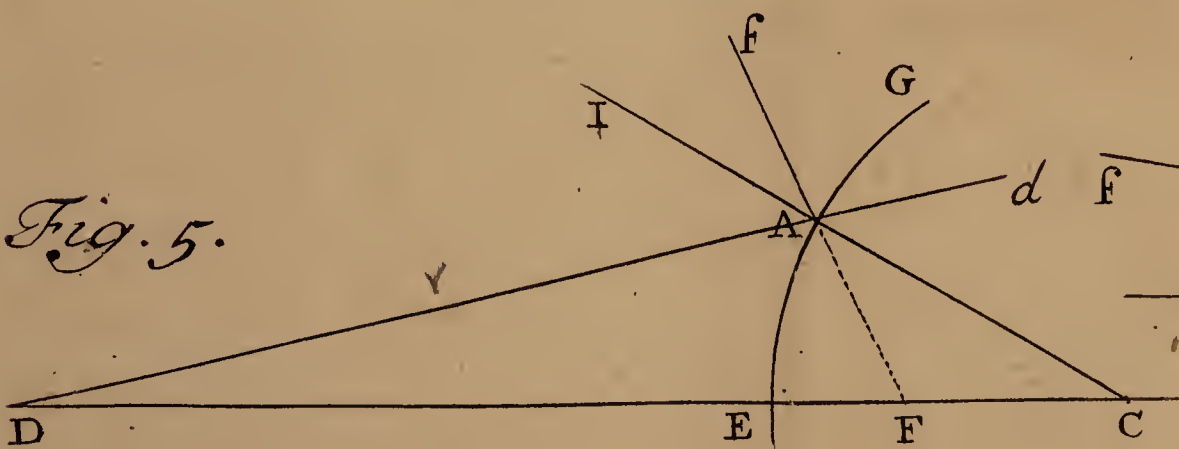


Fig. 6.

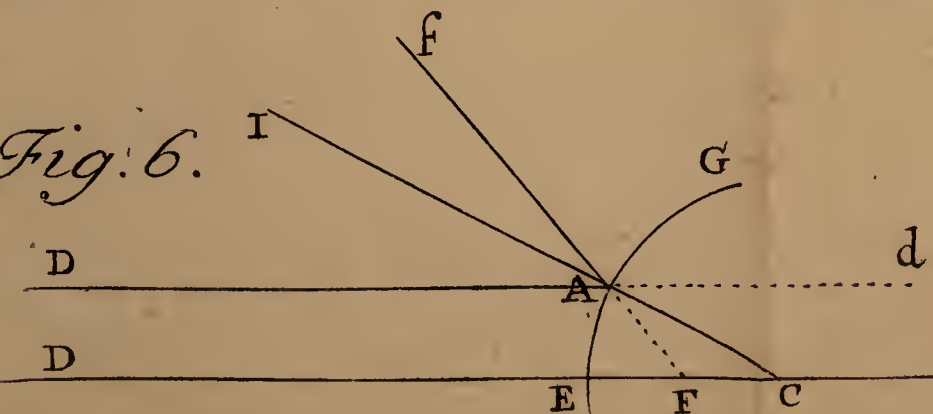


Fig. 7.

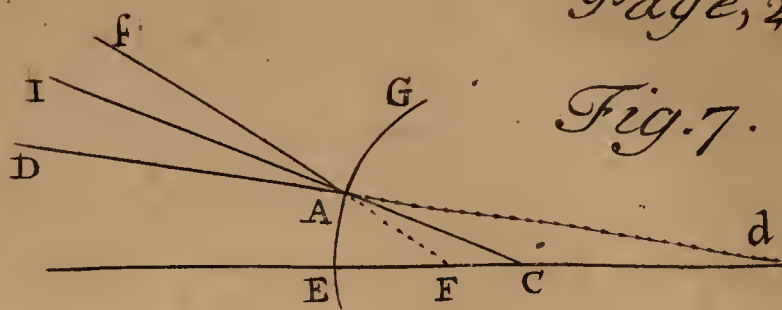


Fig. 8.

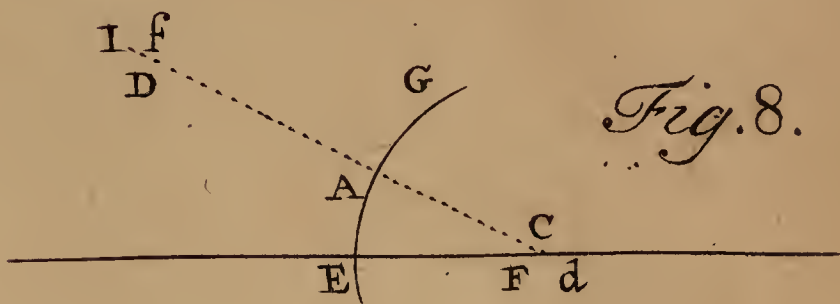


Fig. 9.

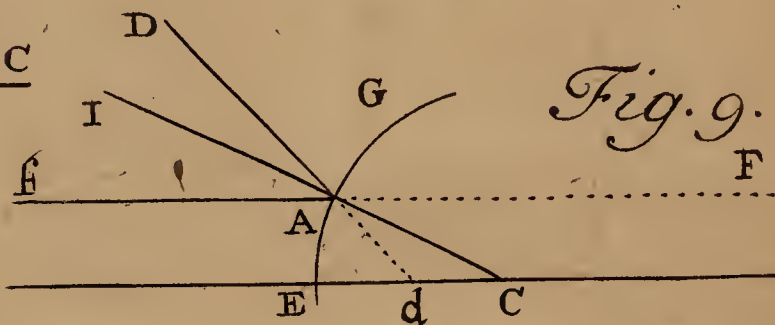


Fig. 10.

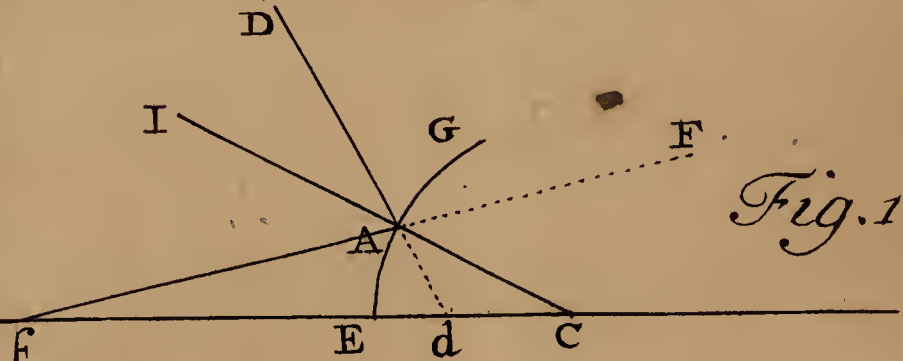


Fig. 11.

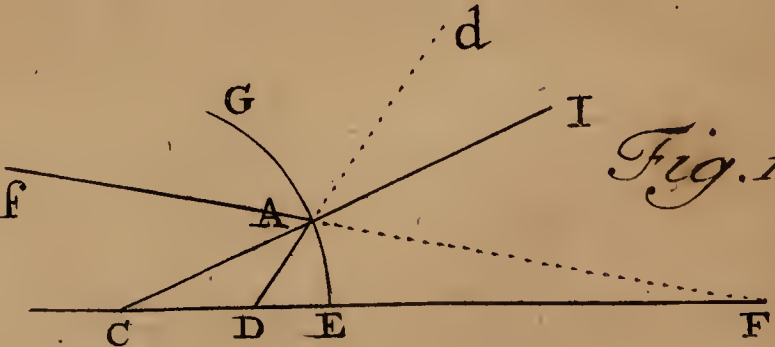
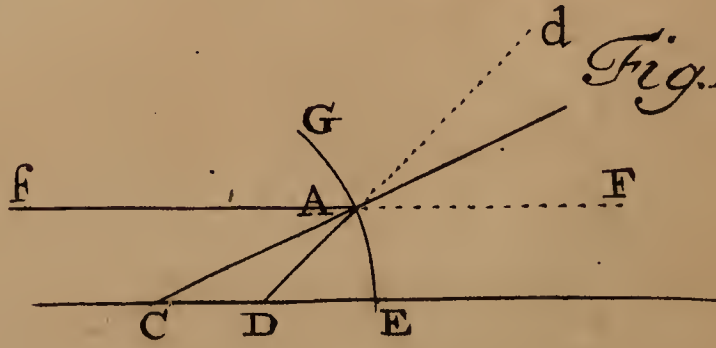


Fig. 12.





the Point of Convergence d on the other Side of the *Speculum*; and the Distance Ed or d must therefore have a negative Sign, or be $-d$; in this Case then, r and d being both *negative*,

the Theorem $\frac{-dr}{2d-r}$ will become $\frac{dr}{-2d-r}$

$= f$. Theor. 16. And since the Divisor $-2d-r$ is wholly *negative*, the Quotient or *Focus* f or Ef , will likewise be always *negative*, or before the Mirrour GE .

$$19. \text{ If } d = r, \text{ then } \frac{dr}{-2d-r} = \frac{rr}{-3r} =$$

$$\frac{r}{-3} = -f. \text{ Theor. 17.}$$

$$\text{If } d = \frac{1}{2}r, \text{ then } \frac{dr}{-2d-r} = \frac{\frac{1}{2}rr}{-2r} =$$

$$\frac{r}{-4} = -f. \text{ Theor. 18.}$$

$$\text{If } d = \frac{1}{3}r, \text{ then } \frac{dr}{-2d-r} = \frac{\frac{1}{3}rr}{-\frac{2}{3}r-r} =$$

$$= \frac{r}{-5} = -f. \text{ Theor. 19.}$$

$$\text{If } d = \frac{1}{4}r, \text{ then } \frac{dr}{-2d-r} = \frac{\frac{1}{4}rr}{-\frac{1}{2}r-r} =$$

$$\frac{r}{-6} = -f. \text{ Theor. 20.}$$

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PLAIN MIRRORS.

20. In Regard of *Plain Mirrours*, or Looking-Glasses, the Radius $CE = r$ is *infinite*; for we may consider a small Arch of Circle described with a *vastly great* or *infinite Radius*, as a *Plane*; and therefore the finite Quantity $2d$ vanishing, the Theorem $\frac{dr}{2d+r}$ becomes

$$\frac{dr}{r} = f.$$

21. CASE I. Of DIVERGING RAYS.

The Theorem is $\frac{dr}{r} = d = f$. Theor. 21.

Fig. 5.

22. CASE II. Of PARALLEL RAYS.

In this Case both d and r are infinite, and therefore the fundamental Equation becomes infinite in every Part; and $\frac{dr}{r} = d = f$ infinite, Theor. 22. Fig. 6.

23. CASE III. Of CONVERGING RAYS.

Here d or Ed being the Negative, the Equation is $\frac{-dr}{r} = -d = -f$. Theor. 23.

Fig. 7.

24. We

24. We have hitherto made Use of the *fundamental Equation* for finding the *Focus* of all Sorts of Rays, incident on any Kind of Mirrours: But sometimes the *Focus* is given, and either the Distance of the Object DE, or the Radius of Convexity or Concavity of the Mirrour CE is required for some particular Purposes; and for either Case the said Equation $dr = 2df + fr$ (in Step. 6.) is sufficient.

25. For suppose the Radius CE = r , and the Focus EF = f , were given to find the Distance ED = d ; we have from the said Equation $dr - 2df = rf$, and therefore $\frac{rf}{r - 2f} = d$, Theor. 24, for a *Convex Mirrour*.

26. For a *Concave Mirrour*, the Radius being *Negative*, or $-r$, the Equation will be $2df + dr = fr$; and so it will be $\frac{fr}{2f + r} = d$. Theor. 25.

27. In the two last Theorems, the Focus is supposed to be *Affirmative*, or behind the Mirrour; but if we suppose it *Negative*, or on the same Side with the Radiant D, the Equation, for a *Convex*, will be $dr + 2df = -fr$, and $\frac{-fr}{r + 2f} = d$. Theor. 26.

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28. For a Concave, the Equation will be $2df - dr = fr$; (for here both f and r are Negative) therefore we have $\frac{fr}{2f - r} = d$.

Theor. 27.

29. If the Distance of an Object $ED = d$, and the Focus $E = f$ be given to find the Radius of the Convexity or Concavity $EC = r$; the original Equation $dr = 2df + rf$ will be for this Case $dr - rf = 2df$; and so we have for a Convex $\frac{2df}{d - f} = r$. Theor. 28.

30. For the Radius of a *Concave Mirrour*, the Equation will be $fr - dr = 2df$; and so $\frac{2df}{f - d} = r$. Theor. 29.

31. But here again if we suppose the Focus *Negative*, or before the Mirrour, we shall have for a *Convex*, $dr + rf = -2df$; and $\frac{-2df}{d + f} = r$. Theor. 30.

32. And for a *Concave*, the Equation will be $-dr - fr = -2df$; and $\frac{-2df}{-d - f} = r$. Theor. 31.

C H A P.

C H A P. IV.

The Theory of CATOPTRICS continued, for determining the mutual Proportion of the Object and it's Image.

I. **L**ET DE be the Portion of a *Convex* *Mirroure*, (as Fig. 8.) or of a *Concave*, (as Fig. 9. Plate 3.) C the Center, V the Vertex ; O B an Object, and I M it's Image ; it is required to find the Proportion between the Object and Image, or the Lines O B and I M.

2. From the Center C let fall on the Radiant or Object, the perpendicular C A ; and from the extreme Points of the Objects O, B, draw O C and B C, meeting the Mirroure in the Points D and E ; these shall be the Axis in which will be the Focus's of Rays proceeding from the Points O and B.

3. From O let a Ray O V pass to the Vertex of the Mirroure, and make the Angle $F V A = O V A$, then shall V F be the reflected Ray, by Princip. 6. which tending to the Point I, in the Axis C O, shall there represent the Image of the Point O of the Radiant, by Princip. 10 and 11. Accordingly the Ray B V will be reflected into V G, which intersecting

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intersecting the Axis CB in M, will there represent the Image of the Point B in the Radiant. Consequently, all the Points between O and B in the Object will be represented between the Points I and M; and therefore IM will be the Image of the Radiant or Object OB.

4. If we suppose the Radiant OB very small, or at a great Distance, the Arch or Portion of the Mirrour ED will also be very small, and not sensibly different from a right Line, and consequently will be parallel to the Radiant BO, for CA is perpendicular to both BO and ED. Also since the Distances OD, AV, and BE, are very nearly equal, as being very near each other, it is plain their focal Distances DI, Va, and EM, will also be nearly equal; *and therefore the Image IM will be very nearly a Right Line, and parallel to the Radiant OB; and also perpendicular to CA.*

5. The Angle $OVA = AVF = aVI$, and the Angle $BVA = AVG = aVM$, from the Nature of Reflection, Princip. 6: therefore $OVA + BVA = aVI + aVM$; that is, the Angle $OVB = IVM$; that is, *The Radiant BO and it's Image IM are seen from the Vertex of the Speculum V under equal Angles.*

6. The Triangles AVO and aVI are equiangular and similar, for the Angle $OVA = aVI$, and the Angles at A and a , are right ones;

ones ; consequently $VA : Va :: AO : aI$. For the same Reason $VA : Va :: AB : aM$; therefore $VA : Va :: OA \div AB : Ia \div aM :: OB : IM$; that is, *The Distance of the Object is to the Distance of it's Image, from the Vertex V, as the Length of the Object to the Length of the Image.*

7. Moreover, since $OA : Ia :: AB : aM$, (for $OA : Ia :: AV : Va :: AB : aM$;) therefore $OA : AB : Ia : aM$; that is, *The Radiant OB and it's Image IM, are cut in the same Proportion by a Right Line CA, drawn from the Center perpendicular to each.*

8. From what has been premised, it is easy to raise a Theorem, to shew at what Distance any Object ought to be placed to bear any given Proportion to it's Image. Let the Object be O, it's Image I, and the given Proportion be as $O : I$; then $O : I :: d : f$, (by the Sixth of

this) and therefore $\frac{Id}{O} = f$. But the funda-

mental Theorem for the Focus is $\frac{dr}{2d \div r}$

$= f$; consequently $\frac{Id}{O} = \frac{dr}{2d \div r}$, and so

$2ddI \div Id r = drO$, that is, $2dI \div Ir = rO$; and $2dI = Or - Ir$; and consequently for *Convex Mirrours*, the Theorem

will be $\frac{Or - Ir}{2I} = d$. Theor. 1.

9. In

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9. For *Concave Mirrours* where r is negative, or $-r$, the Equation will be $2dI =$

$$Ir - Or, \text{ and so } \frac{Ir - Or}{2I} = d. \text{ Theor. 2.}$$

10. For a *Plain Mirrour*, or Looking-Glass, where r is infinite, the finite Part of the Equation $2dI$, will vanish, or be 0; and then $Or - Ir = 0$, and $Or = Ir$, that is, $O = I$. Theor. 3.

11. If the Focus be required *negative*, or on the same Side of the Glass with the Object; the Equation will be for a *Convex*, (f being *negative*) $2dI = -Or - Ir$, and

$$\frac{-Or - Ir}{2I} = d. \text{ Theor. 4.}$$

12. But for a *Concave*, where r and f are now both negative, the Equation will be

$$2Id = Or + Ir, \text{ and therefore } \frac{Or + Ir}{2I} = d. \text{ Theor. 5.}$$

13. We have hitherto considered the Object and it's Image as *Right Lines*, which have a *simple* Proportion to each other; but if the *Object* be a *Superficies*, or a Plane Figure, the Proportion between it and it's Image, will be duplicate of what it was before; for as in Lines, it was $O : I :: d : f$; in Superficies it

$$\text{will be } O : I : d^2 : f^2; \text{ and hence } \frac{\sqrt{Id^2}}{O} = f \\ = dr$$

$= \frac{dr}{2d+r}$; and squaring each Side, we have

$$\frac{I d^2}{O} = \frac{d^2 r^2}{2d+r^2}, \text{ and } I \times \frac{1}{2d+r^2} = O \times r^2,$$

and therefore $\frac{1}{2d+r^2} = \frac{O}{I} \times r^2$, and put-

ting $\frac{O}{I} = p$, and extracting the square Root

we have $2d+r = \sqrt{p r^2}$; and consequent-

ly, $d = \frac{\sqrt{p r^2} - r}{2}$. Theor. 6.

14. For a Concave Mirrour, where r is negative, $d = \frac{\sqrt{p r^2 + r}}{2}$. Theor. 7.

15. In the same Manner, if the Object and Image are *Solids*, and consequently their Proportion *triplicate*, viz. $O : I :: d^3 : f^3$; it is shewn, that, for *Convex* Mirrours, $d = \frac{\sqrt[3]{p r^3 - r}}{2}$. Theor. 8.

16. Also, for Concave Mirrours, where it is $-r$, we have $d = \frac{\sqrt[3]{-p r^3 + r}}{2}$, Theor. 20.

17. If the Distance d , and the Proportion of the Object and Image O and I , be given the Radius r of the Convexity or Concavity of the Mirrour, for that Proportion is to be had

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had from the same Equation, *viz.* $O r = 2 I d + I r$: for $O r - I r = 2 I d$, and $\frac{2 I d}{O - I} = r$.

Theor. 10.

18. For the Radius of a Concave Speculum, the Equation is $I r - O r = 2 I d$, and $\frac{2 I d}{I - O} = r$. Theor. 11.

19. If the Focus be required negative, or before the Mirrour, the Equation for a *Convex* will be $O r + I r = - 2 I d$; and therefore $\frac{- 2 I d}{O + I} = r$. Theor. 12.

20. And for a *Concave*, the Equation is $O r + I r = 2 I d$, and $\frac{2 I d}{O + I} = r$. Theor. 13.

21. Again, from the Equation $O r = 2 I d + I r$, when the Distance of the Object d , and the Radius of the Mirrour r are known, the Proportion between the Object and Image, *viz.* $O : I$ is also known; for, in Case of a *Convex*, $O : I :: 2 d + r : r$. Theor. 14.

22. And for a *Concave*, the Equation being $O r - 2 I d = O r$, we have $O : I :: r - 2 d : r$. Theor. 15.

23. If the Focus be negative, or $-f$, the Equation for a *Convex* will be $O r = - 2 I d - I r$; and so, $O : I :: - 2 d - r : r$. Theor. 16.

24. For

Fig. 1.

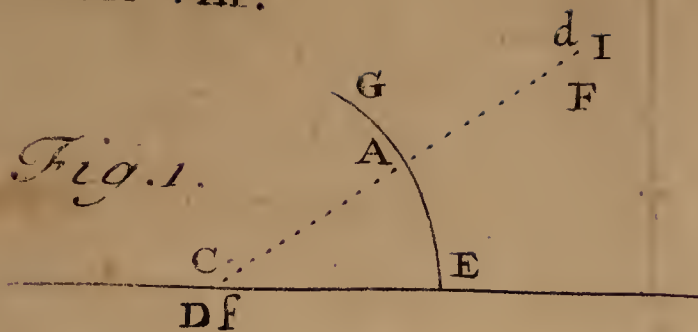


Fig. 2.

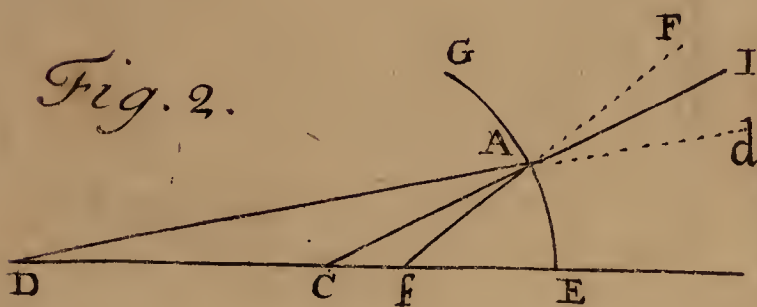


Fig. 3.

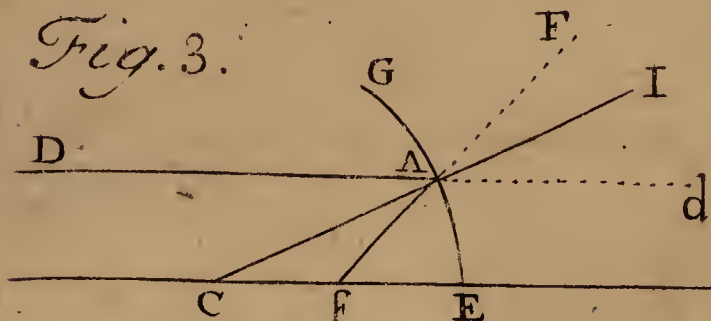


Fig. 4.

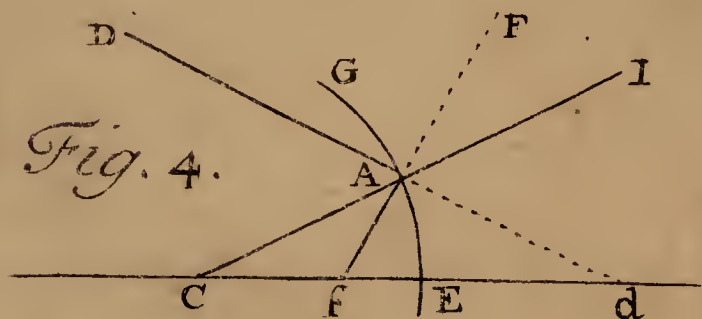


Fig. 5.

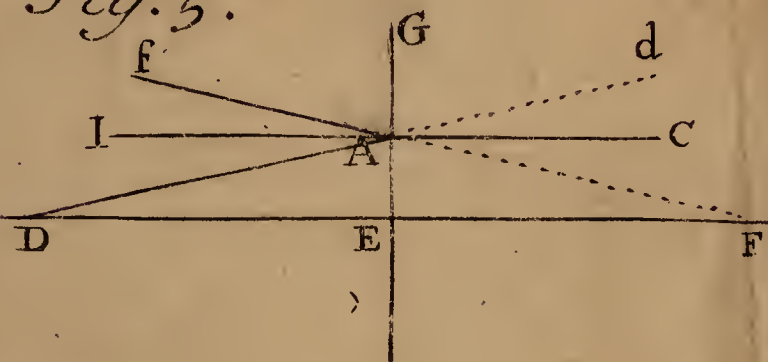


Fig. 6.

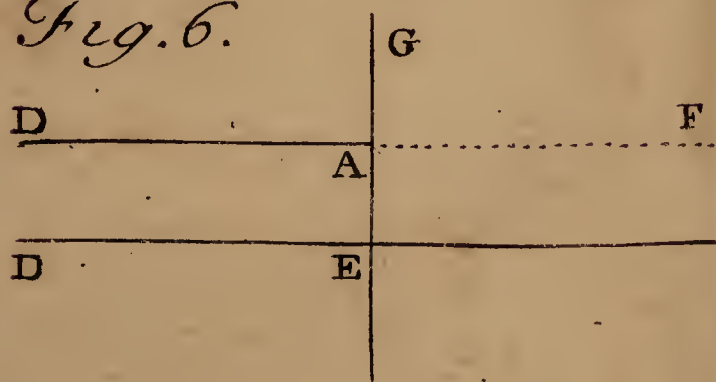


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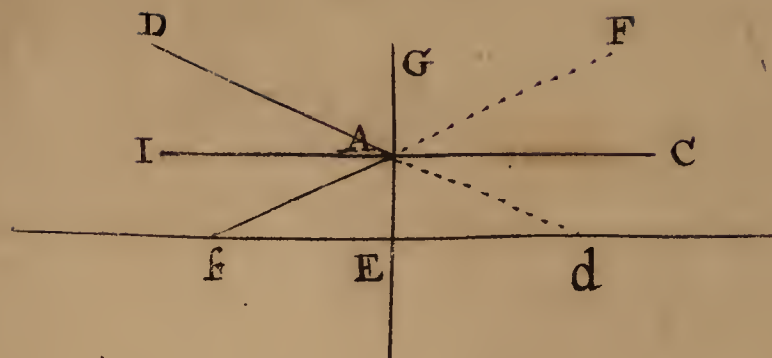


Fig. 8.

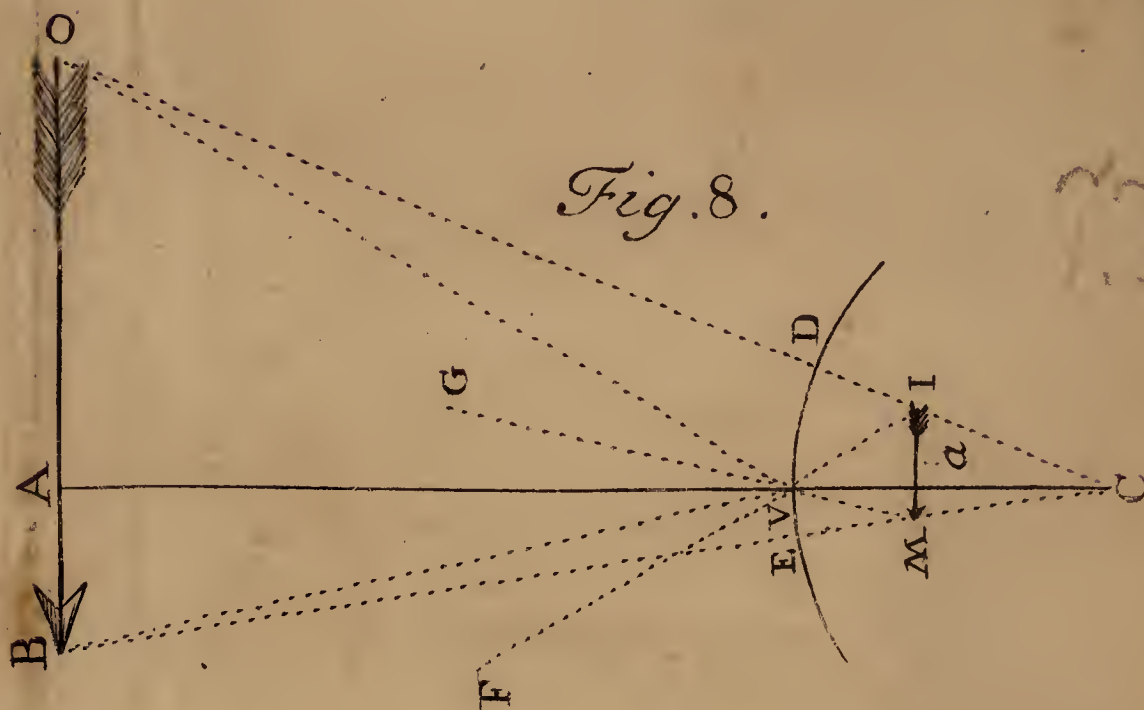


Fig. 9.



24. For a Concave Speculum, the Equation being $O r = 2 I d - I r$, we have $O : I :: 2 d - r : r$. Theor. 17.

25. These are the principal Theorems in the *Theory of Catoptrics*, and most of them very curious, and of great Use. I shall conclude this Theory with a very notable Observation, *viz.* That the musical or harmonical Proportion is the Grounds of this whole Theory: or, that the Center C, the Focus F, the Vertex of the Mirrour V, and the Radiant D, are harmonical Points, by which the Axis or Line CD, is harmonically divided in the Points D, E, F, C; so that $DC : DE :: FC : FE$.

26. For if we take it for granted, we shall have the following Analogy, $DC (d + r) : DE (d) :: FC (r - f) : FE (f)$; and therefore $df + rf = dr - df$, and so $2df + rf = dr$; and $\frac{dr}{2d + r} = f$, which is the same as the first fundamental Theory, Chap. III. Art. 6, and this holds equally in *Concave* as in *Convex Mirrours*. This admirable Property of Mirrours in reflecting the Light, was first published by Mr *Ditton*, in N^o. 295, of the *Philos. Transactions*.

C H A P. V.

*The foregoing Theory of CATOPTRICS
explained and illustrated, by familiar
Rules and Examples.*

1. **I** SHALL here explain and illustrate the foregoing Theorems in Words at Length, and give an Example, in all the principal ones, in Numbers, shewing the Distance of the *Focal Point*, real or virtual, of all Sorts of Rays, reflected from all Sorts of Mirrours.

2. I shall likewise shew how every thing happens, by Schemes, as large as the Life; and therefore, in Plate IV, I have given three Schemes of Rays falling on a Convex Mirrour, where you are to observe, that A B is the *Mirrour*; C E the *Radius* thereof $1\frac{1}{2}$ Inch; E the Vertex or middle Point thereof; D the radiant Point; and F the Focus to which the Rays tend after Reflexion.

3. Note, the *black Lines* in these Schemes denote the Rays of the Sun's Light, as they are seen to go to and from the Mirrour in a darkened Chamber; and the *dotted Lines* denote only the Course or Tendency of the reflected Rays, were they not intercepted by the Mirrour.

4. In

4. In the following Examples, I shall suppose an Inch divided into *Ten equal Parts*, which Parts are expressed by the Numbers I shall there use, every 10 of which will therefore express the Length of an Inch, and the Remainders will be the *tenth Parts* of an Inch.

For CONVEX MIRROURS. Plate IV.

5. CASE I. For *Diverging Rays*, Scheme I. Let there be given DE, the Distance of the radiant Point from the Mirrour, and CE, the Radius of Convexity, to determine the Distance of the Focus F E.

R U L E.

Multiply the Distance and Radius together, divide that Product by the Sum of the Radius added to twice the Distance, the Quotient will be the Distance of the Focus required. (*per Theor. 1. Chap. III.*)

E X A M P L E.

$$\text{Let } \begin{cases} CE = 15, & \text{and } CE = 15 \\ DE = 30, & 2 DE = 60 \end{cases}$$

Produce 450 ;

Sum 75 ;

then 75) 450 (6 = FE

450

C . . .

So

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So that in this Case, the Focus F of the reflected Rays GH, will be 6 Tenths of an Inch behind the Vertex E.

6. If the Distance DE be equal to the Radius CE, the Distance of the Focus EF will be $\frac{1}{3}$ of the Radius CE, *per* Theor. 2. If DE be $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of CE, then shall EF be $\frac{1}{4}$, $\frac{1}{5}$, or $\frac{1}{6}$ of CE, till at last the Radiant D, and Focus F, both coincide in the Vertex of the Mirrour E. See Theor. 3, 4, 5.

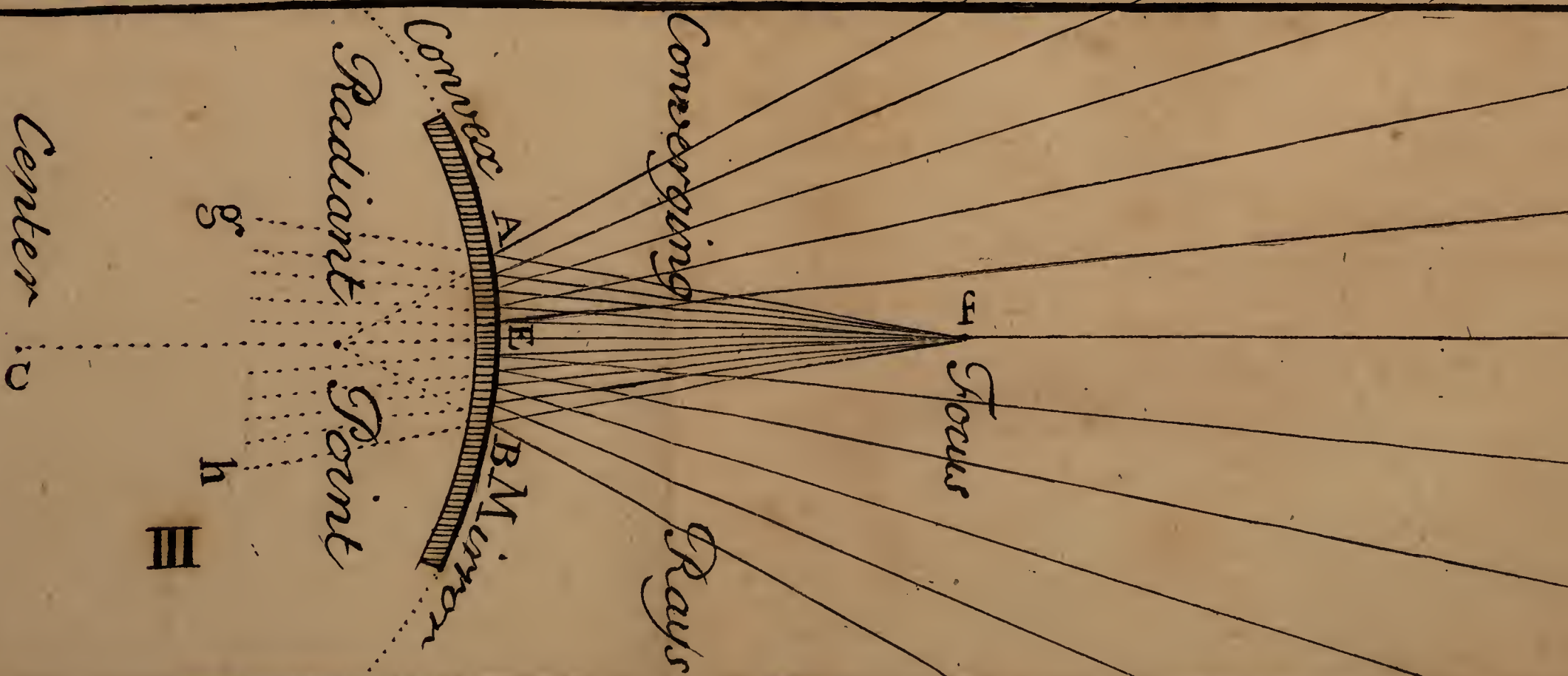
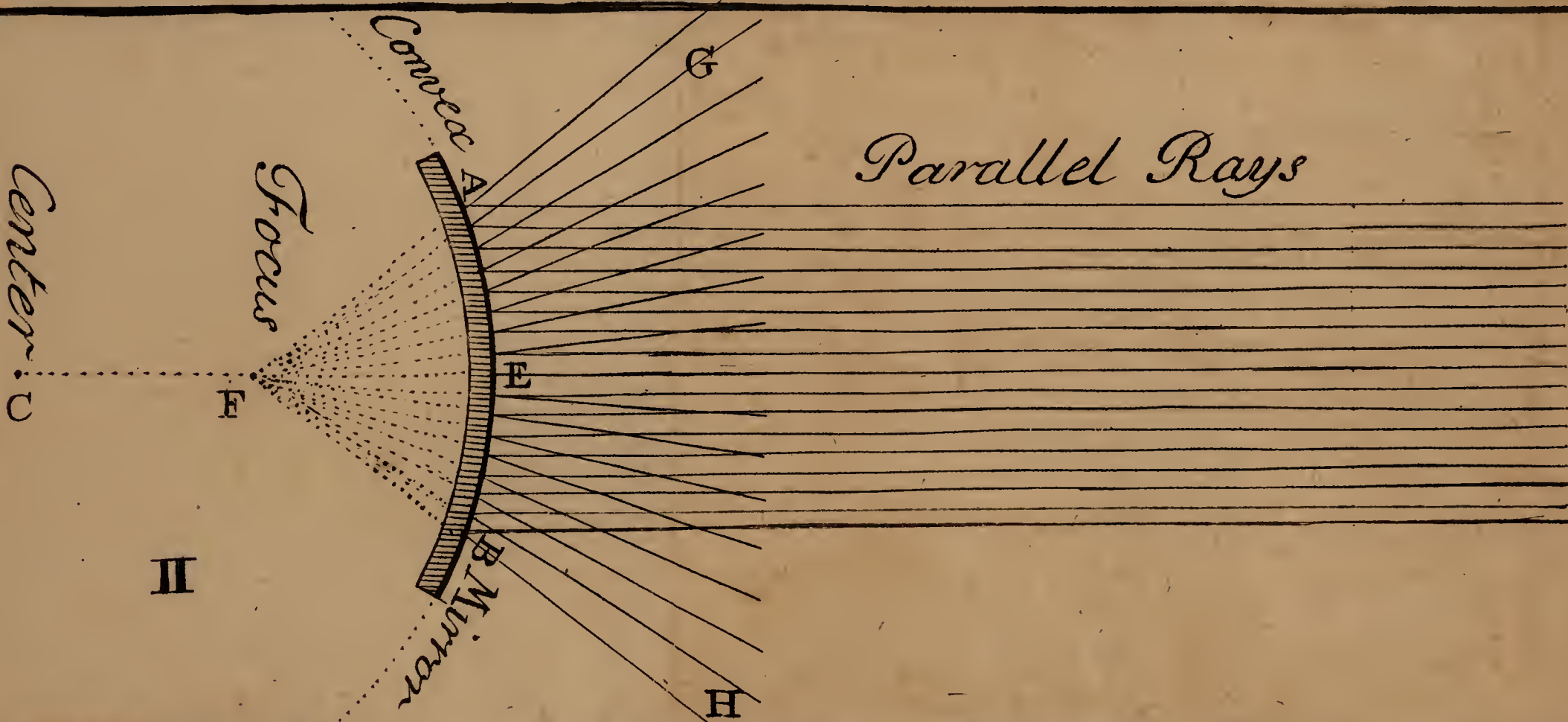
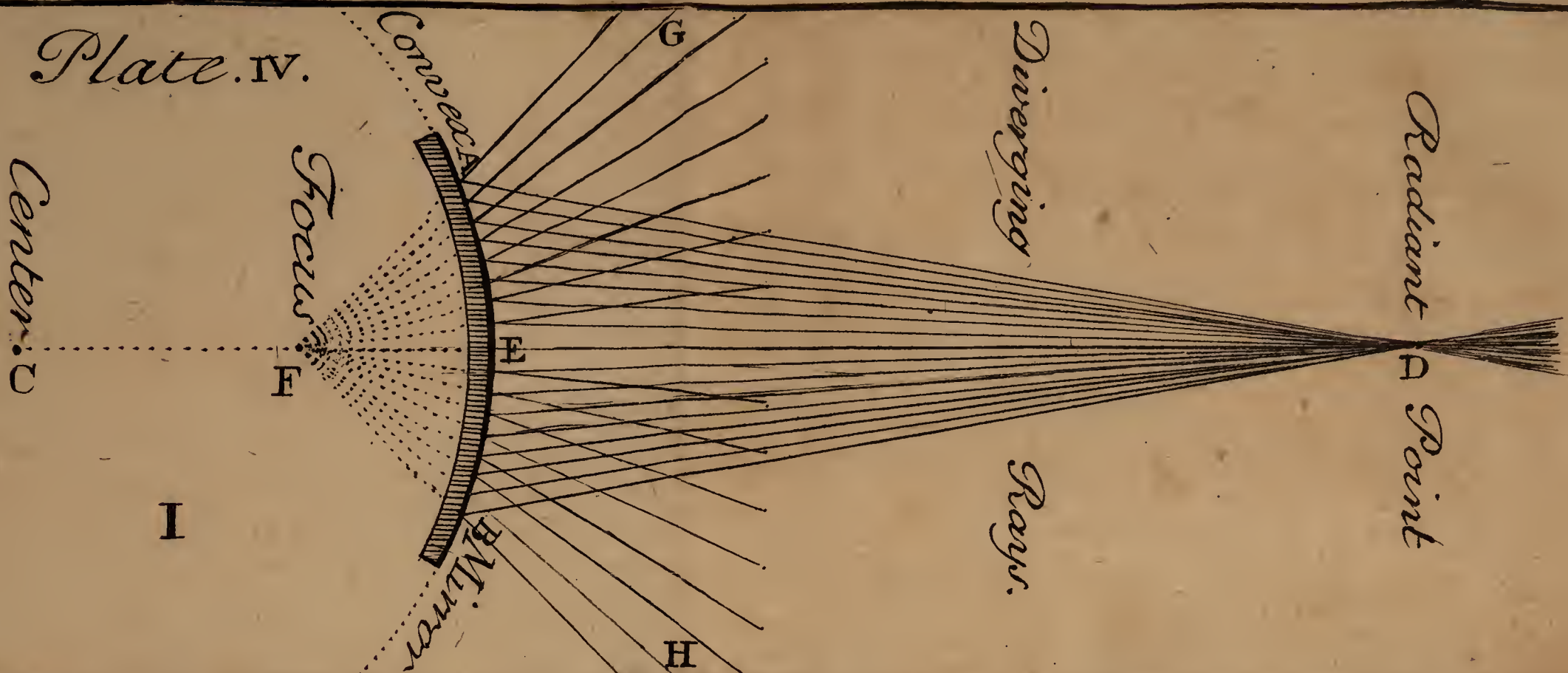
7. CASE II. For *Parallel Rays*, Scheme 2. When Rays fall parallel upon a Convex Mirrour, they are reflected in such a Manner, as to have their focal Distance FE just equal to half the Radius CE; that is, $EF = CF$, *per* Theor. 6. Thus, for Example, if CE be 15, then will FE be $7\frac{1}{2}$ Tenths of an Inch.

8. CASE III. For *Converging Rays*, Scheme III. In this Case, if the Distance of the radiant Point (which here is *virtual*, or behind the Mirrour) and the Radius be given, the Rule for finding the Distance of the Focus is thus.

R U L E.

Multiply the given Distance and Radius together, and divide that Product by the Difference between the Distance doubled and the Radius

Plate IV.



Radius; the Quotient is the Distance of the Focus required, *per*. Theor. 7.

Note, if the double Distance of the Radiant exceed the Radius, the Focus will be behind the Glass; if it be less than the Radius, the Focus will be before the Glass.

9. If the Distance be equal to the Radius, the focal Distance will also be equal to the Radius; that is, those Rays which converge towards the Center C of the Mirrour, will be reflected back again upon themselves, *per* Theor. 8.

10. If the double Distance of the Radiant be equal to the Radius, or the said Distance be equal to half the Radius, then will the focal Distance be infinite; that is, Rays converging to a Point in the Axis equally distant from the Center C, and Vertex E, will be reflected parallel to each other; *per* Theor. 9.

11. Let the double Distance of the Radiant be less than half the Radius; for Example,

$$\text{Let } \begin{cases} DE = 15, & \text{and } CE = 15, \\ dE = 5, & 2dE = 10, \end{cases}$$

$$\text{Product } 75; \quad \text{Diff.} = 5;$$

$$\text{then } 5) 75 (15 = Ef$$

$$\underline{75}$$

..

C 2

Here

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Here because $2\ dE$ is less than CE , the Focus f will be before the Mirrour AB ; and since dE is $\frac{1}{3}$ of CE , the focal Distance Ef is equal to the Radius CE . See Scheme 3.

12. From hence, and a due Consideration of the Properties of the first and third Cases, it is plain they are converse by the same; the *incident Rays, Radiant and Focus* in one being but the *reflected Rays, Focus and Radiant* in the other.

For CONCAVE MIRROURS. Plate V.

13. CASE I. For *Diverging Rays*, Scheme 1. Having given the Distance of the Radiant DE , and Radius CE , the focal Distance is found by the same Rule as in the third Case of a Convex. Only here observe, that if the double Distance of the Radiant be less than the Radius, the Focus will be behind the Mirrour; if greater, it will be before it.

E X A M P L E.

$$\begin{array}{ll} \text{Let } DE = 30, & \text{and } 2\ DE = 60, \\ CE = 15, & CE = 15, \end{array}$$

$$\begin{array}{ll} \text{Product } 450; & \text{Diff. } 45; \end{array}$$

$$\text{then } 45) 450 (10 = FE.$$

$$\begin{array}{r} 450 \\ \hline \end{array}$$

$$\begin{array}{r} : : : \\ \hline \end{array}$$

That is, the Focus F is 1 Inch before the Mirrour A B.

14. If the Distance of the Radiant D E be equal to the Radius C E, then will the focal Distance be equal to the Radius also, *per* Theorem 13. That is, if an Object be placed in the Center of a Concave *Speculum*, the Image will be reflected upon the Object, or they will seem to meet and embrace each other in the Center; which agreeable Phænomenon is easily tried by any Mirrour of this Sort.

15. If $2\ D E = C E$, that is, if the Distance of the Radiant be equal to half the Radius, it's Image will be reflected to an infinite Distance; for the reflected Rays will be parallel, *per* Theor. 12. Hence, if a luminous Body be placed at the Distance of half the Radius from a Concave, it will enlighten Places directly before it at the greatest Distances. And hence appears their Use when placed behind a Candle in a Lantern, and in several other like Cafes.

16. CASE II. For *Parallel Rays*, Scheme 2. In this Case the focal Distance F E is always equal to half the Radius C E, and before the Mirrour A B; *per* Theor. 15. And since the Sun-Beams are parallel among themselves, if they are received on a Concave Mirrour, they will all be reflected to that Point, and there burn

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in proportion to the Quantity of Rays collected by the Mirrour.

17. CASE III. For *Converging Rays*, Scheme 3. The Rule for finding the *Focus* from the Distance of the Radiant Point, and the Radius given, is here the same as in the first Case of the Convex; only as there the Focus F was always behind the Mirrour, here it is ever before it.

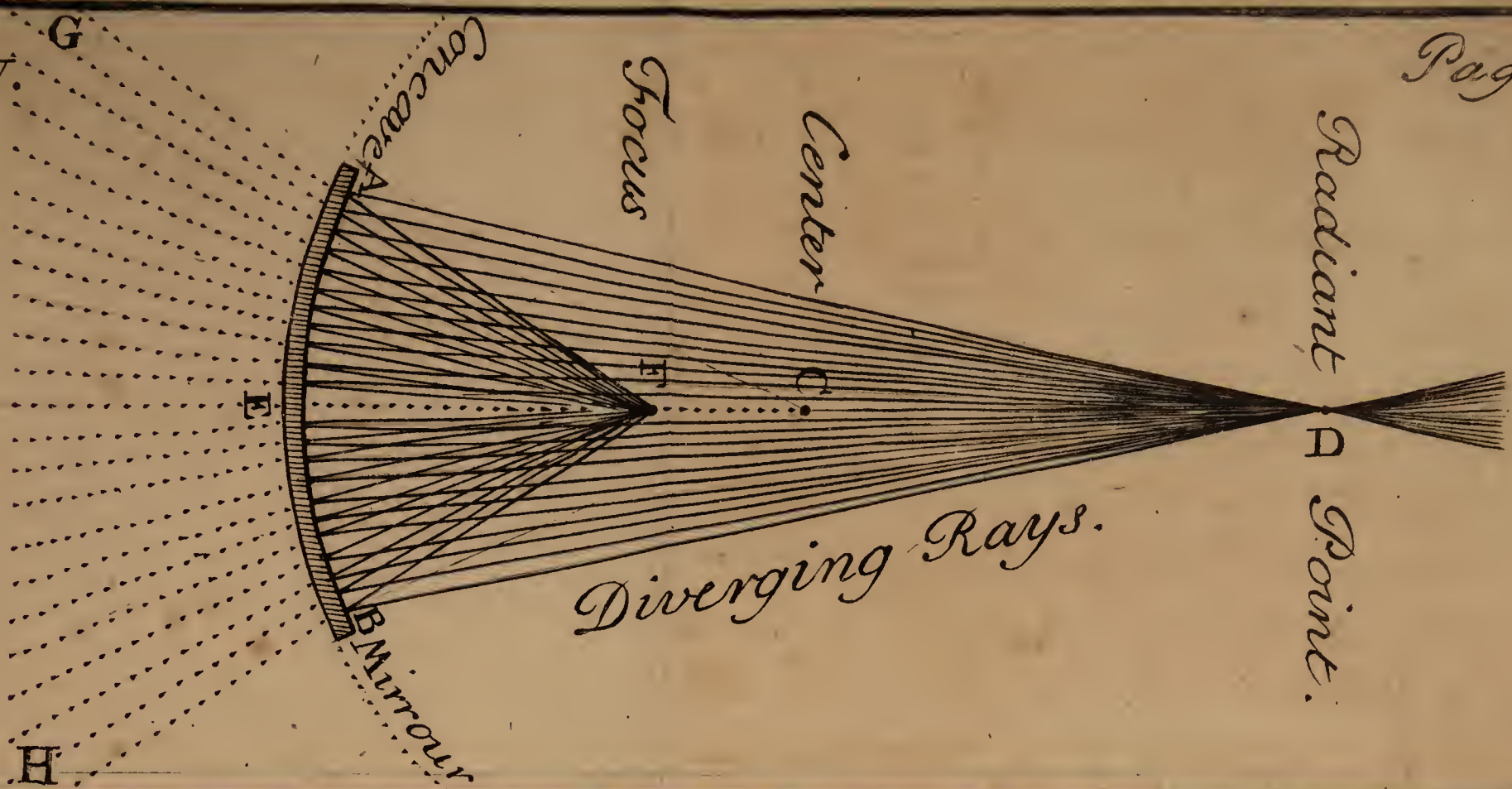
E X A M P L E.

$$\begin{array}{rcl}
 \text{Let } dE = 15, & \text{and } 2dE = 30, \\
 CE = 15, & CE = 15, \\
 \hline
 \text{Product } 225; & \text{Sum } 45; \\
 \text{then } 45) 225 (5 = FE \\
 \quad \underline{225} \\
 \quad \dots
 \end{array}$$

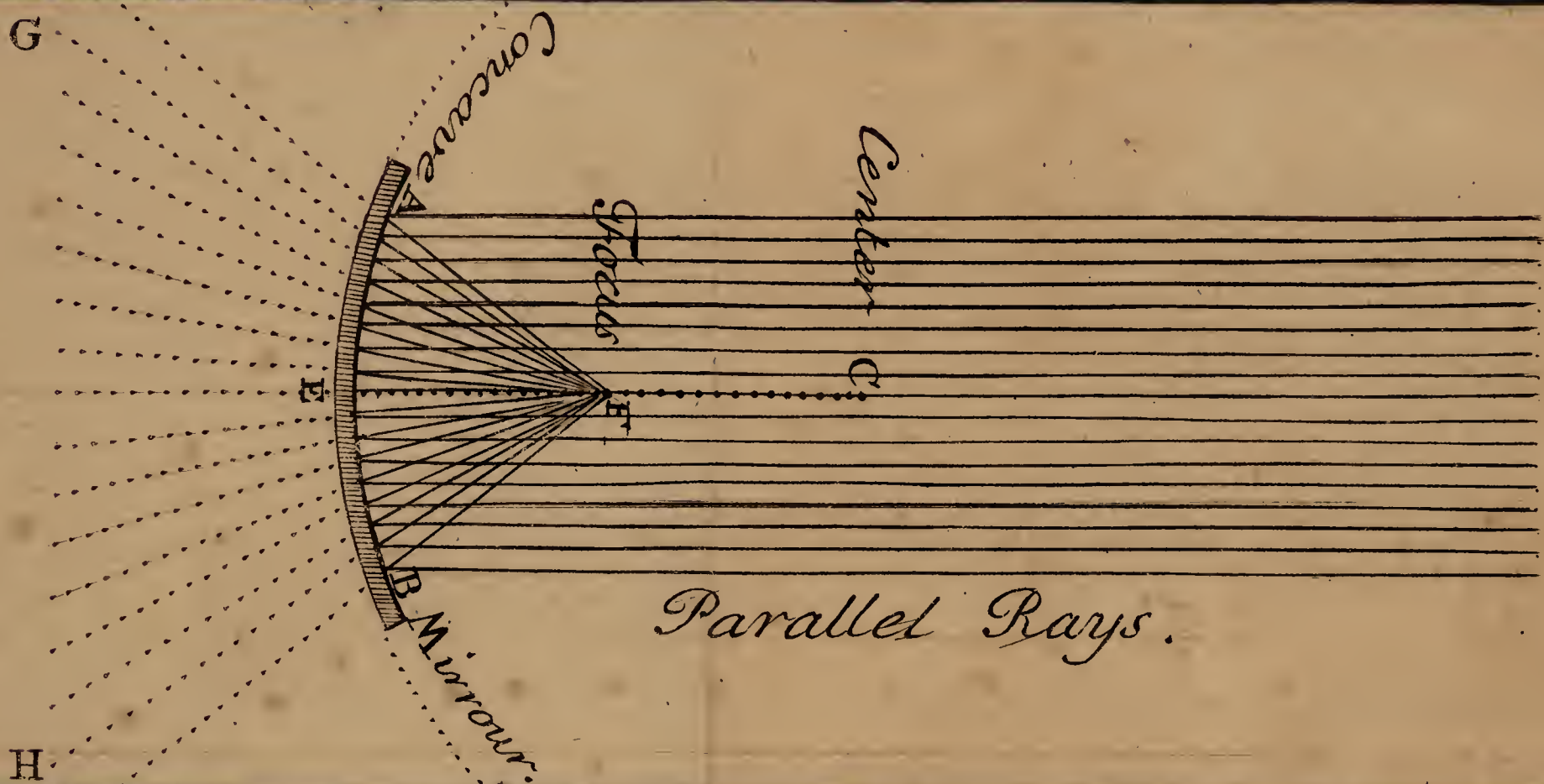
18. If dE be equal to CE , or $\frac{1}{2} CE$, or $\frac{2}{3} CE$, or $\frac{1}{4} CE$, &c. then shall the focal Distance FE be equal to $\frac{1}{3} CE$, $\frac{1}{4} CE$, $\frac{1}{5} CE$, or $\frac{1}{6} CE$, &c. As per Theor. 17, 18, 19, 20.

19. From what has been hitherto said of the Properties of these Mirrours, and from a View of the Schemes, it is plain the Cases of a *Concave*, are but the *Reverse* of those of a *Convex* in an inverse Order. And it is moreover to be observed, that in all those Cases the *Focus*, and *Radiant Point* mutually respect each

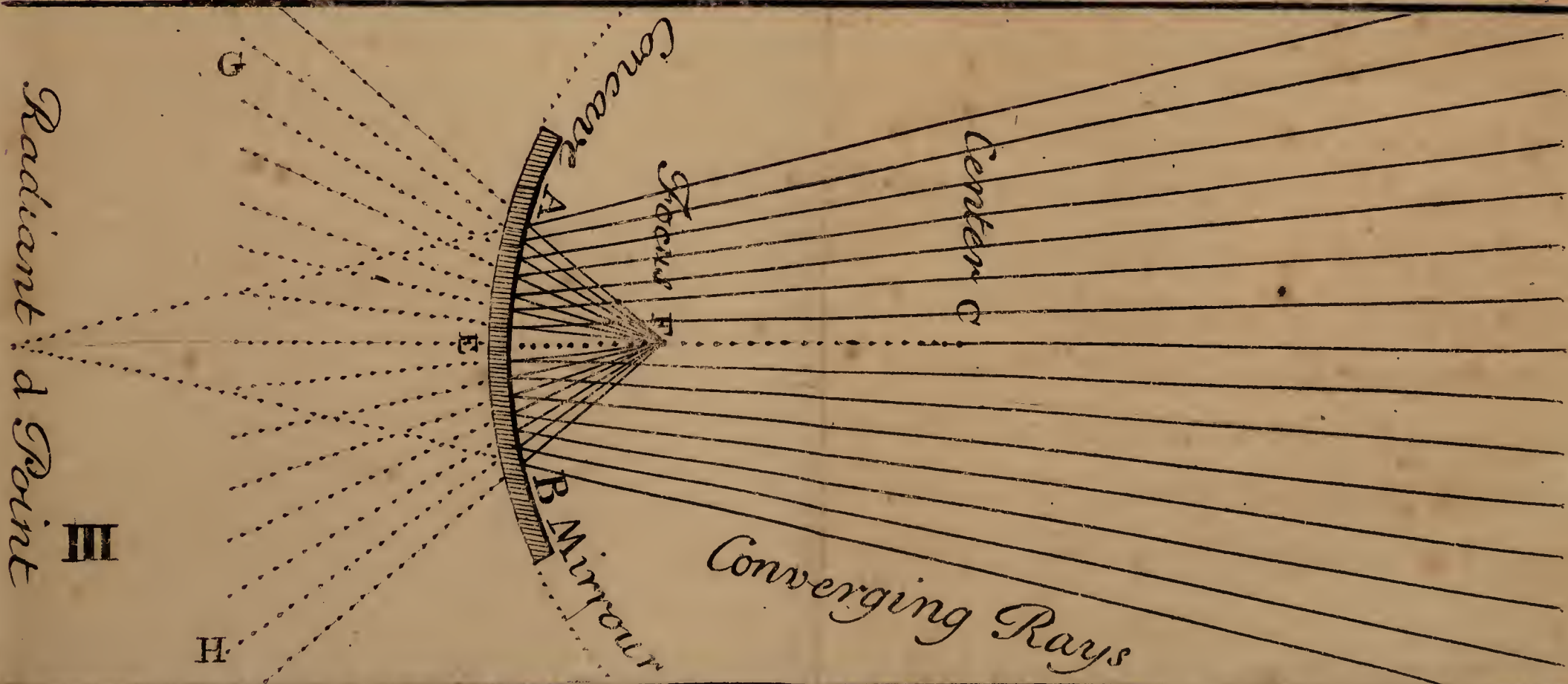
I



II



III





each other, and may be interchangeably taken one for another ; that is, in any Case, if we take F for the Radiant Point, then will D be the Focus of the reflected Rays.

FOR PLAIN MIRROURS. Plate VI.

20. CASE I. Of *Diverging Rays*, Scheme 1. In these Mirrours there is no Radius to be considered, and the Distance of the Radiant Point D being given, the focal Distance is also given, as being equal thereto, *per* Theor.

21. That is, if D be the Point whence Rays proceed, diverging to the Mirrour AB, they will be so reflected towards GH, as if proceeded from the Point F, which Point or Focus F, will always be just as far behind the Mirrour AB, as the Point D is before it, and also on the same Side with it. And this is the known Property of a Plain Mirrour or *Looking-Glass*.

21. CASE II. Of *Parallel Rays*, Scheme 2. If Parallel Rays fall on a plain Mirrour, they are reflected parallel ; and as the *Radiant Point* D is infinitely distant, so also is the *Focus* F, *per* Theor. 22. Hence the Sun being viewed by Reflection in a *Looking-Glass*, appears as vastly distant behind the Glass, as he really is before it.

22. CASE III. Of *Converging Rays*, Scheme 3. In this Case the Radiant Point d is supposed to be behind the Glass, and the Rays incident thereon, will be reflected to a real Focus f , at an equal Distance before it, *per Theor. 23.* So that this Case is but the reverse of the first.

23. Hitherto we have given Rules for finding the *Focus*; but if the Distance of the Focus be given, and either the Radius of the Mirrour, or Distance of the Radiant Point, the other may as easily be found. Thus,

24. Let the focal Distance and Radius be given, to find the Distance of the Radiant Point; this is the Rule for a *Convex Mirrour*.

R U L E.

From the Radius take twice the focal Distance, and with the Remainder divide the Product of the focal Distance by the Radius, the Quotient will be the Distance of the Object, *per Theor. 24.*

25. For a *Concave Mirrour* the Rule is this;

R U L E.

Divide the Product of the Radius into focal Distance, by the Sum of twice that Distance added to the Radius, the Quotient is the Distance of the Radiant Point required, *per Theor. 25.*

26. Both

26. Both these Rules regard the Focus behind the Mirrour, if it be before the Mirrour, the Problem will be impossible for a *Convex Speculum*, *per* Theor. 26. And for a Concave, the Rule is this.

R U L E.

From twice the focal Distance take the Radius, and with the Remainder divide the Product of the focal Distance into the Radius, the Quotient will be the Distance of the Radiant Point, *per* Theor. 32.

27. If the Distance of the Radiant and the Focus be given, we may find the Radius of the *Convex Mirrour* by this

R U L E.

From the Distance of the Radiant take the focal Distance, and with the Remainder divide twice the Product of the focal Distance into the Distance of the Radiant; the Quotient is the Radius of the Convexity required, *per* Theor. 28.

28. The Rule for finding the Radius of Concavity is this:

R U L E.

R U L E.

From the focal Distance take the Distance of the Radiant, and with the Remainder divide twice the Product of those Distances into each other; the Quotient will be the Radius of Concavity sought, *per* Theor. 29.

29. If the given Focus be before the Mirror, the thing is impossible for a Convex, *per* Theor. 30; but is ever possible for a Concave, and is found by this

R U L E.

With the Sum of the Distances of Focus and Radiant, divide the double Product of those Distances; the Quotient is the Radius of Concavity, *per* Theor. 31.

30. I presume it is needless to give either Examples or Schemes for the Illustration of these Rules, if those for finding the Focus be well understood. I shall therefore proceed to the Rules for determining the Distances necessary for any given Proportion of the Object and Image, in the next Chapter.

Plate VI.

I

Focus

F

Plain

A

B

Mirror

G

H

Radiant Point

D

Diverging Rays.

II

F

Plain

A

B

Mirror

G

H

D

Parallel Rays

III

d. Radiant Point

g

h

Plain

A

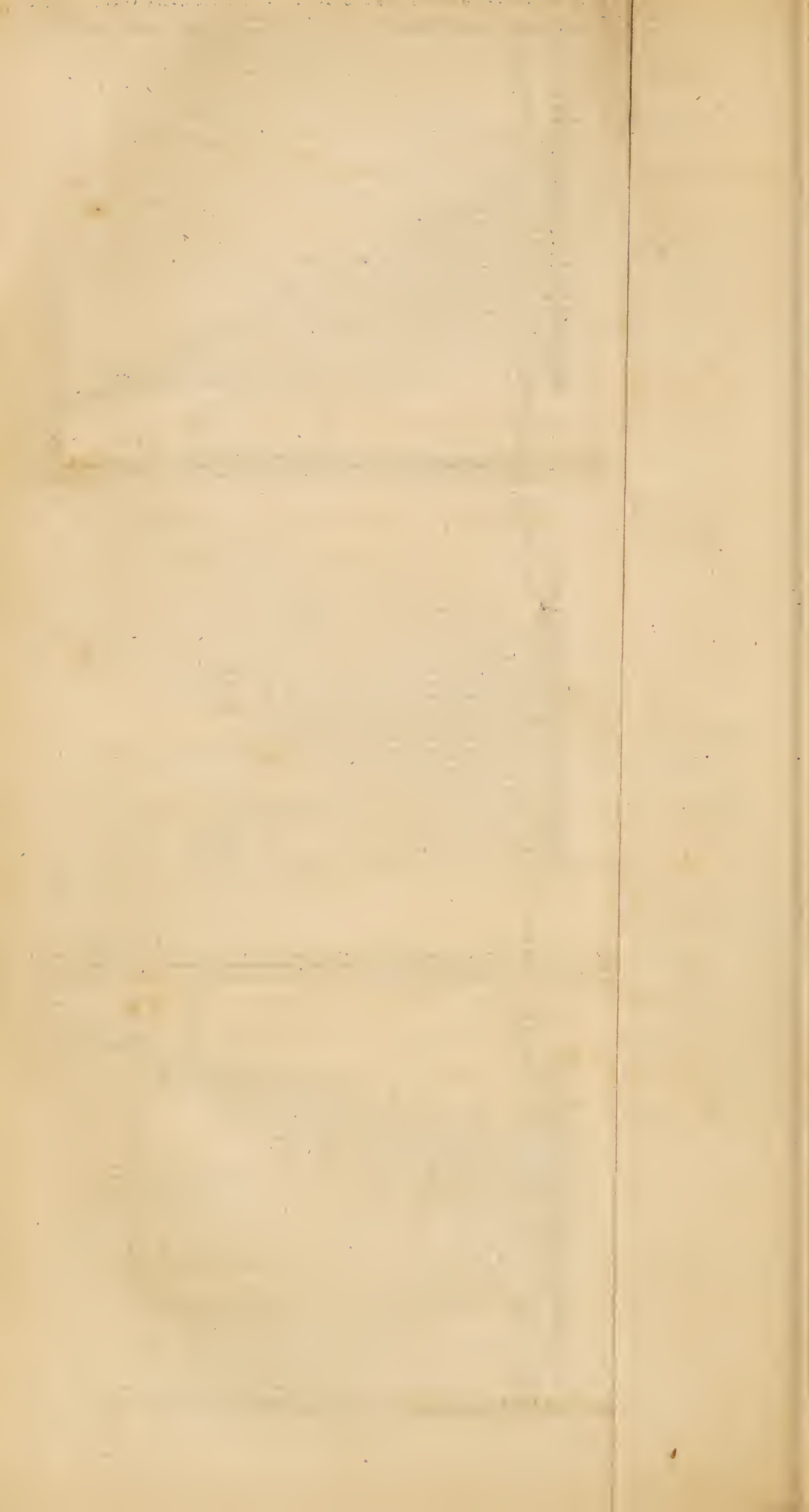
F

B

Mirror

F Focus

Converging Rays



CHAP. VI.

The RULES for determining the Distance of an Object, that shall bear any assigned Proportion to it's Image.

I. **W**E have now done with *Radiant Points*, and are come to treat of *Lines, Superficies, and Solids*, as they appear by Reflection from all Sorts of Mirrours; and first,

Of CONVEX MIRROURS. Plate VII.

2. Let OB be an Object placed before a Convex Mirrour FG , whose Center is C , and Radius CE ; the Image of the Object will be IM , as is evident from what has been taught in Chap. IV. Art. 1, 2, 3.

3. Now to find at what Distance the Object OB ought to be placed, that it may bear any assigned Proportion to the Image IM , we have this Rule for *Lines*.

R U L E.

From the Length of the Object take the Length of the Image, multiply the Remainder by

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by the Radius of Convexity, and divide that Product by the double Length of the Image, the Quotient will be the Distance of the Object required, *per* Theor. 1. of Chap. IV.

E X A M P L E.

Let the Radius $CE = 20$, and the Proportion of OB to IM be as 5 to 1, to find the Distance of the Object OB .

From $OB = 5$
take $IM = 1$

Remains 4

Multiplied by $CE = 20$

Product 80

Then $IM = 1$

Multiplied by 2

Dividend $2) 80$ ($40 = AE$
 80
..

The Numbers here made use of are Tenths of an Inch; and hence it appears, that an Object OB must be placed 4 Inches before a Convex Mirrour of 2 Inches Radius, that it may be in proportion to it's Image as 5 to 1.

4. If the Object be a *Superficies* or *Plane* of any Sort, the Rule for finding it's Distance is as follows.

R U L E.

R U L E.

Divide the Object by the Image, multiply that Quotient by the Square of Radius; then from the square Root of that Product take the Radius, half the Remainder is the Distance sought, *per* Theor. 6. Chap. IV.

E X A M P L E.

Let the Radius be 20, and the given Proportion of the Object to the Image be as 25 to 1. Here

The Object divided by the Image is 25

Multiply by the Square of Radius 400

Extract the square Root — — 10000 (100
the square Root

Subtract the Radius — — — — 20

The Remainder — — — — 80

The half thereof is — — — — 40
the Distance of the Object as required.

5. The Rule is the same for a *Solid*, as for a *Superficies*, if instead of *Square*, and *Square Root*, we use the *Cube* and *Cube Root*, as *per* Theor. 8.

E X A M P L E.

Let the Radius be 20, and the Proportion of the Object to the Image be that
of

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of 125 to 1; to find the Distance of the Object in that Case.

The Object divided	}	125
by the Image is	}	
Multiplied by the	}	8000
Cube of Radius	}	

Extract the Cube	}	1000000	(100 Cube
Root of Prod.	}		(Root
Subtract Radius	— — —	20	

The Remainder	— — —	80
---------------	-------	----

Half thereof is — — — — 40, the Distance of the Object required.

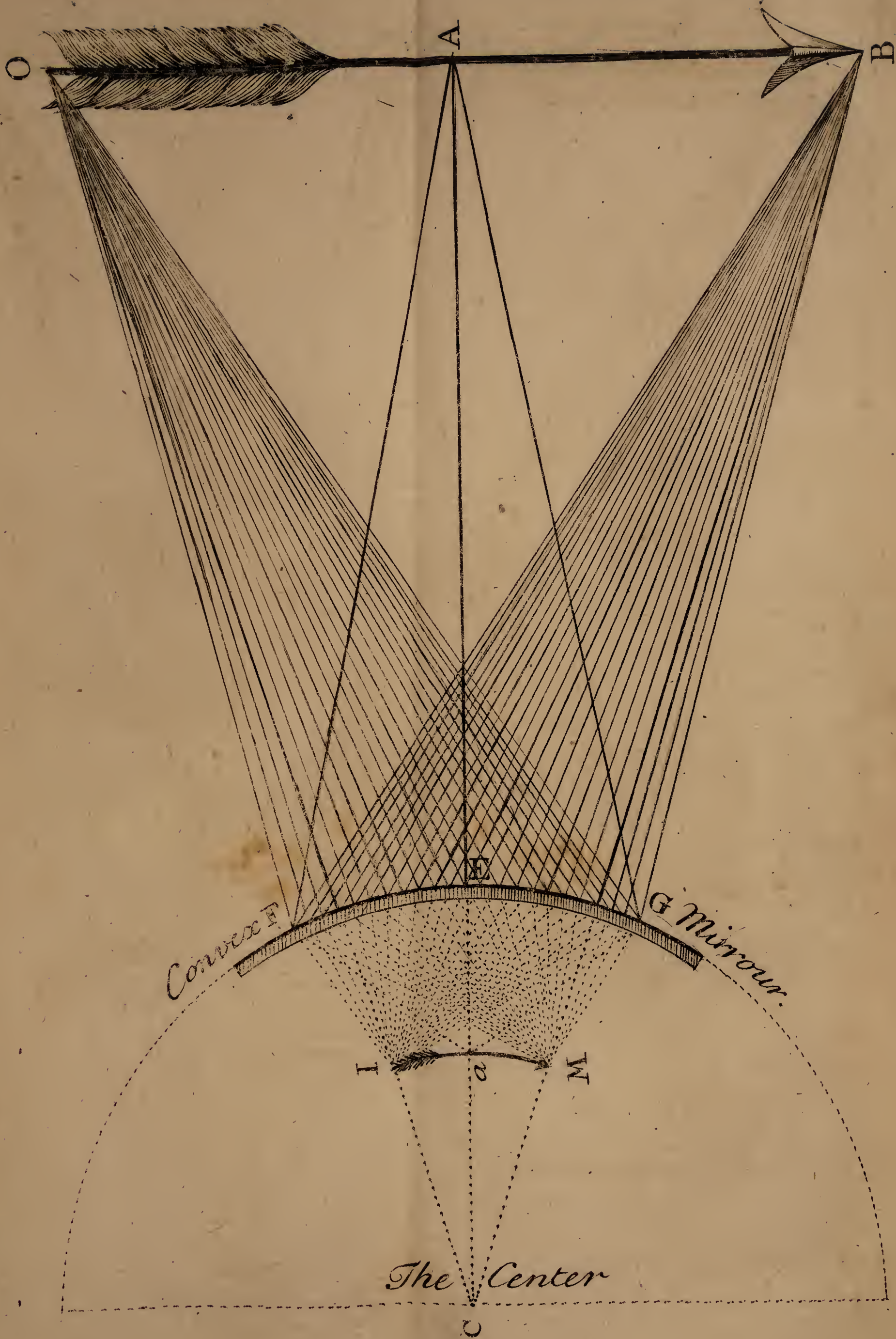
6. Thus it appears that an Object placed 4 Inches before a Convex Mirrour, of 2 Inches Radius, will

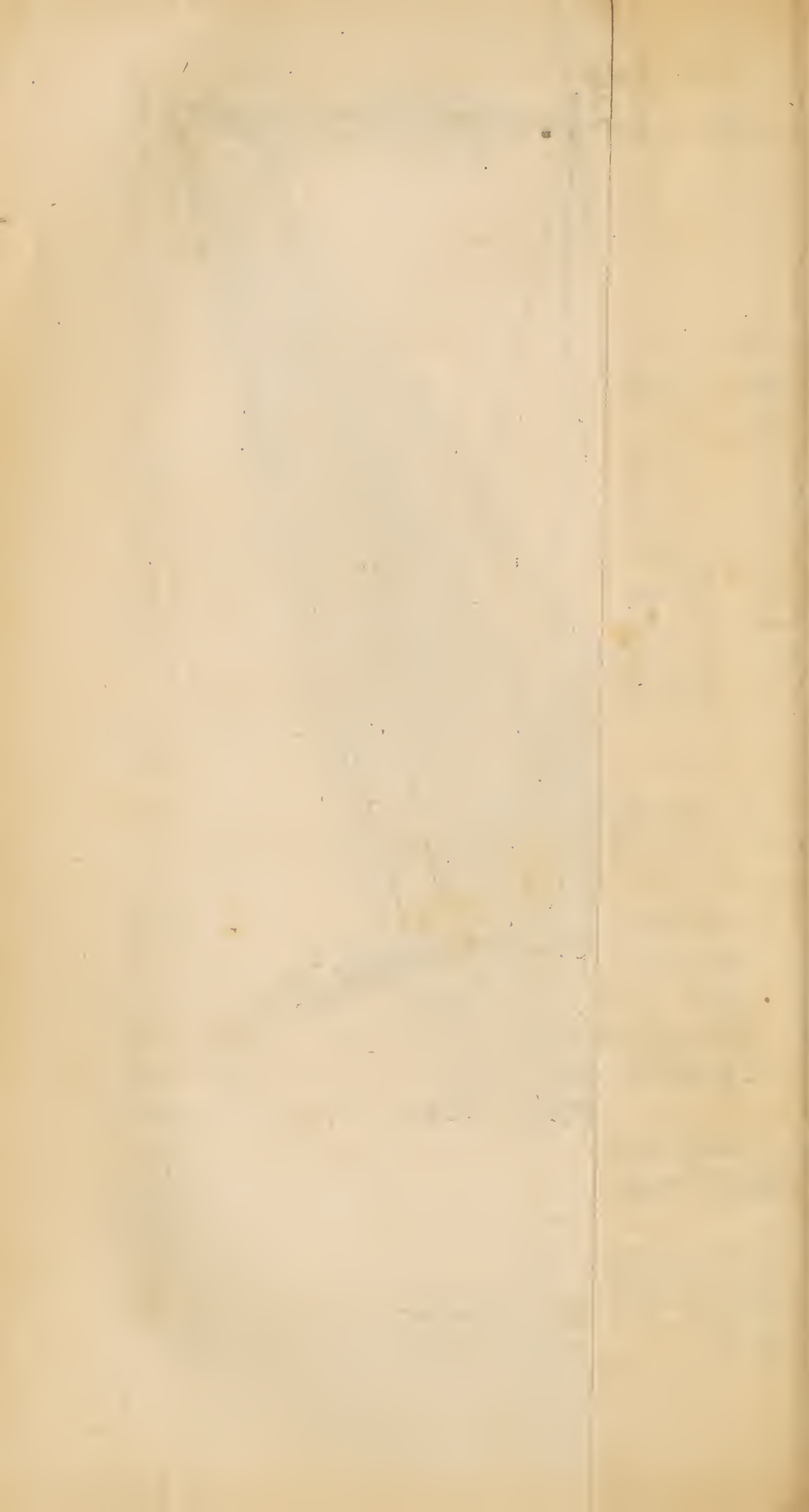
If it be	{	A Line	}	be in Propor-	{	5 to 1
	{	A Superficies	}	tion to it's	{	25 to 1
	{	A Solid	}	Image as	{	125 to 1

x That is, the Diminutions of such Objects are in the *Simple Square*, and *Cubic* Proportions, at equal Distances from the Mirrour, as they ought to be by the Rules of Geometry, which likewise proves the Truth of the Rules here laid down.

x 7. From the 4th Theorem it is evident it is impossible to form an Image of any Object before a Convex Mirrour.

x 8. Also it is impossible to magnify an Object by a Convex Mirrour, because the Image being





being greater than the Object, cannot be taken from it, as the Rule above (in Art. 3.) requires. Nor can the Image be ever equal to the Object, but when they both meet or coincide in the Vertex of the Mirrour.

9. It is evident likewise, if the Object be large in respect of the Mirrour, that it's Image will not be strait or plain, but curv'd, yet not circular or concentric with the Mirrour by what has been taught in Chap. IV.

10. Again, the Image formed by a Convex Mirrour will be *erect*, as is easily understood from Chap. IV, and is evident from the Scheme of this Plate.

CONCAVE MIRROURS. Plate VIII.

Let the Radius be 20, and the Proportion of the Object to the Image be as 1 to 5, to find at what Distance the Object must be placed; this is the

R U L E.

From the Image subtract the Object, and multiply the Remainder by Radius; divide that Product by twice the Image, the Quotient will be the Distance of the Object sought, *per* Theor. 2.

E X A M P L E.

E X A M P L E.

From the Image $IM = 5$

Take the Object $OB = 1$

The Remainder $— — 4$

Mult. by Radius $CE = 20$

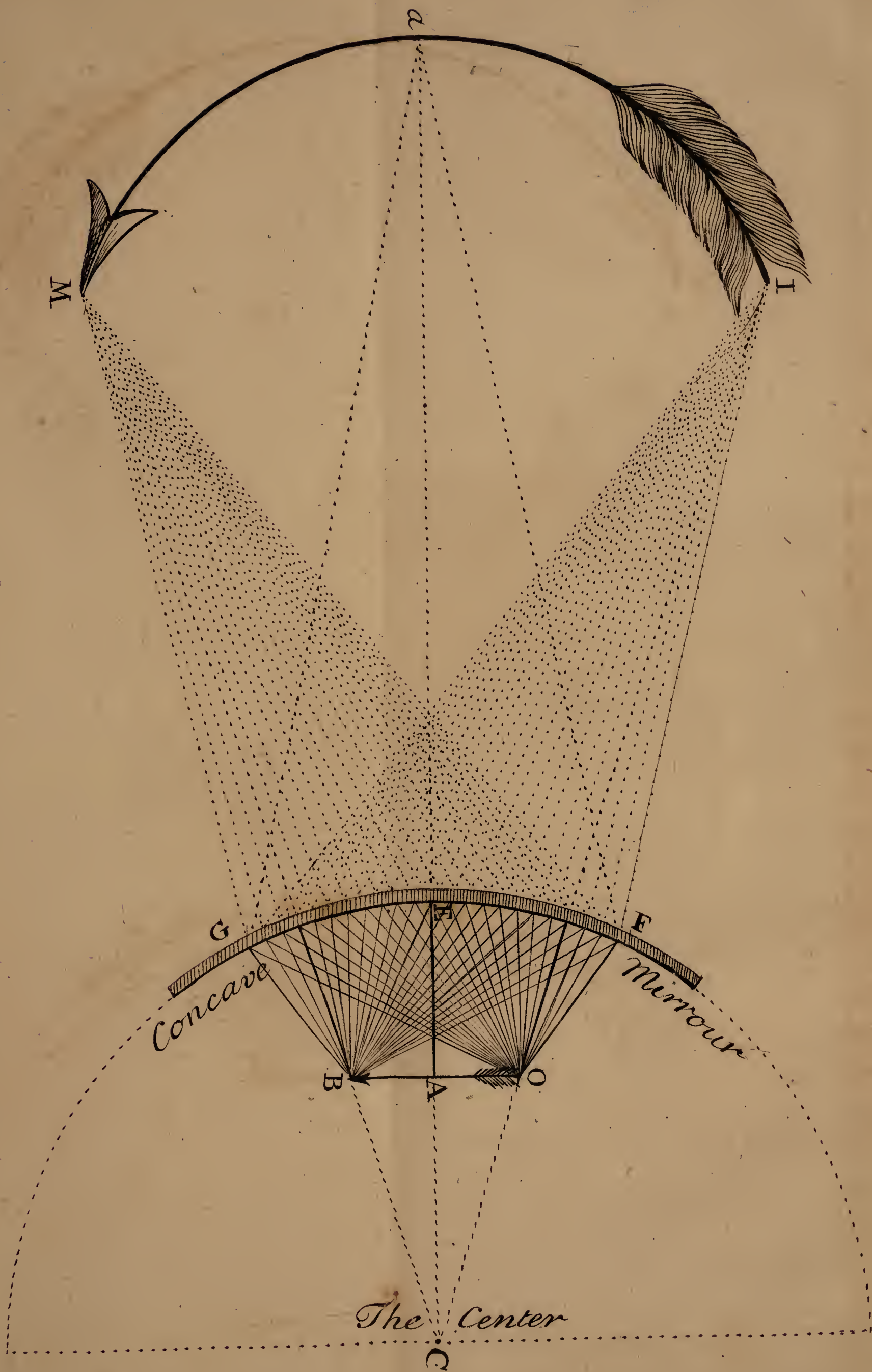
Divide by 2 $IM = 10$) 80 ($8 = AE$ the Di-
 80
 $—$
 $..$

That is, a Line placed 8 Tenths of an Inch before a Concave Mirrour of 2 Inches Radius, will appear 5 Times as big as it is; and were it a Plane, it would be 25 Times as big, if a solid 125 Times as big, at that Distance. For the Rules relating to Surfaces and Solids are here the same as in the Convex before laid down, with this Difference only, that there the Radius was *subtracted* from the Root; but here it must be *added* thereto.

12. While the Focus is behind the Glass, the Object must be *magnified* from the Nature of the Theorem on which this Rule is founded. It will also appear *curv'd* and *erect*, as in the Convex.

13. If the Focus be required before the Glass, the Rule for finding the Distance of the Object, having the same things given as before, will be as follows.

R U L E.



R U L E.

To the Object add the Image, and multiply that Sum by the Radius, divide this Product by twice the Image, the Quotient will be the Distance of the Object, *per*. Theor. 5.

According to this Rule, any Object may, in this Case, be magnified or diminished in any Proportion we please.

E X A M P L E.

Let the Radius be 20, and the Proportion of the Object to the Image be as 5 to 1, to find the Distance of the Object. See Plate IX.

To the Object — $OB = 5,$

Add the Image — $IM = 1,$

The Sum is — — — $6;$

Multiply by Radius $CE = 20,$

The Product is — — — $120;$

Then $2\ IM = 2) 120 (60 = AE$, the Distance of the Object OB required, *viz.* Six Inches.

14. But if IM be supposed the Object, then will OB be the Image, 5 Times larger than it; and the Distance AE will be found 12, or one Inch and two Tenths,

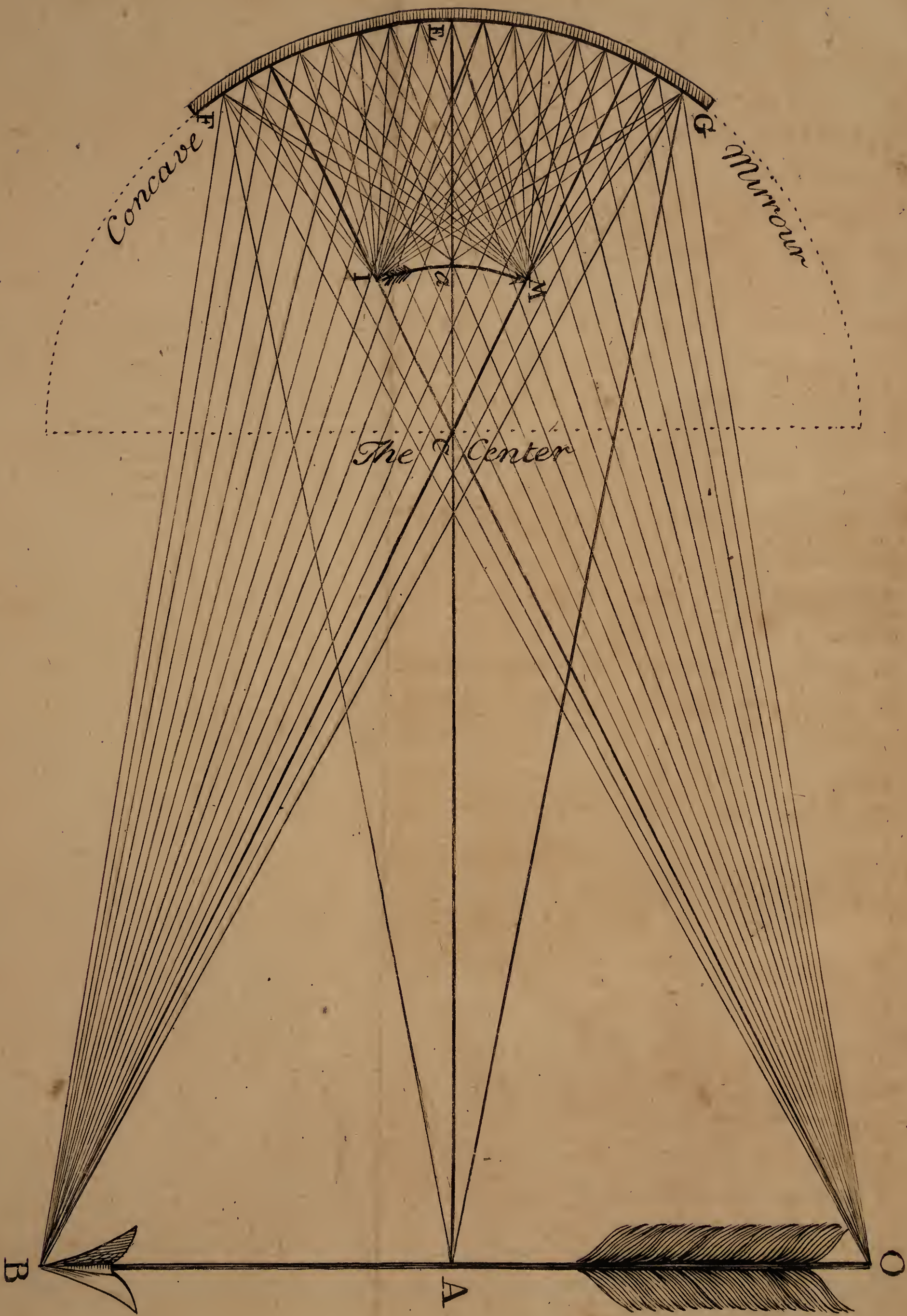
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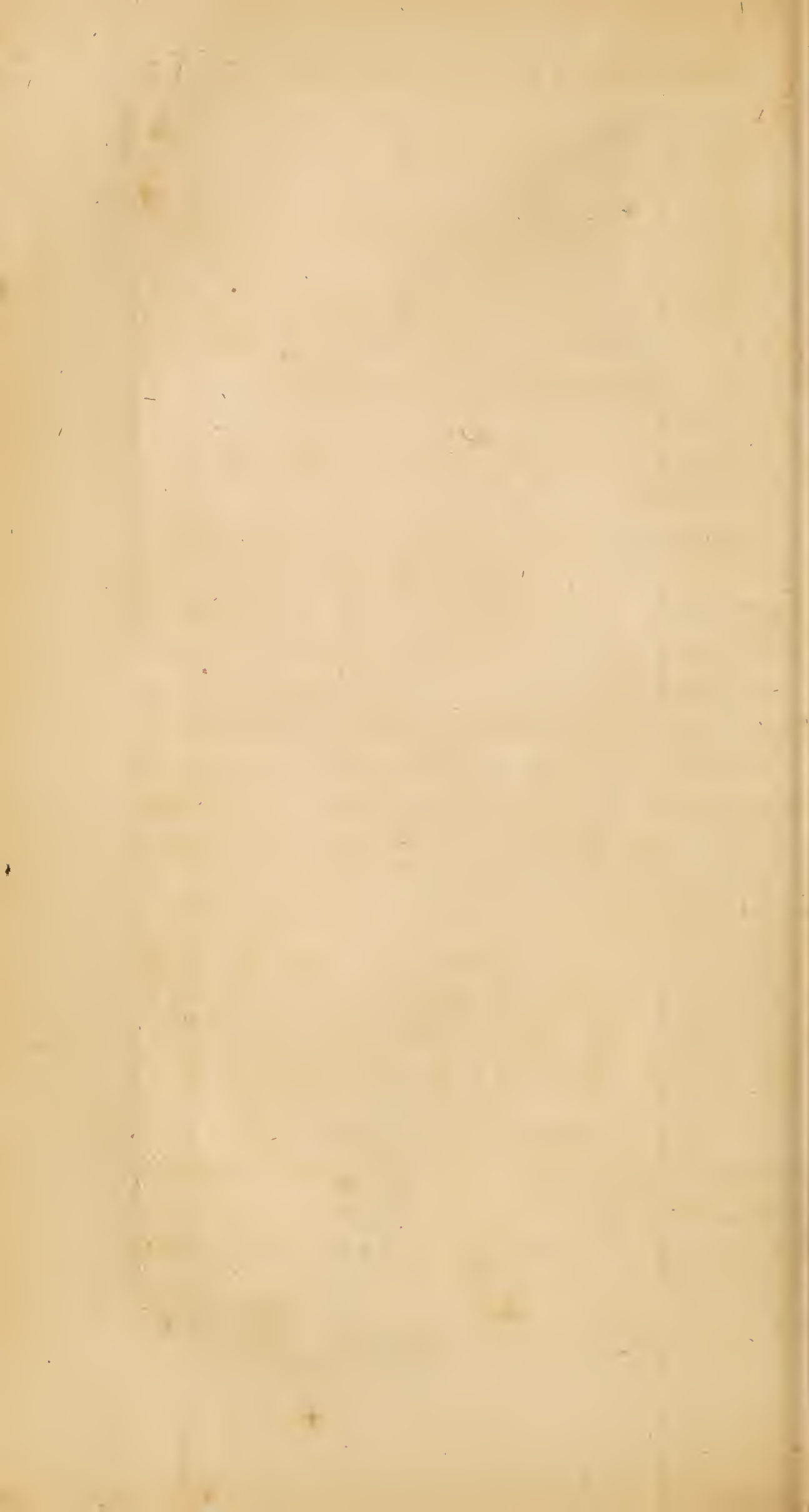
15. If

15. If the Object and Image are desired equal, the Object must be placed in C, the very Center of the Mirrour; this is easily understood from the Rules, and is visible by Experiment, and is, by the Way, a good Expedient for finding the Center of the Concavity of a Mirrour.

16. The Image will here also be *curved*, and it will be now *inverted*, because the Rays cross each other in the Center, and form it on the other Side; the Reason is plain in the Scheme, and from what has been taught in Chap. IV. of the Theory.

17. It will now be no difficult thing to understand how *Concave Mirrours* make such excellent *Burning-Glasses*. For suppose the Radius of Concavity be 2 Feet, or 24 Inches, and the Diameter of the Mirrour 12 Inches. It has been shewn, that the Image of the Sun would be formed by this Mirrour at the Distance of $\frac{1}{2}$ the Radius, or 12 Inches from the Vertex. See Chap. V. Art. 16. Also the Image being seen under the same Angle, as the Object from the Vertex of the Mirrour (*per* Chap. IV. Art. 5.) will there subtend an Angle of only 32 Minutes. The Diameter therefore of this Image, by the Rules of Trigonometry, will be found about 1 Tenth of an Inch, and the Diameter of the Mirrour being 12 Inches, the Squares of these Diameters will be as 1 to 14400; and consequently the Density of the Sun's Rays in this Image (where





(where they are all collected) will be to their Density on the Surface of the Glass, as 14400 to 1, and their Heat will be in Proportion. That is, the Heat of the Sun's Ray, in the Focus of such a Glass, will be *fourteen thousand and four hundred* Times greater than before; and therefore must needs burn very furiously, and produce very great Effects.

PLAIN MIRROURS. Plate X.

18. In Plain Mirrours, or *Looking-Glasses*, the Image I M will always be equal to the Object O B, at what Distance soever it be placed, *per* Theor. 3. It will also be *erect*, and as far behind the Glass as the Object is before, as hath been already shewn, Chap. V. Art. 20.

19. Hence their excellent Use in representing Objects *every Way like the Life*; but this is too common to be insisted upon. I shall only just observe, that the Mirrour being but at half the Distance of the Image, will completely receive an Image of twice it's own Length; and therefore a Man of 6 Foot height may view himself entirely in a Looking-Glass of 3 Feet Length, and half his own Breadth.

20. If the Proportion of the Object and Image be given, and the Distance of the Object from the Mirrour, the Radius of the Convexity may be found for that Purpose by this Rule.

R U L E.

Multiply twice the Distance of the Object by the Image, divide that Product by the Difference between the Object and Image, the Quotient is the Radius of the Mirrour, *per* Theor. 10.

Note, the Focus is here behind the Glass, and it is impossible to have it before, *per* Theor. 12.

21. The same things given, the Radius of Concavity is found by this

R U L E.

Multiply twice the Distance by the Image, and divide that Product by the Difference between the Image and Object, the Quotient is the Radius required, *per* Theor. 11.

The Focus here being behind the Mirrour, the Image will be greatest. But if the Focus be required before the Glass, the same Rule finds the Radius, if, instead of the *Difference*, you take the *Sum* of the Object and Image, *per* Theor. 13.

22. In the last place, if the Radius of the Mirrour, and the Distance of the Object be given, then may the Proportion between the Object and Image be found for a *Convex*, by this

A N A L O G Y.

A N A L O G Y.

As twice the Distance added to the Radius,
is to the Radius ;

So is the Object to the Image, *per* Theor. 14.

23. For a Concave, the Proportion will be
found by this

A N A L O G Y.

As the Radius lessened by twice the Di-
stance, is to the Radius ;

So is the Object to the Image, *per* Theor. 15.

24. If the Focus be required before the
Glas, the Analogy is this.

A N A L O G Y.

As twice the Distance, lessened by Radius,
is to the Radius ;

So is the Object to the Image, *per* Theor. 17.

25. I shall exemplify this last Case of a Con-
cave Mirrour, as being the most frequent and
curious.

Let the Radius be 20, and the Distance of
an Object from the Mirrour be 130 ; then
twice 130 is 260, from which take Radius
20, there will remain 240 ; then shall the
Object be to the Image as 240 to 20, or as

D 3

12

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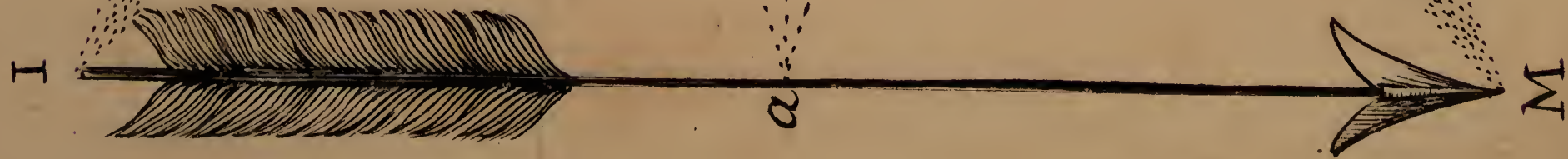
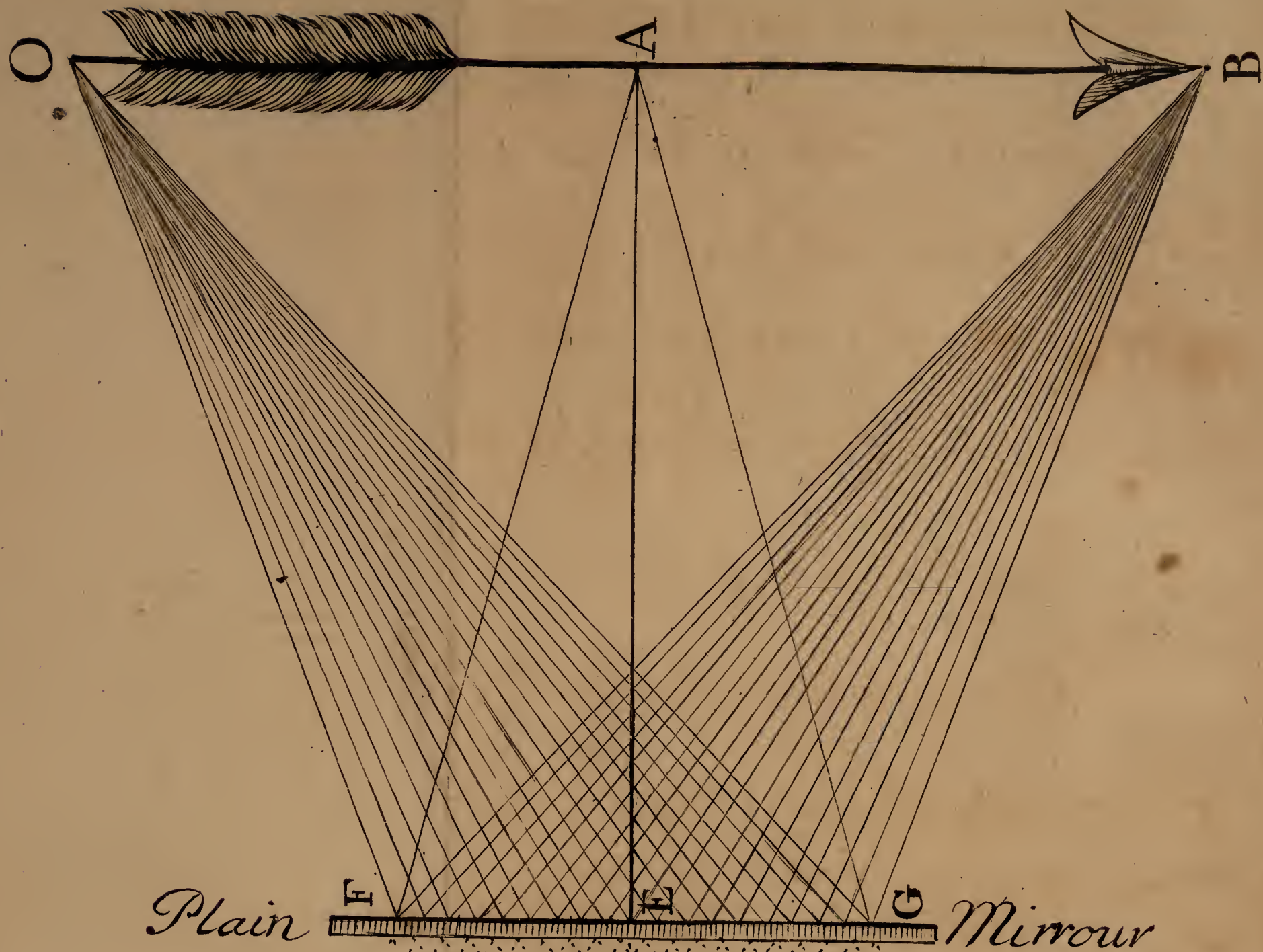
12 to 1 ; that is, it will be 12 Times as large as the Image. Now if you can tell the Length of the Object, as suppose 36, then is the Length of the Image known ; for

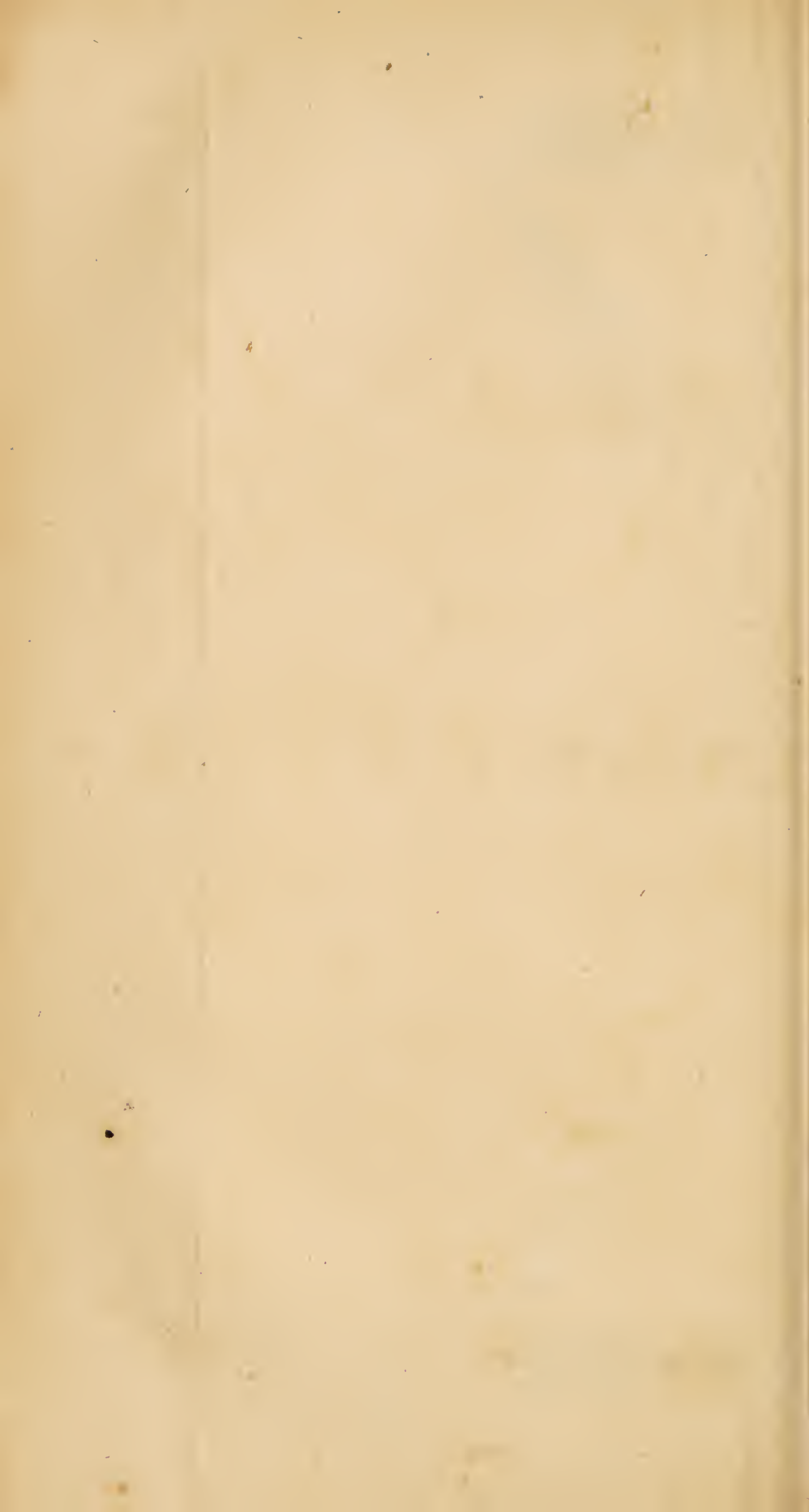
As 240 : 20 :: 36 : 3 = the Length of the Image, as appears by the Work.

$$\begin{array}{r}
 36 \\
 \hline
 120 \\
 60 \\
 \hline
 240 \overline{) 720} (3 \\
 \underline{720} \\
 \dots
 \end{array}$$

26. And thus I have, I hope, sufficiently explained the Doctrine of *Catoptrics*, and illustrated the Theory by plain and practical Rules, Examples, and Schemes. I shall now proceed to do the same by *Dioptrics*, the more noble and useful Part of this Science.





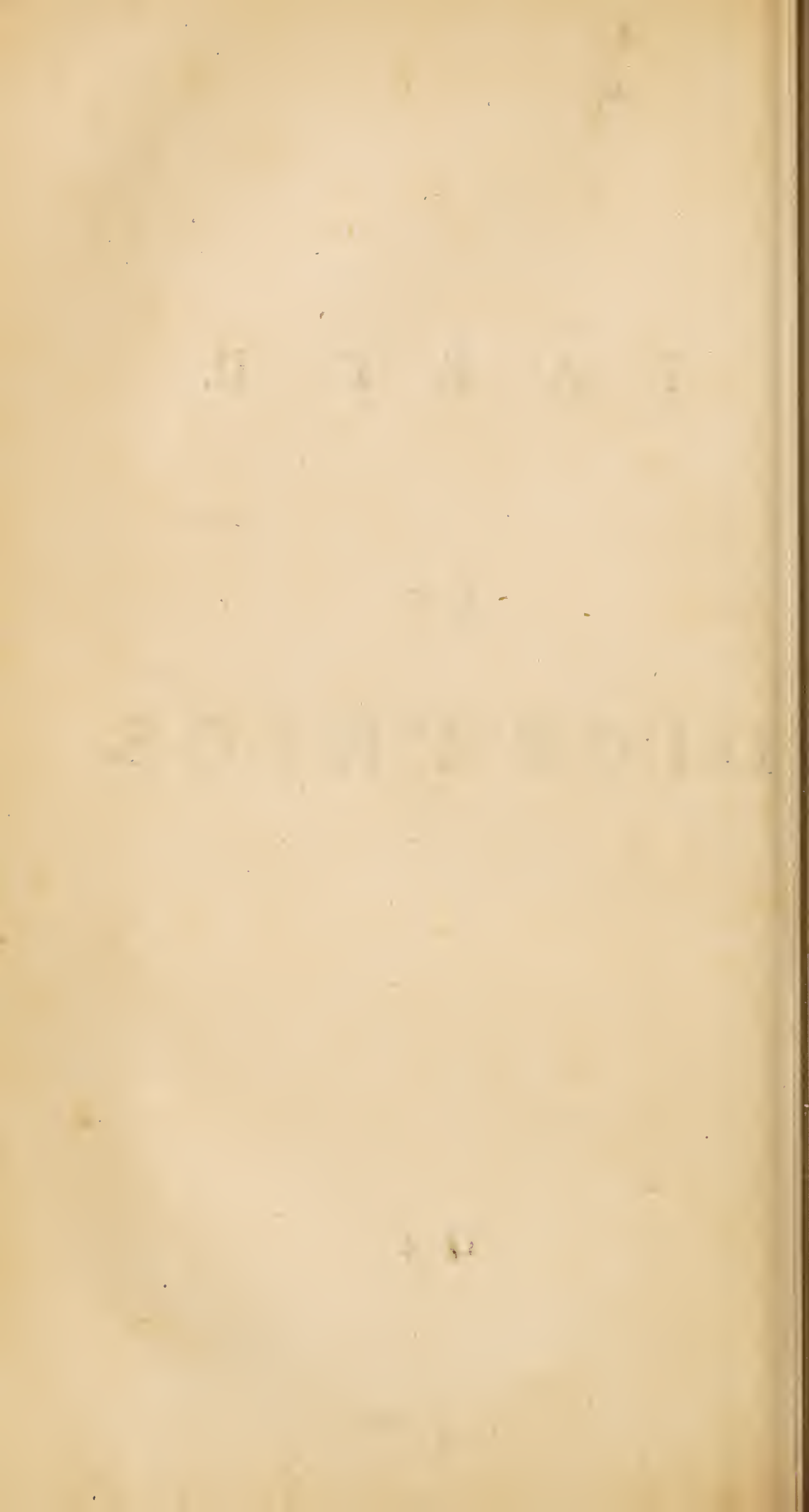


P A R T II.

O F

DIOPTRICS.

D 4



CHAP. I.

The Theory of DIOPTRICS.

1. **I**N Plate XI, Fig. 1. Let LN represent a convex Lens, Df it's Axis, D a Radiant Point therein, DA a Ray proceeding from thence to A, a Point in the Surface LBN, C is the Center of the Convexity of that Surface, CG being drawn through the Point A, is a perpendicular to that Surface in the Point A. CA or CB is the Radius, Af is the refracted Ray, and f the Point where it meets the Axis after the first Refraction.

2. Let $DB = d$, $CA = r$, $EB = t$, the Thickness of the Lens; and let the Sine of the Angle of Incidence DAG be called I, and the Sine of the Angle of Refraction CAf, or GAH, be called R.

3. Now since the Point A is supposed to be very near to the Vertex B, DA may be esteemed equal to $DB = d$, by Princip. 4; and in the Triangle CAD, we shall have AD to AC, as the Angle C to the Angle D, *per* Princip. 1. That is, $d : r :: C : D$.

4. Also

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4. Also $DB + BC = d + r$, will be as the opposite Angle CAD or DAG , for the Sine of both is the same.

5. Then as $I : R :: d + r : \frac{dR + rR}{I}$,

which will be as the Angle CAf ; this taken from the Angle $ACD = d$, leaves the Angle

$$AfD = \frac{dI - rR - dR}{I}.$$

6. Lastly, As the Angle $f : D :: AD$ or $DB : Af$ or Bf ; that is, as $\frac{dI - rR - dR}{I}$

$$:: r :: d : \frac{rdI}{dI - rR - dR} = Bf, \text{ the Di-}$$

stance of the Point f , in the Axis after the first Reflection.

7. But since there is a second Surface LEN of the Lens, there must necessarily be a second Refraction of the Ray AD , to some other Point in the Axis, as F ; see Fig. 2. Let K be the Center of this second Surface, and KI a perpendicular thereto in the Point a , from whence the said Ray is refracted to F . In this Case, the Refraction being out of a dense into a rarer Medium, the Sine of Incidence will be to that of Refraction, the reverse of what it was before, *viz.* as R to I ; that is, the Sine of Iaf is to the Sine of IaF , as R to I .

8. Here let Ka be called r , $Ef = d$; and then will be, as $d : r :: K : f$. And $Ef + EK$

$EK = d + r$; and this will be as the Angle $f a K$, or it's Complement $I a f$; therefore $R :$
 $I :: d + r : \frac{dI + rI}{R}$, which will exprefs
the Angle $I a F$.

$$\begin{aligned} 9. \text{ Then } IaF - aKF &= \frac{dI + rI}{R} - d \\ &= \frac{dI + rI - dR}{R} = aKF. \text{ Now as } F : \\ K :: Ka \text{ or } KB : aF \text{ or } EF; \text{ that is, as } \\ \frac{dI + rI - dR}{R} : d :: r : \frac{Rdr}{dI + rI - dR} \\ &= EF. \end{aligned}$$

$$\begin{aligned} 10. \text{ But } Bf - BE &= \frac{drI}{dI - rR - dR} \\ - t = d = Ef, \text{ per Art. 6.} \end{aligned}$$

$$11. \text{ Therefore, putting } I - R = O, \text{ we shall} \\ \text{have } d = \frac{drI}{dO - rR} - t = \frac{drI - dOt + rRt}{dO - rR}.$$

$$12. \text{ Also } dRr = \frac{drIRr - dotRr + rtRRr}{dO - rR}.$$

$$\begin{aligned} 13. \text{ Again } dI + rI - dR &= dO + rI; \\ \text{if then we multiply the Equation, Art. 11. by} \\ O, \text{ and add thereto } rI; \text{ we shall have } dI + rI \\ - dR &= \frac{drIo - doto + rRto + dorI}{dO - rR} \\ &\quad - rRIr \end{aligned}$$

14. Then

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$$14. \text{ Then } \frac{d R r}{d I + r I - d R} = \frac{d r I R r - d o t R r + r t R R r}{d r I o - d o t o + r R t o + d o r I - r R I r} = E F.$$

15. This last Equation may be abridged, by substituting p for $\frac{R}{O}$, that is, for $\frac{R}{I - R}$, then

$$\text{shall we have } \frac{p d r I r - p d o t r + r t R r p}{d r I - d o t + r R t + d r I - p r I r} = E F.$$

16. Lastly, if now we take $R = p O$, in $p d o t r$; and $I - R = O$ in $d o t$; this Equation will be finally reduced to this fundamental Equation for all *Dioptrics*, viz.

$$\frac{p d r r I - d t r R + r t r p R}{d r I - d t I + d t R + r t R + d r I - p r r I} = E F = f.$$

17. The Ratio of I to R being in Glafs as 3 to 2, we shall have $\frac{R}{I - R} = \frac{2}{3 - 2} = 2 = p$, for a Glafs Lens.

18. In *Water*, the Ratio of I to R being as 4 to 3, we shall have $p = 3$; and in *Diamond*, the

the Ratio being as 5 to 2, p will be $= \frac{2}{3}$. Wherefore, if instead of p , I, and R, we write their Value in Numbers in the foregoing Equation, it will then be suited for a Lens of Glass, *Water*, or *Diamond*. But I shall only regard that of Glass, for which it will

stand thus; viz.
$$\frac{6 d r r - 2 d t r + 4 t r r}{3 d r + 3 d r - d t + 2 r t - 6 r r} = E F = f.$$

19. We shall now adapt this Theorem to the Case of a *Glass Globe*, *Glass Hemispheres*, and all Sorts of *Glass Lenses*, for all Kind of Rays, as follows.

FOR A GLOBE.

20. CASE I. *Diverging Rays*, Fig. 3. Here $r = r$, and $t = 2 r$; that is, both the Radius's being equal, and t the Thickness being equal to twice the Radius; the Equation in Art. 18. will be reduced to this Form

$$\frac{d r + 4 r r}{2 d - r} = f. \text{ Theor. 1.}$$

21. CASE II. Of *Parallel Rays*, Fig. 4. In this Case d being infinite, the Parts of the Equation will vanish where it is not found; consequently it will be $\frac{d r}{2 d} = \frac{r}{2} = f.$ Theor. 2.

22. CASE

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22. CASE III. Of *Converging Rays*, Fig. 5. Here d being negative, or $-d$, the Equation will be $\frac{-dr + 4rr}{-2d - r} = f$. Theor. 3.

For an HEMISPHERE.

23. CASE I. Of *Diverging Rays*, Fig. 6. If the convex Part be towards the Radiant Point D, then will the Radius r be infinite, and $t = r$; and the Equation will become $\frac{-4dr + 4rr}{3d - 6r} = f$. Theor. 4.

24. CASE II. Of *Parallel Rays*, Fig. 7. Here d being infinite, we shall have $\frac{4dr}{3d} = \frac{4}{3}r = f$. Theor. 5.

25. CASE III. Of *Converging Rays*, Fig. 8. Here d being negative, or $-d$, the Equation will be $\frac{-4dr + 4rr}{-3d - 6r} = f$. Theor. 6.

26. If the plain Part of the Hemisphere be exposed to the Radiant, there will be for

CASE

CASE I. Of *Diverging Rays*, Fig. 9.

$$\frac{6dr + 4rr}{3d - 4r} = f. \text{ Theor. 7.}$$

27. CASE II. Of *Parallel Rays*, Fig. 10.

$$\frac{6dr}{3d} = \frac{6}{3}r = 2r = f. \text{ Theor. 8.}$$

28. CASE III. Of *Diverging Rays*, Fig. 11.

$$\frac{-6dr + 4rr}{-3d - 4r} = f. \text{ Theor. 9.}$$

29. Note, if from the focal Distance $2r$ in Art. 27, we take $\frac{2}{3}r$ in Art. 24, there will remain $\frac{2}{3}r$, which will be the Difference in turning the Convex and plain Sides of the Hemisphere towards the Sun-Beams.

FOR DOUBLE CONVEX LENSES.

30. If the Lens be convex on both Sides, and the *Radii* of the Convexities unequal, and the Thickness considered, we shall have for

CASE I. Of *Diverging Rays*, the fundamental Equation it self, viz.

$$\frac{6drr - 2dtr + 4trr}{3dr + 3dr - dt + 2rt - 6rr} = f. \text{ Theor. 10.}$$

31. CASE

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31. CASE II. Of *Parallel Rays*, where d is infinite, $\frac{6dr - 2dt}{3dr + 3dr - dt} = \frac{6rr - 2tr}{3r + 3r - t} = f$. Theor. 11.

32. For *Converging Rays*, where d is negative, or $-d$; we have

$$\frac{-6drr + 2dtr + 4trr}{-3dr - 3dr + dt + 2rt - 6rr} = f.$$

Theor. 12.

33. If the *Thickness* of the Lens be neglected, as inconsiderable, which it very well may, and always is in common Use; then all those Parts of the Equation where t is found will vanish, and it will become for

CASE I. Of *Diverging Rays*, Fig. 12.

$$\frac{2drr}{dr + dr - 2rr} = f. \text{ Theor. 13.}$$

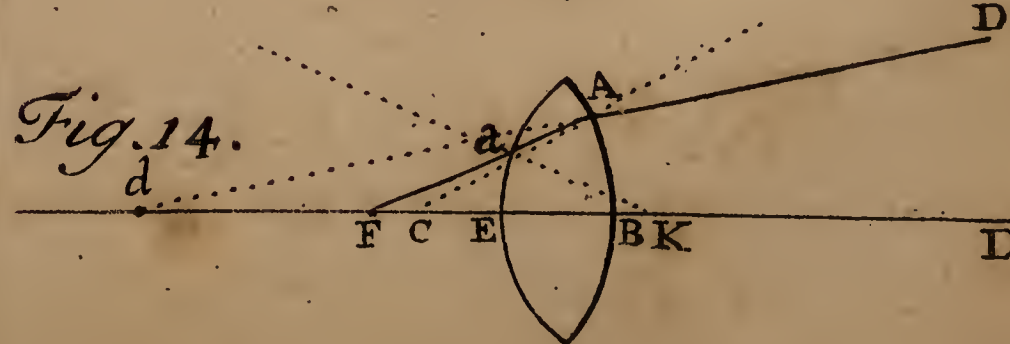
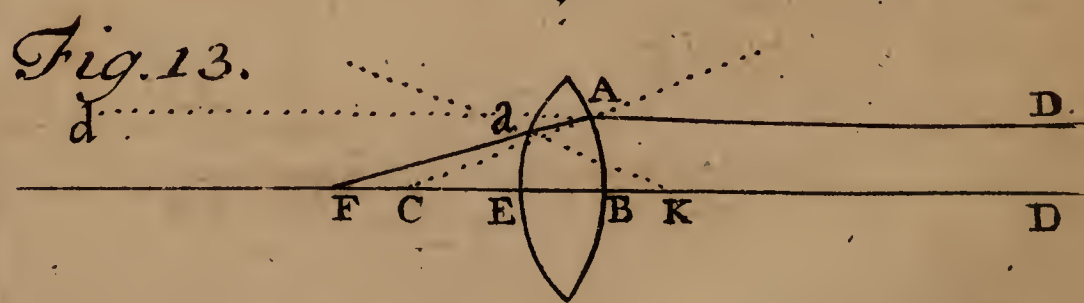
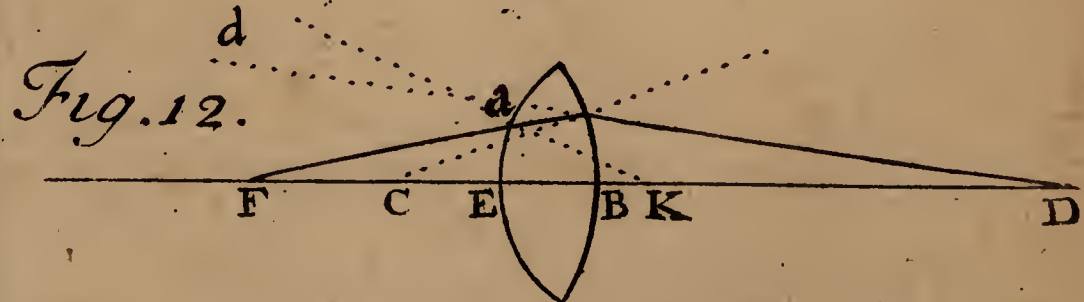
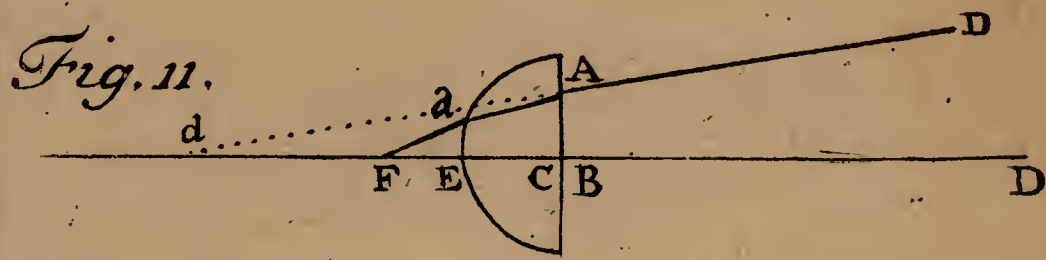
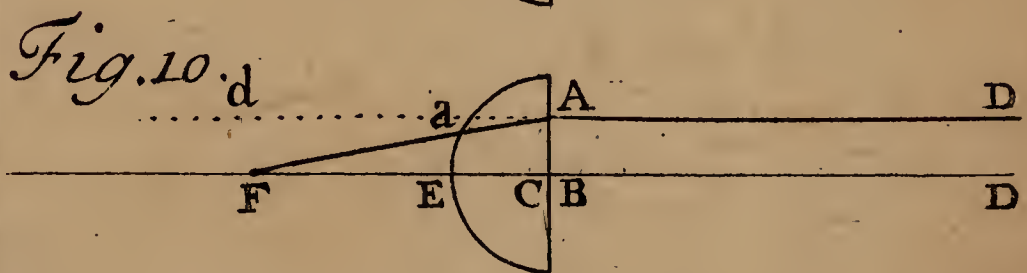
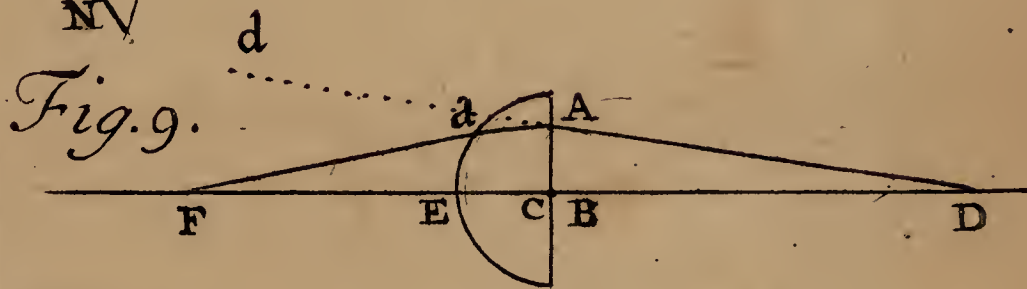
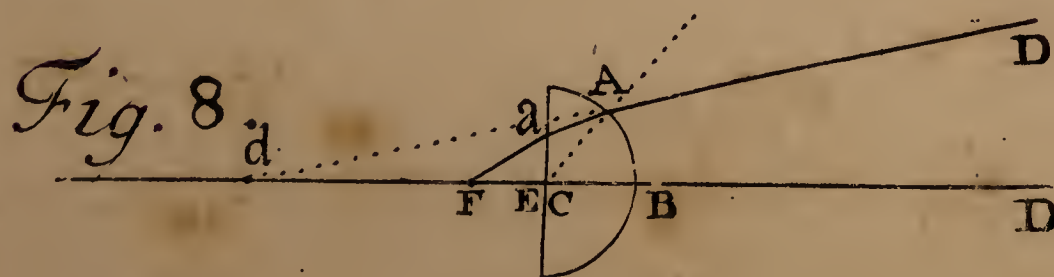
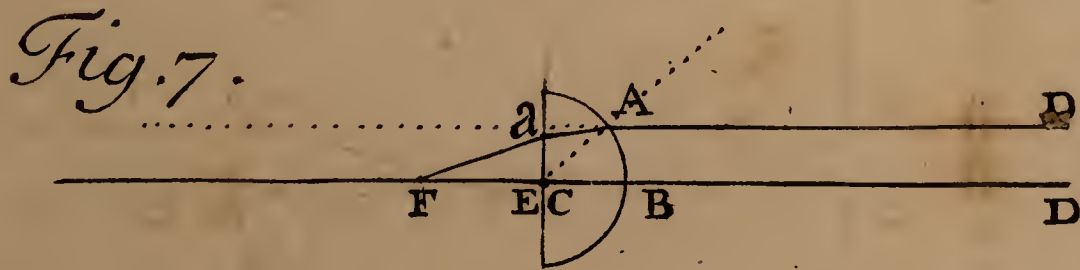
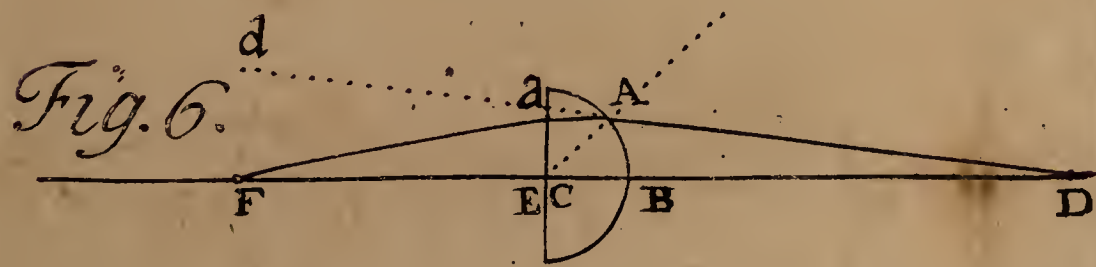
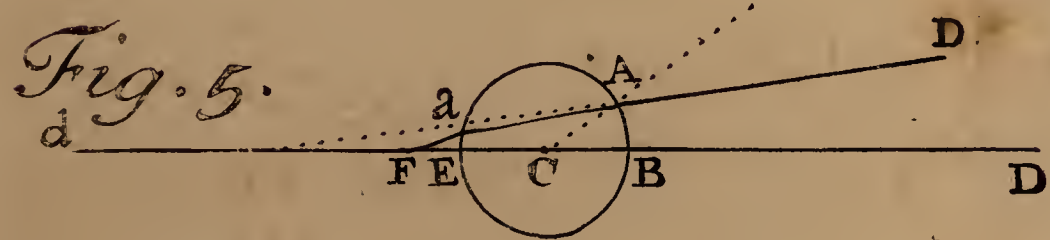
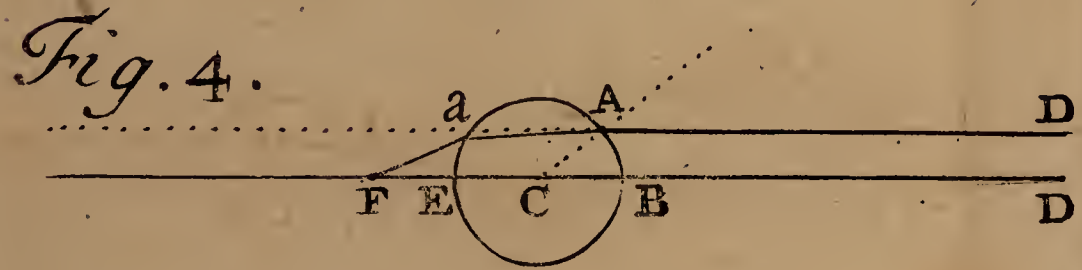
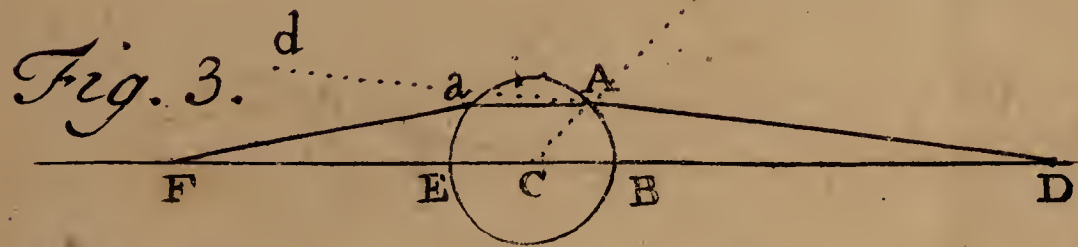
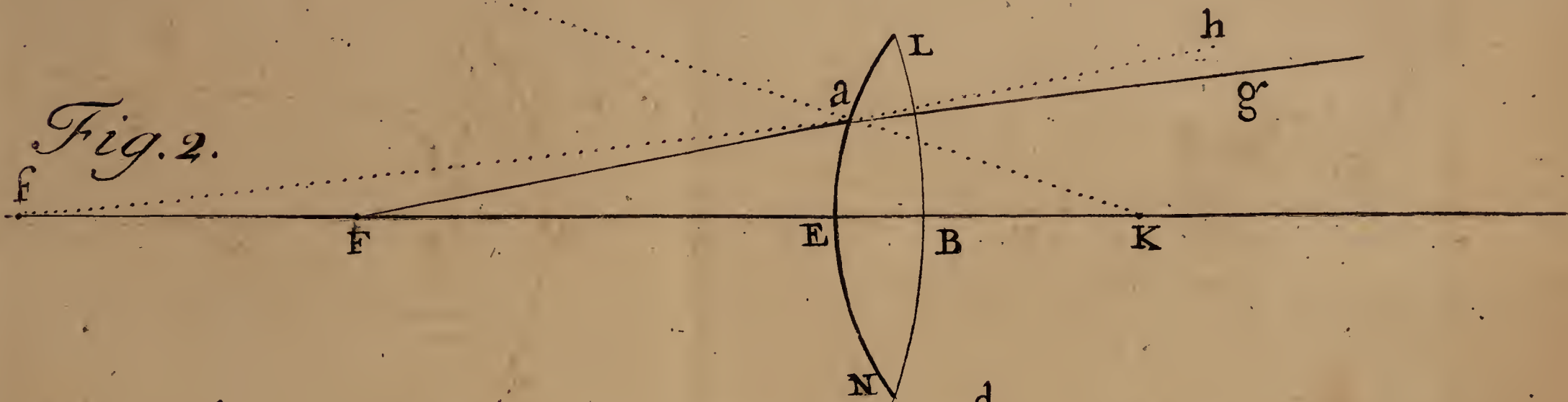
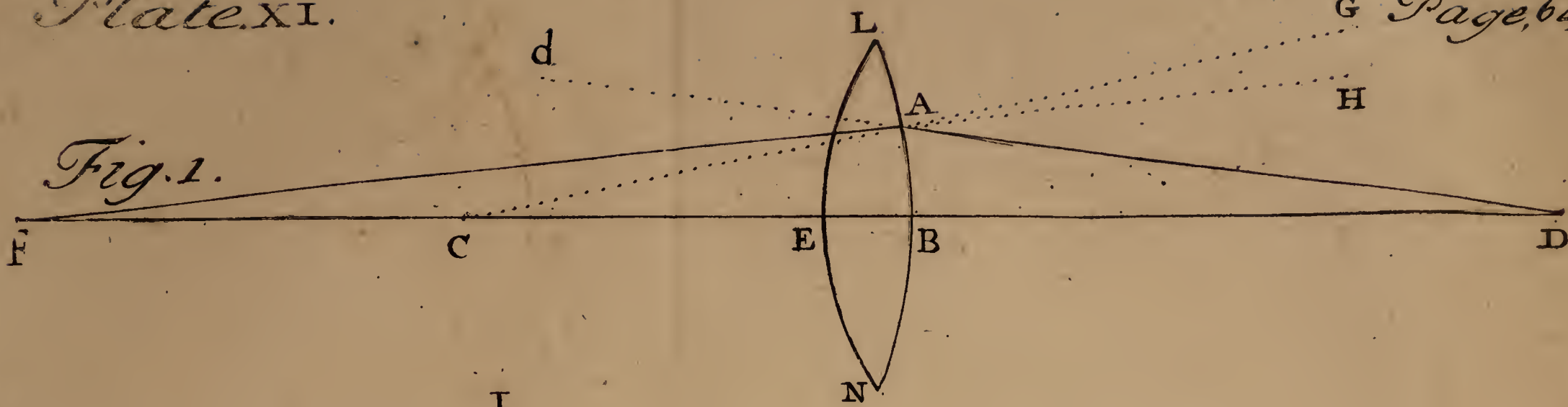
34. CASE II. For *Parallel Rays*, Fig. 13.

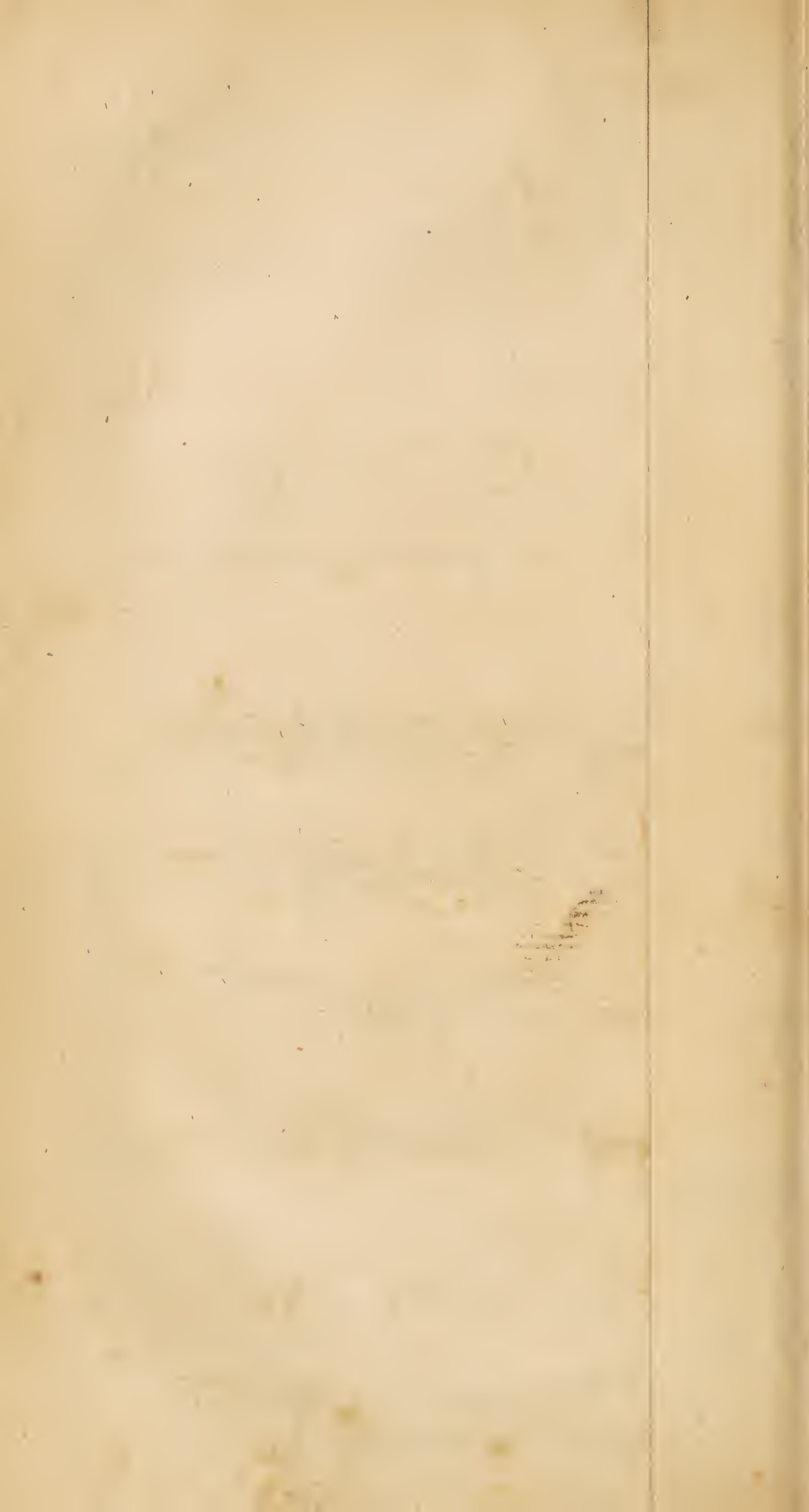
$$\frac{2drr}{dr + dr} = \frac{2rr}{r + r} = f. \text{ Theor. 14.}$$

35. CASE III. For *Converging Rays*, Fig. 14.

$$\frac{2drr}{dr + dr + 2rr} = f. \text{ Theor. 15.}$$

36. If





36. If the Radii of the Convexities are equal, that is, $r = r$, and the Thickness of the Lens, or t^* , be neglected; then will the Equation be very simple, *viz.*

CASE I. For *Diverging Rays*, Fig. 15.

Plate XII. $\frac{dr}{d-r} = f$. Theor. 16.

37. CASE II. For *Parallel Rays*, Fig. 16.

$\frac{dr}{d} = r = f$. Theor. 17.

38. CASE III. For *Converging Rays*,

Fig. 17. $\frac{dr}{d+r} = f$. Theor. 18.

For a PLANO-CONVEX LENS.

39. If the *Convex Surface* be exposed to the Radiant, r being infinite, the Equation will be for

CASE I. Of *Diverging Rays*, Fig. 18.

$\frac{dr - 2dt + 4tr}{3d - 6r} = f$. Theor. 19.

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40. CASE II. For *Parallel Rays*, Fig. 19.

$$\frac{6r - 2t}{3} = 2r - \frac{2}{3}t = f. \text{ Theor. 20.}$$

41. CASE III. For *Converging Rays*,

Fig. 20.
$$\frac{-6dr + 2dt + 4tr}{-3d - 6r} = f. \text{ Theor. 21.}$$

42. If the Thickness of the Lens t be neglected, as is usual, the Theorems will be more concise, *viz.*

CASE I. For *Diverging Rays*, $\frac{2dr}{d - 2r} = f.$
Theor. 22.

CASE II. For *Parallel Rays*, $\frac{2dr}{d} = 2r = f.$ Theor. 23.

CASE III. For *Converging Rays*, $\frac{2dr}{d + 2r} = f.$ Theor. 24.

43. If the plain Surface be turned to the Radiant, in which Case r will be infinite, we shall have

CASE

CASE I. For *Diverging Rays*, Fig. 21.

$$\frac{6dr + 4tr}{3d + 2t - 6r} = f. \text{ Theor. 25.}$$

44. CASE II. For *Parallel Rays*, Fig. 22.

$$\frac{6dr}{3d} = 2r = f. \text{ Theor. 26.}$$

45. CASE III. For *Converging Rays*, Fig. 23.

$$\frac{-6d + 4tr}{-3d + 2t - 6r} = f. \text{ Theor. 27.}$$

46. If the Thickness, or t , be neglected, we shall have

CASE I. For *Diverging Rays*, $\frac{2dr}{d - 2r} = f.$
Theor. 28.

CASE II. For *Parallel Rays*, $\frac{2dr}{d} = 2r$
 $= f.$ Theor. 29.

CASE III. For *Converging Rays*, $\frac{2dr}{d + 2r}$
 $= f.$ Theor. 30.

47. From the Theorems in Art. 40 and 44, it appears, that the focal Distance is greater
E 2 by

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by $\frac{2}{3} t$, when the plain Side of the Lens is turned towards that of the Sun Beams, than when the convex Side is.

For CONCAVE LENSES.

48. If it be a double Concave, whose Radii of Concavities are unequal, and the Thickness considered; each Radius having, in this Case, a negative Sign, the Equation will be for

CASE I. Of *Diverging Rays*,

$$\frac{6dr + 2dtr + 4trr}{-3dr - 3dr - dt - 2rt - 6rr} = f.$$

Theor. 31.

49. CASE II. Of *Parallel Rays*,

$$\frac{6rr + 2tr}{-3r - 3r - t} = f.$$

Theor. 32.

50. CASE III. Of *Converging Rays*,

$$\frac{-6dr - 2dtr + 4trr}{3dr + 3dr + dt - 2rt - 6rr} = f.$$

Theor. 33.

51. If the Thickness of the Lens, or t , be neglected, which in Concaves is most inconsiderable; the Theorems will be much shortened, and stand thus;

CASE

CASE I. For *Diverging Rays*, Fig. 24.

$$\frac{2dr}{-dr - dr - 2rr} = f. \text{ Theor. 34.}$$

52. CASE II. For *Parallel Rays*, Fig. 25.

$$\frac{2rr}{-r - r} = f. \text{ Theor. 35.}$$

53. CASE III. For *Converging Rays*, Fig. 26.

$$\frac{-2dr}{dr + dr - 2rr} = f. \text{ Theor. 36.}$$

54. Moreover, if we suppose the Radii of Concavities equal, that is, $r = r$, we shall have the Theorems still more abridged thus ;

CASE I. For *Diverging Rays*, Fig. 27.

$$\frac{dr}{-d - r} = f. \text{ Theor. 37.}$$

55. CASE II. For *Parallel Rays*, Fig. 28.

$$\frac{dr}{-d} = -r = f. \text{ Theor. 38.}$$

56. CASE III. For *Converging Rays*, Fig. 29.

$$\frac{-dr}{d - r} = f. \text{ Theor. 39.}$$

Of a PLANO-CONCAVE LENS.

57. If the concave Surface be towards the Radiant, r being infinite, we shall have the Equation for

CASE I. Of *Diverging Rays*, Fig. 30.

$$\frac{-6dr - 2dt - 4tr}{3d + 6r} = f. \text{ Theor. 40.}$$

58. CASE II. Of *Parallel Rays*, Fig. 31.

$$\frac{-6r - 2t}{3} = -2r - \frac{2}{3}t = f. \text{ Theor. 41.}$$

59. CASE III. Of *Converging Rays*, Fig. 32.

$$\frac{6dr + 2dt - 4tr}{-3d + 6r} = f. \text{ Theor. 42.}$$

60. If the Thickness of the Lens be neglected, the Theorems will be thus abridged.

CASE I. For *Diverging Rays*, $\frac{-2dr}{d + 2r} = f.$
Theor. 43.

CASE II. For *Parallel Rays*, $\frac{-2dr}{d} =$
 $-2r = f. \text{ Theor. 44.}$

CASE

Fig.15. d

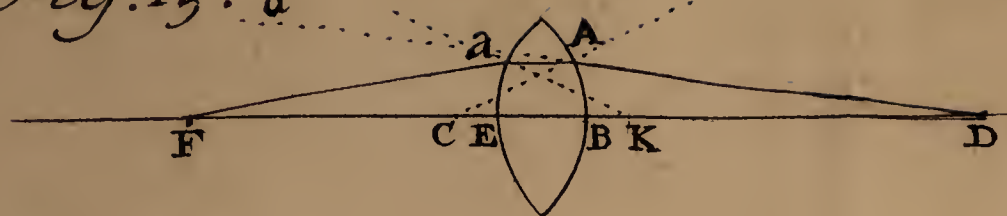


Fig.16.

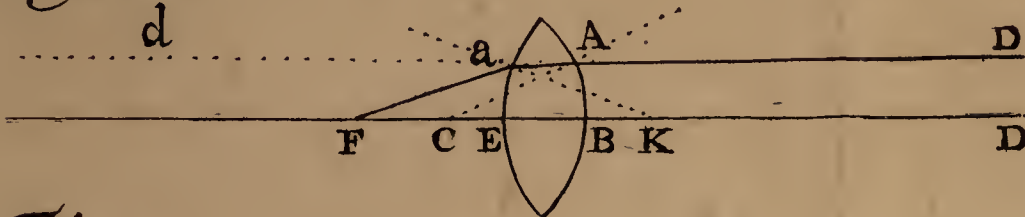


Fig.17.

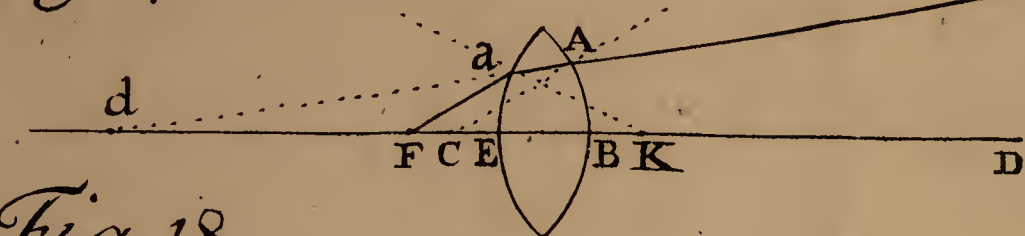


Fig.18.

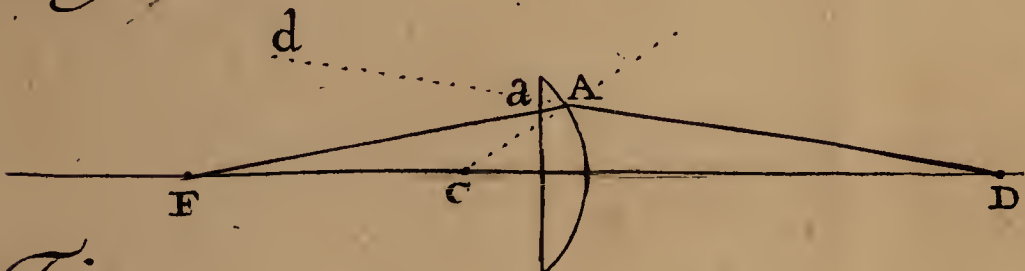


Fig.19.

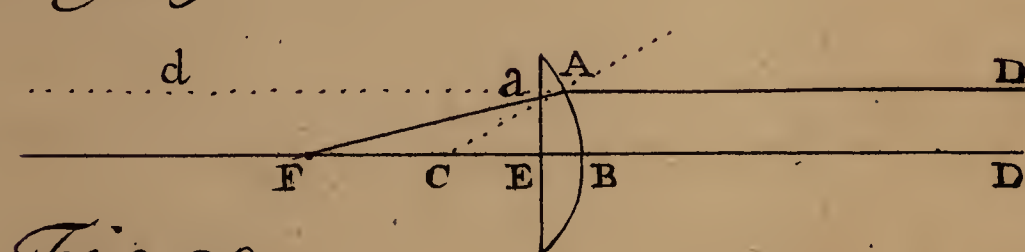


Fig.20.

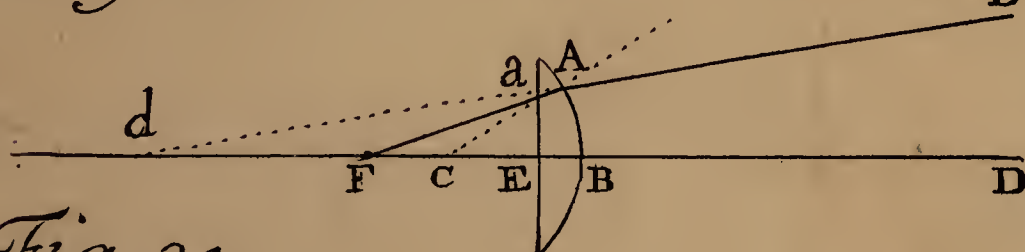


Fig.21.

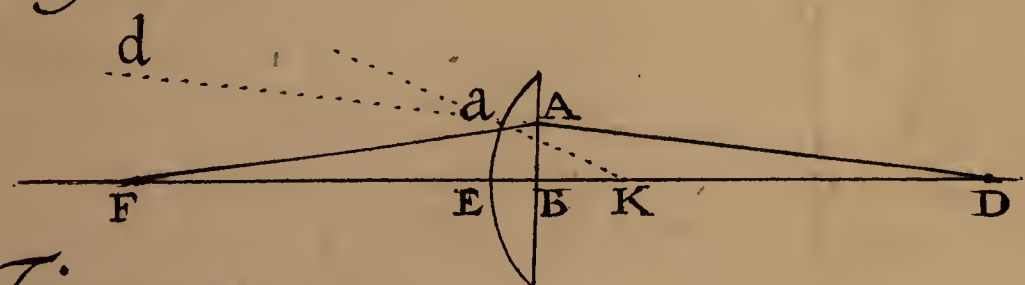


Fig.22.

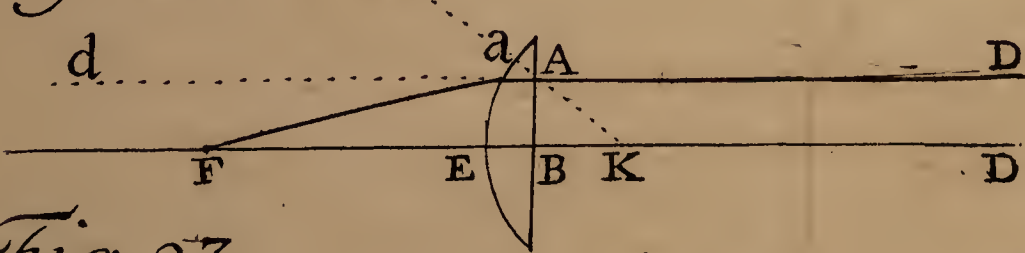


Fig.23.

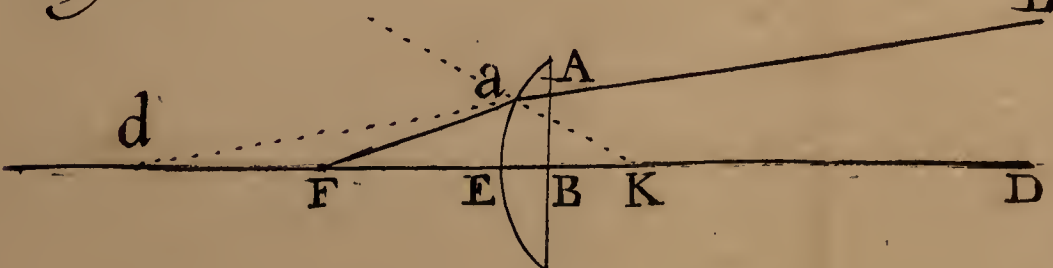


Fig.24 d

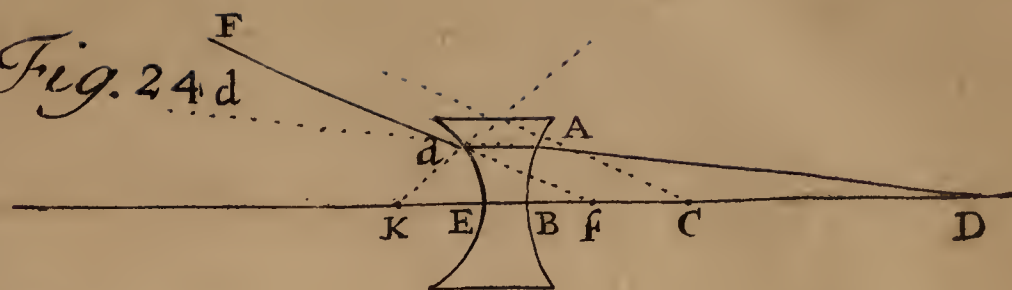


Fig.25.

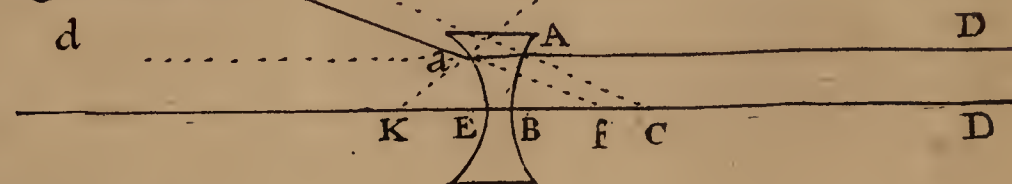


Fig.26.



Fig.27.

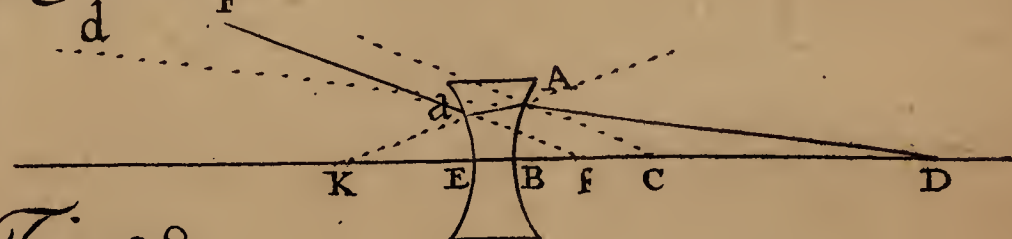


Fig.28.

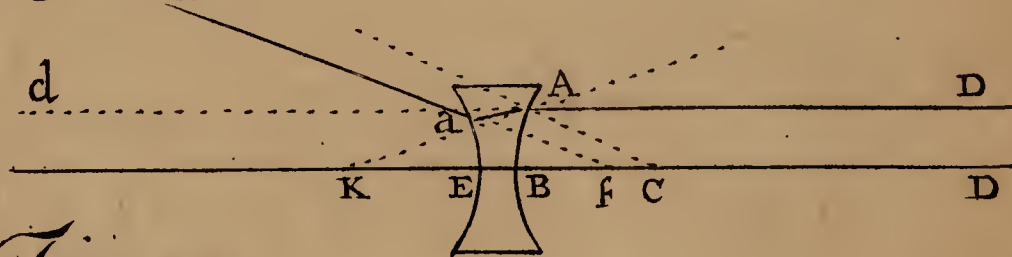


Fig.29.

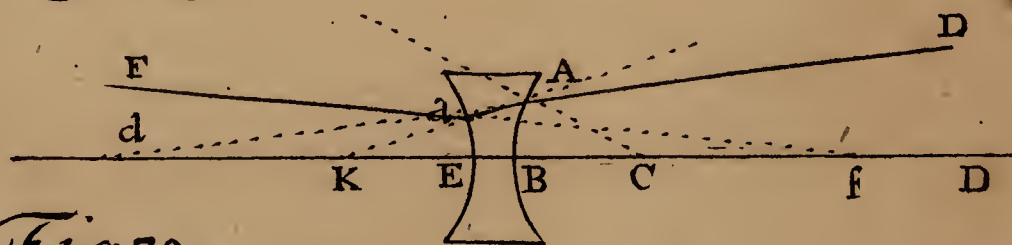


Fig.30.



Fig.31.

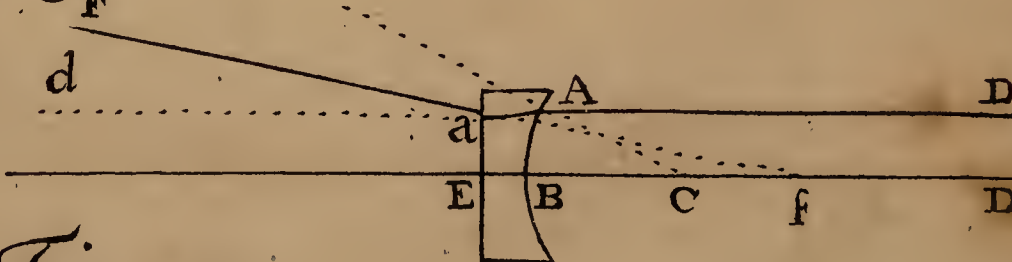
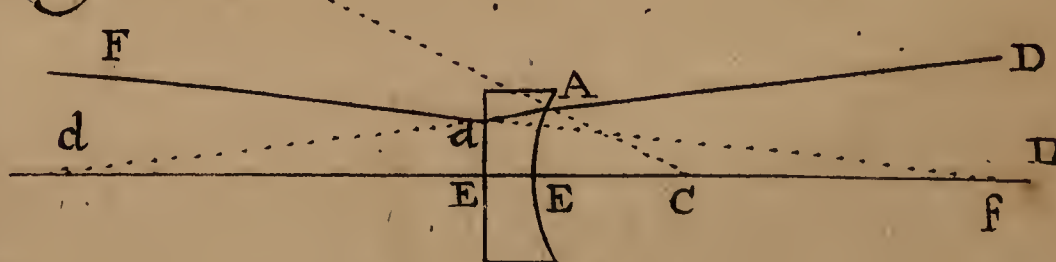
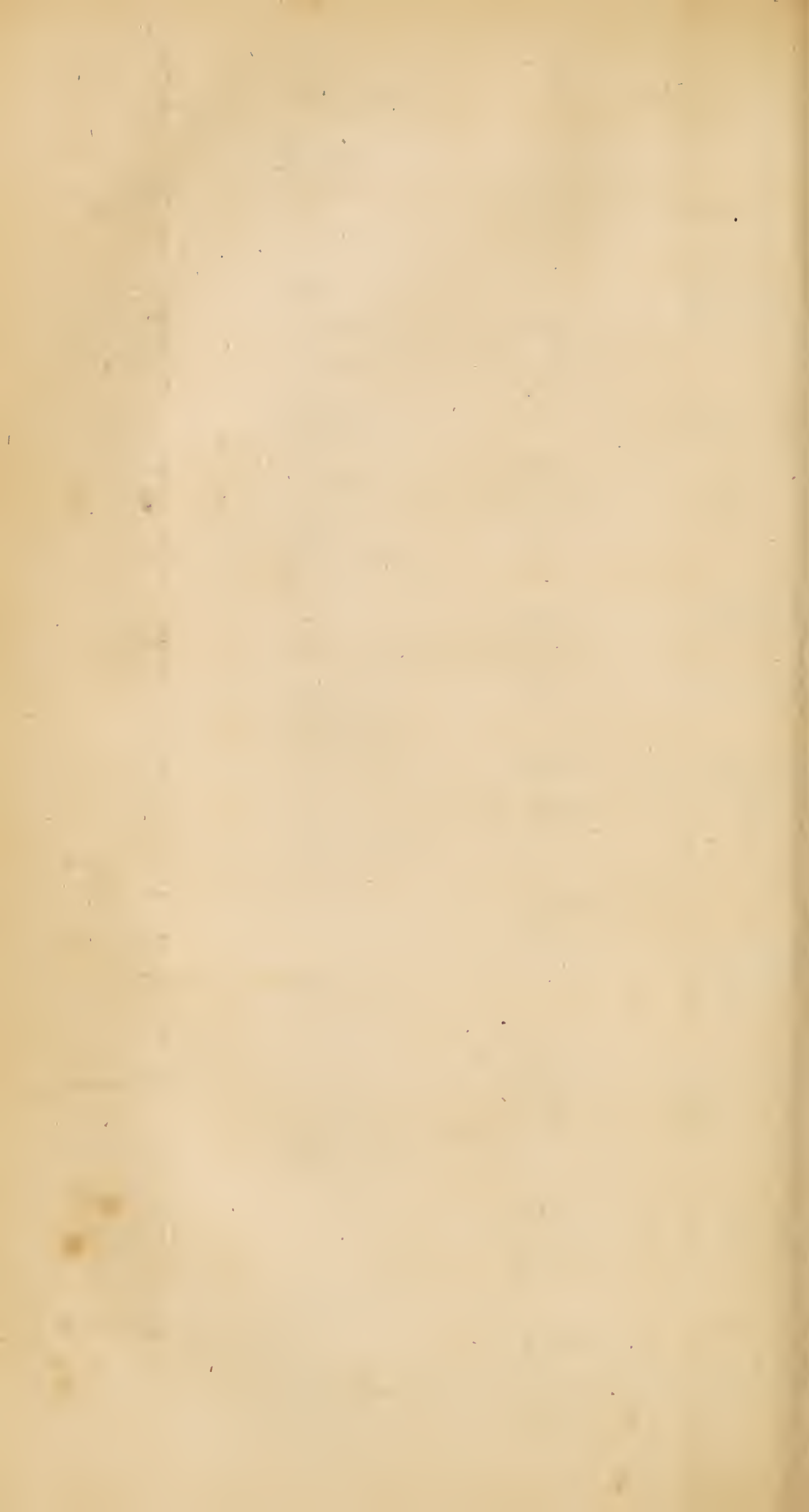


Fig.32.





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CASE III. For *Converging Rays*, $\frac{2dr}{2r-d}$
 $=f$. Theor. 45.

61. If the plain Surface be towards the Radiant, r will be infinite; and the Theorems will be for

CASE I. Of *Diverging Rays*, Fig. 33.
 Plate XIII. $\frac{-6dr-4tr}{3d+2t+6r} = f$. Theor. 46.

62. CASE II. Of *Parallel Rays*, Fig. 34.
 $\frac{6dr}{-3d} = -2r = f$. Theor. 46.

63. CASE III. Of *Converging Rays*, Fig. 35.
 $\frac{6dr-4tr}{-3d+2t+6r} = f$. Theor. 47.

64. If the Thickness t be neglected, these Theorems will become for

CASE I. Of *Diverging Rays*, $\frac{-2dr}{d+2r} = f$.
 Theor. 48.

CASE II. Of *Parallel Rays*, $\frac{-2dr}{d} =$
 $-2r = f$. Theor. 49.

CASE III. Of *Converging Rays*, $\frac{2dr}{2r-d}$
 $=f$. Theor. 50.

For a MENISCUS.

65. If the Radii of the Surfaces are unequal, and the convex Side exposed to the Radiant, in which Case r will be negative, we shall have the Equation for

CASE I. Of *Diverging Rays*,
 $\frac{-6dr + 2dt - 4tr}{3dr - 3dr - dt + 2rt + 6rr} = f$. Theor. 51.

66. CASE II. Of *Parallel Rays*,
 $\frac{-6rr + 2tr}{3r - 3r - t} = f$. Theor. 52.

67. CASE III. Of *Converging Rays*,
 $\frac{6dr - 2dt - 4tr}{-3dr + 3dr + dt + 2rt + 6rr} = f$.
 Theor. 53.

68. If the Thickness t be neglected, the Theorems will be for

CASE I. Of *Diverging Rays*, Fig. 36.
 $\frac{-2dr}{dr - dr + 2rr} = f$. Theor. 54.

69. CASE

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69. CASE II. Of *Parallel Rays*, Fig. 37.

$$\frac{-2rr}{r-r} = f. \text{ Theor. 55.}$$

70. CASE III. Of *Converging Rays*, Fig. 38.

$$\frac{2dr}{dr-dr+2rr} = f. \text{ Theor. 56.}$$

71. If the *Radius* of Convexity be equal to the *Radius* of Concavity, viz. $r=r$; and the Thickness t neglected, the Theorems will be for

CASE I. Of *Diverging Rays*, Fig. 39.

$$\frac{-2dr}{2rr} = -d = f. \text{ Theor. 57.}$$

72. CASE II. Of *Parallel Rays*, Fig. 40.

$$\frac{-2rr}{0} = f. \text{ Theor. 58.}$$

73. CASE III. Of *Converging Rays*, Fig. 41.

$$\frac{2dr}{2rr} = d = f. \text{ Theor. 59.}$$

74. If the concave Side be exposed to the Radiant, and the Radii of the Surfaces unequal, r being in this Case negative, we shall have the Equation for

CASE

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CASE I. Of *Diverging Rays*,

$$\frac{-6 d r r - 2 d t r - 4 t r r}{-3 d r + 3 d r - d t - 2 r t + 6 r r} = f.$$

Theor. 60.

75. CASE II. Of *Parallel Rays*,

$$\frac{-6 r r - 2 t r}{-3 r + 3 r - t} = f. \text{ Theor. 61.}$$

76. CASE III. Of *Converging Rays*,

$$\frac{6 d r r + 2 d t r - 4 t r r}{3 d r - 3 d r + d t - 2 r t + 6 r r} = f. \text{ Theor. 62.}$$

77. If we neglect the Thickness t , we shall have for

$$\text{CASE I. Of } \textit{Diverging Rays}, \frac{-2 d r r}{d r - d r + 2 r r} = f. \text{ Theor. 63.}$$

$$78. \text{ CASE II. Of } \textit{Parallel Rays}, \frac{-2 r r}{r - r} = f. \text{ Theor. 64.}$$

79. CASE III. Of *Converging Rays*,

$$\frac{2 d r r}{d r - d r + 2 r r} = f. \text{ Theor. 65.}$$

80. If

80. If the Radius of Concavity be equal to the Radius of Convexity, and the Thickness neglected, we have for

CASE I. Of *Diverging Rays*, $\frac{-2 d r r}{2 r r} = -d = f.$ Theor. 66.

81. CASE II. Of *Parallel Rays*, $\frac{-2 r r}{0} = f.$ Theor. 67.

82. CASE III. Of *Converging Rays*, $\frac{2 d r r}{2 r r} = d = f.$ Theor. 68.

For a PLAIN LENS.

83. This being no other than a Piece of common plain Glass, whose Surfaces are parallel Planes, the Radii r and r being both infinite, the Equation in Art. 18, will become for

CASE I. Of *Diverging Rays*, Fig. 42.
 $\frac{6 d + 4 t}{-6} = d + \frac{2}{3} t = -f.$ Theor. 69.

84. CASE II. Of *Parallel Rays*, Fig. 43.
 $\frac{6 d}{0} = f.$ Theor. 70.

85. CASE

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85. CASE III. Of *Converging Rays*, Fig. 44.

$$\frac{-6d + 4t}{-6} = d - \frac{2}{3}t = f. \text{ Theor. 71.}$$

86. If the Thickness t be neglected, the Theorems will be for

CASE I. Of *Diverging Rays*, $\frac{6d}{-6} = -d = f. \text{ Theor. 72.}$

87. CASE II. Of *Parallel Rays*, $\frac{6d}{0} = f. \text{ Theor. 73.}$

88. CASE III. Of *Converging Rays*, $\frac{-6d}{-6} = d = f. \text{ Theor. 74.}$

89. Thus much for finding the *Foci* of Rays after Refraction. If the Focus be given with either the Distance of the Radiant, Radii of the Convexities or Concavities, or the Ratio of Refraction, the rest may be found by Theorems raised from the same fundamental Equation in Art. 16, which, if t be rejected as inconsiderable, will stand thus,

$$\frac{pdr}{dr + dr - pr} = f. \text{ And when reduced}$$

 we shall have $pdr = drf + drf - prf.$

90. If

90. If now f, r, r, p , are given, d will be thus found, $p r r f = d r f + d r f - p d r r$; and therefore it will be $\frac{p r r f}{r f + r f - p r r} = d$.

Theor. 76.

91. If f, r, d , and p be given, r will be found from this Equation $p r r f + p d r r - d r f = d r f$; for then $\frac{d r f}{p r f + p d r - d f} = r$. Theor. 77.

92. If r be required, the Equation will be $p r r f + p d r r - d r f = d r f$; and so $\frac{d r f}{p r f + p d r - d f} = r$. Theor. 78.

93. If f, r, r , and d be given, the Equation for p will be $p r r f + p d r r = d r f + d r f$; and hence we shall have $\frac{d r f + d r f}{r r f + d r r} = p$. Theor. 79.

94. Having found p , the Ratio of the Refraction is easily known; for $p = \frac{R}{I - R}$, and therefore $I p - R p = R$, and so $I p = R + R p$; whence $I : R :: p + I : p$.

95. If

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95. If the Lens be a double Convex of Glass, then will be $p = 2$, and if $r = r$; then the Theorems for d and r will be as follows.

For the Distance, $\frac{rf}{f-r} = d$. Theor. 80.

For the Radius $\frac{df}{f+d} = r$. Theor. 81.

96. If the Lens be a double equally Concave, then we have for the Distance, $\frac{rf}{r-f} = d$. Theor. 82.

And for the Radius $\frac{-df}{f+d} = r$. Theor. 83.



C H A P. II.

*The Theory of DIOPTRICS continued,
for determining the mutual Proportion
of the Object and Image.*

1. **L**ET LN be a double equally convex Lens, Plate XIII, AC a it's Axis, OB an Object in a Position perpendicular thereto.

2. From the Extremity of the Object O, suppose a Ray OF fall upon the Lens in such a Point F, that by it's Refraction in the Glass, it be made to pass through the middle Point of the Lens C to G; at G it will be so refracted to I, that the Ray GI shall be parallel to OF. This Ray OCI will be the Axis of all the Rays which fall on the Lens from the Point O, and I will be the Focus where they will all be collected, by what has been before taught.

3. In like Manner BHCKM is the Axis of that Pencil of Rays, which proceed from the Extremity of the Object B, and their Focus, suppose at M. Then since all the Points in the Object between O and B, must necessarily

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rily have their *Foci* between I and M, IM will consequently be the *Image* of the *Object* OB, *per* Princip. 10 and 11.

4. From the Vertices of the Lens D and E draw the Lines DO, DB, and EI, EM, and produce the incident Ray OF, till it meet the Axis in d, and the refracted Ray IG, till it cuts the Axis in e; now since Od and Ie are both in the same Plane, and parallel to each other, from the Nature of Refractions, the Angle O d A will be equal to the Angle I e a; and since the Points d and e are both within and near the middle of the Lens, it is evident that if the Thickness of the Lens be very small or neglected, the Angle O d A will become equal to ODA, and I e a equal to IEa; and consequently the Angle ODA will be equal to the Angle IEa. In like Manner it is shewn the Angle BDA = MEa. And therefore the whole Angle ODB = MEI. From whence it follows, *That the Image IM appears from the Vertex of Emerision E, under an Angle equal to that under which the Object appears from the Vertex of Incidence D.*

5. If the Object OB be very small, or at a great Distance, so that the Points O, A, B, are nearly at an equal Distance from the Vertex D; then will their correspondent Points I, a, M, be very nearly in the same Plane; and the Image IM parallel to the Object OB, and perpendicular to the Axis Aa.

6. In

6. In this Case, the Triangles ODA and IEa are similar; for they are equiangular, since the Angle $D = E$, and $A = a$, as being both right ones, and therefore $O = I$; and consequently $DA : Ea : AO : Ia$; for the same Reason $DA : Ea :: AB : aM$. But $DA : Ea :: OA + AB : Ia + aM :: OB : IM$. That is, *The Distance of the Object is to the Distance of the Image, as the Length of the Object to the Length of the Image.*

7. Also $AO : AB :: Ia : aM$; that is, *The Object and it's Image are divided proportionally by the Axis of the Lens Aa .*

8. Call the Object O , it's Image I ; the Distance of the Object and it's Image, d and f , as before; then will it be $O : I :: d : f$, and

therefore $\frac{Id}{O} = f$. But for the Case of a

double and equally convex Lens, whose Thickness is inconsiderable, we before found $f =$

$\frac{dr}{d - r}$, Chap. I. Theor. 16. Therefore $\frac{Id}{O}$

$= \frac{dr}{d - r}$; and so $Id(d - r) = drO$, that

is, $Id - Ir = rO$, and $Id = rI + Or$;

and consequently $\frac{rI + Or}{I} = d$. Theor. 1.

9. If the Object be a *Surface*, the Image will be so too; and their Proportion duplicate of the former; that is, $O : I :: d^2 : f^2$; and

F

hence

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hence $\sqrt{\frac{I d^2}{O}} = f = \frac{d r}{d - r}$; and squaring

each Side, we have $\frac{I d^2}{O} = \frac{d^2 r^2}{d - r} 2$, which re-

solved is, $I \times \frac{d^2}{d - r} = O \times r^2$; hence $\frac{d^2}{d - r} = \frac{O}{I} \times r^2$; put $\frac{O}{I} = p$, then $\frac{d^2}{d - r} = p r^2$, and $d - r = \sqrt{\frac{d^2}{p r^2}}$; and lastly, $d = \sqrt{p r^2} + r$. Theor. 2.

10. If the Object be a *Solid*, the Image will be so; and then it will in like Manner appear (the Proportion being triplicate of the first, viz. $O : I :: d^3 : f^3$) that $d = \sqrt[3]{\frac{O}{p r^3}} + r$. Theor. 3.

11. These three Theorems find the Distance d , at which an Object (whether *Line*, *Surface*, or *Solid*) must be placed that it may bear the same Proportion to it's Image, as O does to I , in Case we use a *double and equally convex Lens*.

12. But the same thing is found for any other Lens, as I shall shew for a *Plano-Convex*, *Double* and *Plano-Concaves*, as follows.

13. In Chap. I. Theor. 22. For a *Plano-*

Convex we had $\frac{2 d r}{d - 2 r} = f$. Therefore $\frac{I d}{O} = 2 d r$

$$= \frac{2dr}{d-2r}; \text{ and consequently } Id - 2Ir = 2rO, \text{ and } Id = 2rO + 2Ir; \text{ and hence}$$

$$\frac{2rO + 2Ir}{I} = d. \text{ Theor. 4.}$$

14. If the Object be a Superficies, then

$$\frac{Id^2}{O} = \frac{4d^2r^2}{d-2r}2, \text{ and } I \times \frac{d^2}{d-2r} = 4r^2 \times O;$$

that is, $\frac{d^2}{d-2r} = 4r^2 \times \frac{O}{I} = 4r^2p;$

wherefore $d - 2r = \sqrt{4r^2p};$ and $d = 2r + \sqrt{4r^2p}.$ Theor. 5.

15. In the same Manner, for Solids, we shall have $d = 2r + \sqrt[3]{8r^3p}.$ Theor. 6.

16. For a double and equally concave Lens, the Theorem was $\frac{dr}{d-r} = f.$ See Theor.

37. And since in this Case $O : I :: d : -f;$ we shall have $\frac{Id}{-O} = f = \frac{dr}{d-r},$ and so $Id + Ir = rO;$ and therefore $Id = Or - rI;$ and consequently $d = \frac{Or - rI}{I}.$

Theor. 7.

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17. For a Superficies it will be $d = \sqrt{r^2 p} - r$. Theor. 8.

18. And for a Solid, $d = \sqrt[3]{p r^3} - r$. Theor. 9.

19. In Case of a *Plano-Concave*, it will be also $\frac{Id}{-O} = f = \frac{-2dr}{d + 2r}$, (per Theor. 43.) and therefore $Id + 2Ir = 2rO$; and hence $d = \frac{2rO - 2rI}{I}$. Theor. 10.

20. For a Superficies, $d = \sqrt{4r^2 p} - 2r$. Theor. 11.

21. For a Solid, $d = \sqrt[3]{8r^3 p} - 2r$. Theor. 12.

22. In a *Meniscus*, whose Radii of Convexity and Concavity are unequal, and the convex Side exposed to the Radiant; we

have $\frac{Id}{O} = \frac{-2dr}{dr - dr + 2rr}$, (per Theor. 54.)

whence we find $d = \frac{2rrO + 2rrI}{rI - rI}$. Theor.

13.

23. If

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23. If the concave Side be exposed to the Radiant, we have $d = \frac{2rrO + 2rrI}{rI - rI}$.

Theor. 14.

24. If the Radius of Convexity be equal to that of the Concavity, we shall have (*per*

Theor. 57 and 66.) $\frac{Id}{O} = d$; so that $Id =$

$O d$, that is, $I = O$. Theor. 15. Which shews, that in this Case a Meniscus Lens can neither magnify nor diminish an Object.

25. In all the foregoing Theorems we had Regard only to *affirmative Focus's*; but they may be as easily adapted to *negative ones*, if it be required to have the Image on the same Side with the Object. Thus for a *double equally convex Lens*, for an affirmative Focus,

the Theorem is $\frac{dr}{d-r} = +f$; but for a ne-

gative Focus, it will be $\frac{dr}{r-d} = f$; also it

will be $O : I :: d : -f$; and therefore $\frac{Id}{O}$

$= \frac{dr}{r-d}$; from whence we have $d = \frac{Ir - Or}{I}$.

Theor. 16.

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26. In this Manner Theorems are raised for other Glasses, where it is to be observed, that the Terms of the Equations are the same as for *affirmative Focus's*, the *Signs* only are changed: But this is a Matter not worth insisting on, being of little Use.

27. If it be required to form an Image equal to the Object, we shall have $O = I$; and the Distance required will be for the several Lenses, as follows.

28. For a double and equally convex Lens, we have $\frac{Ir + Or}{I} = \frac{2Ir}{I} = 2r = d.$

Theor. 17.

29. For a *Plano-Convex*, $\frac{2rO + 2Ir}{I} = \frac{4Ir}{I} = 4r = d.$ Theor. 18.

30. For a double and equally concave Lens, we have $\frac{Or - Ir}{I} = \frac{O}{I} = d.$ Theor. 19.

And since in this Case the Focus is always negative, it is evident the Image can never be equal to the Object at any Distance from the Lens, much less can it ever exceed it; and therefore

therefore such a Lens can only diminish, never magnify an Object.

31. The same thing is true, and for the same Reason, in a Plano-Concave, as will appear from Theor. 10 hereof; and also in a Meniscus from Theor. 13 and 14.

32. If the Distance d of the Object, and the Proportion of O to I be given, the Radius r of double and equally convex and concave Lenses, in this Case necessary, is found from the Equation in Art. 8, *viz.* $Id = Ir + Or$;

and therefore for a *double Convex* $\frac{Id}{I + O} = r$.

Theor. 20.

33. For a *Plano-Convex*, the Equation is $Id = 2rO + 2Ir$, and therefore $\frac{Id}{2O + 2I} = r$. Theor. 21.

34. For a double and equally concave Lens, the Equation being $Id = Or - Ir$; we have $\frac{Id}{O - I} = r$. Theor. 22.

35. For a Plano-Concave, the Equation is $Id = 2rO - 2Ir$, and so $\frac{Id}{2O - 2I} = r$. Theor. 23.

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36. If the Distance d of the Object, and the Radius r of the Convexity or Concavity of the Lens be given; the Proportion of the Object to the Image, *viz.* $O : I$, is easily found as follows.

37. For a double and equally convex Lens, the Equation is $Id - Ir = Or$; whence this Analogy, $d - r : r :: O : I$. Theor. 24.

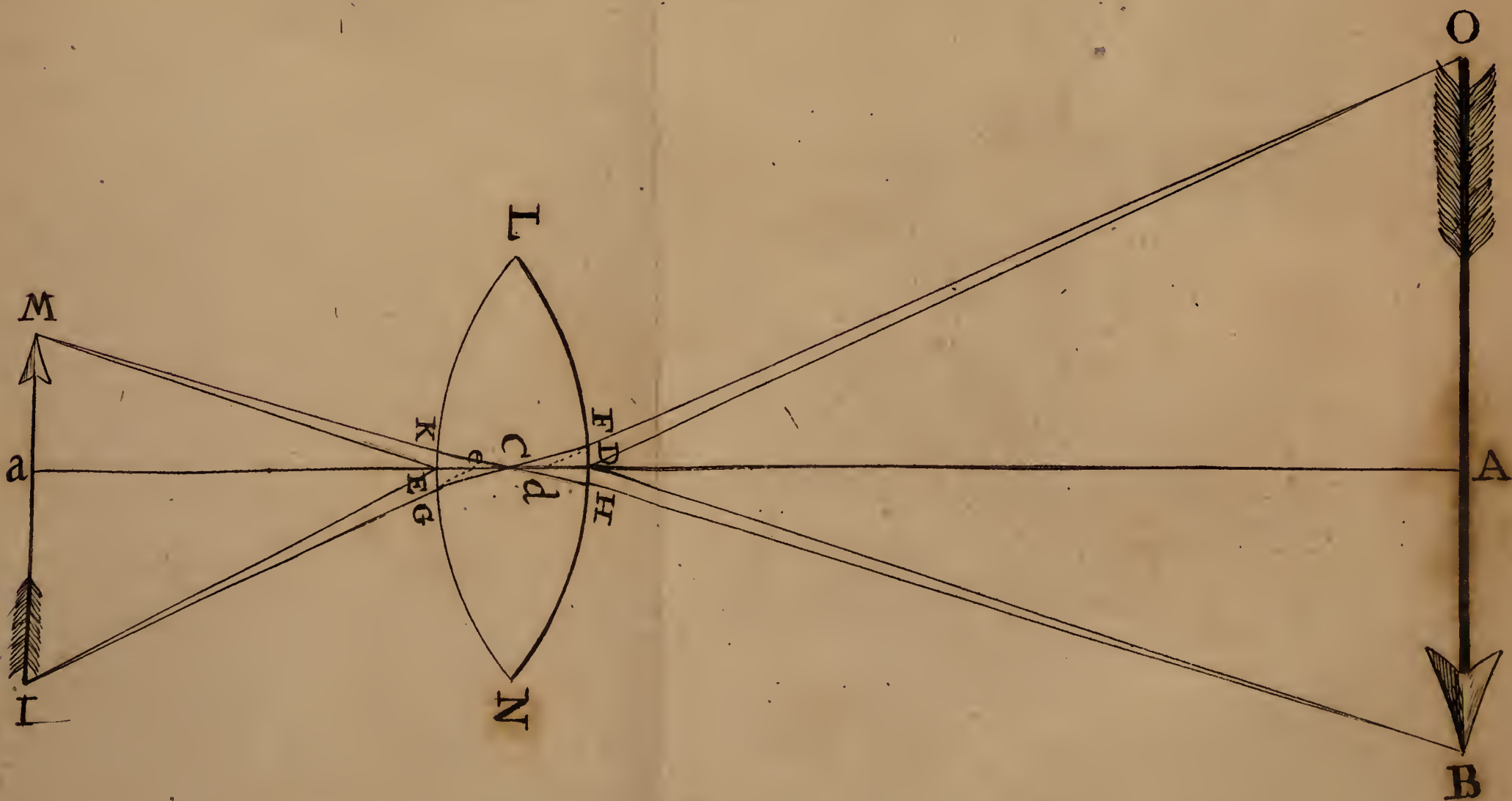
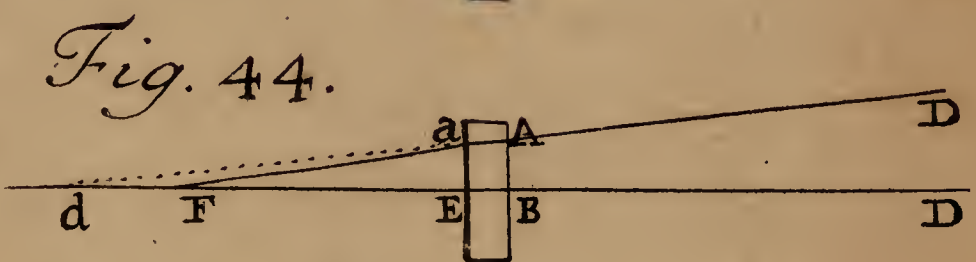
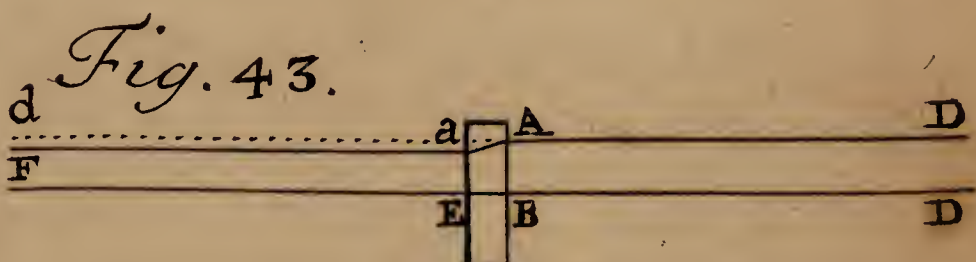
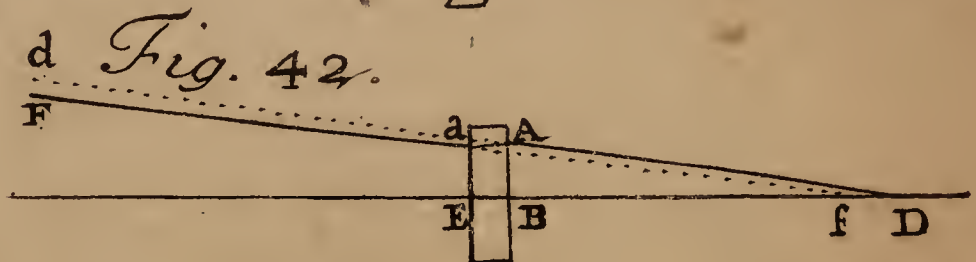
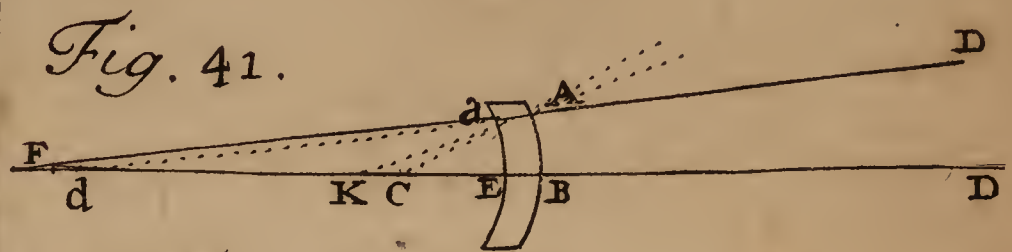
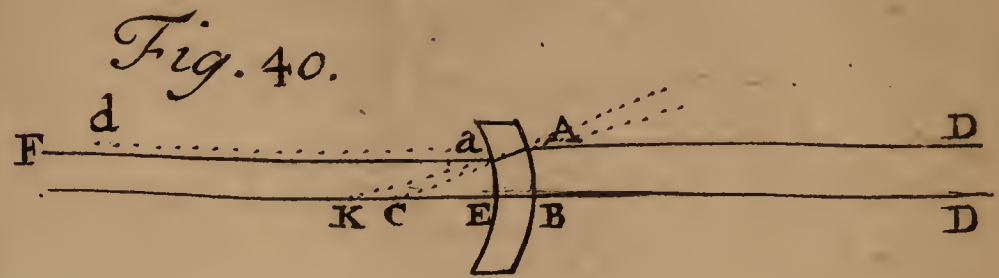
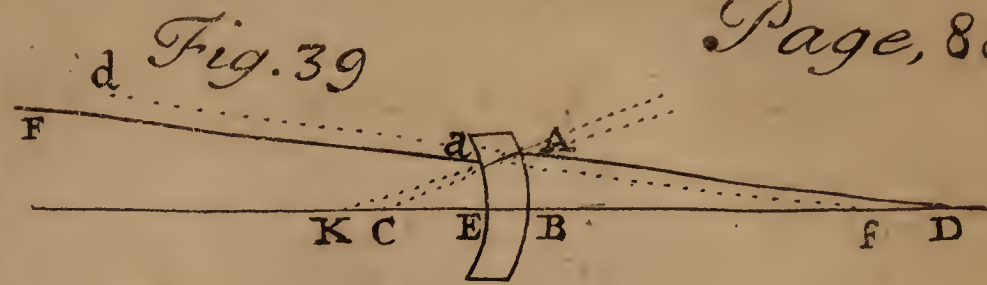
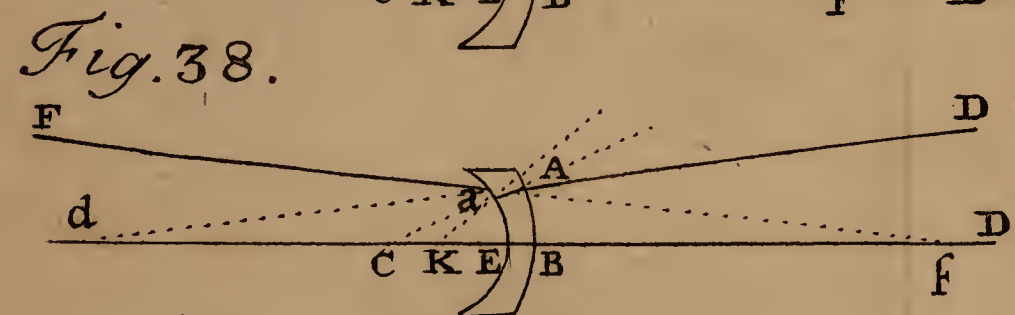
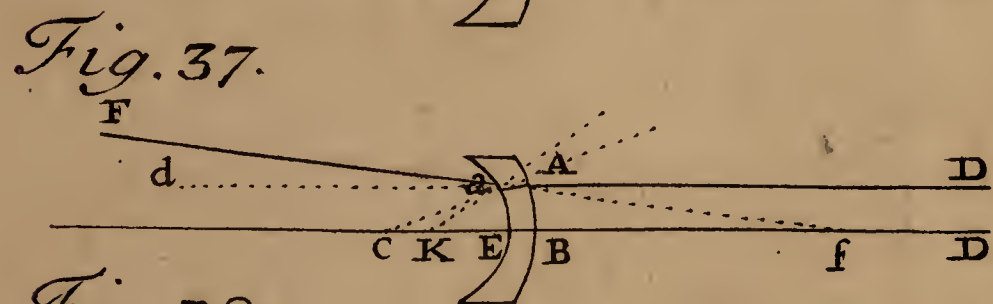
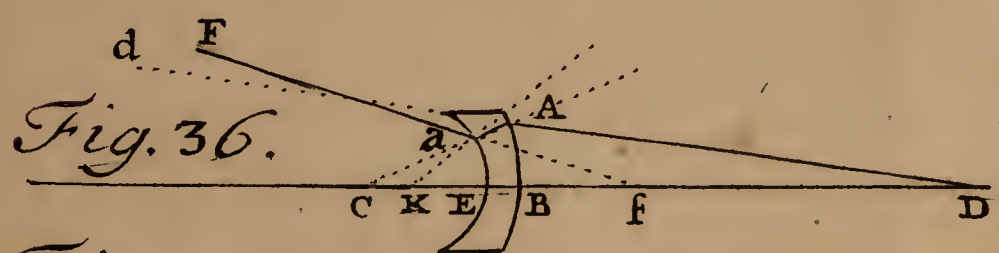
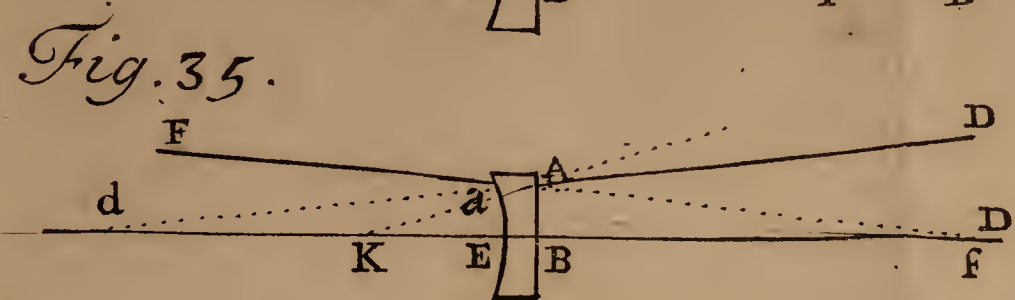
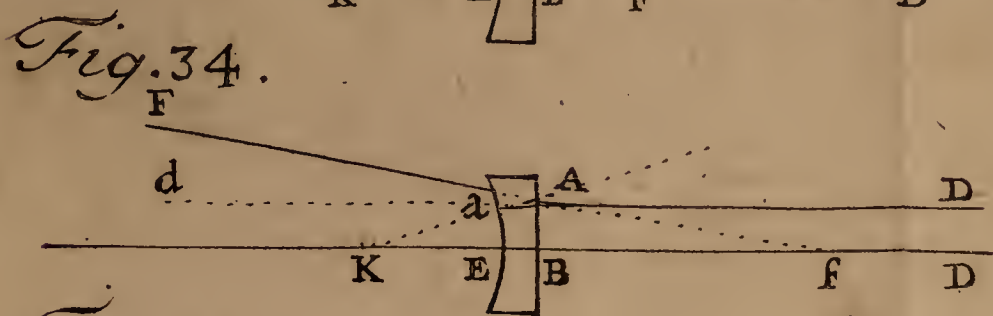
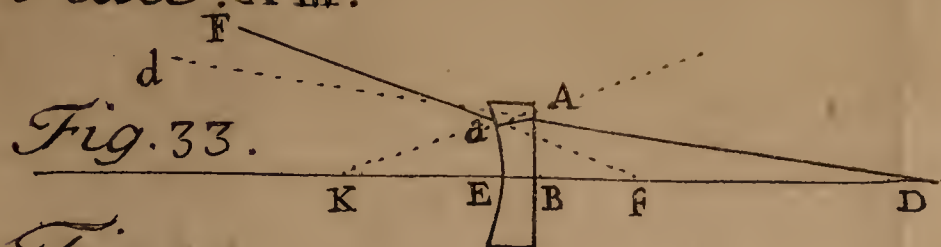
38. For a Plano-Convex it will be $Id - 2Ir = 2rO$; and so $d - 2r : 2r :: O : I$. Theor. 25.

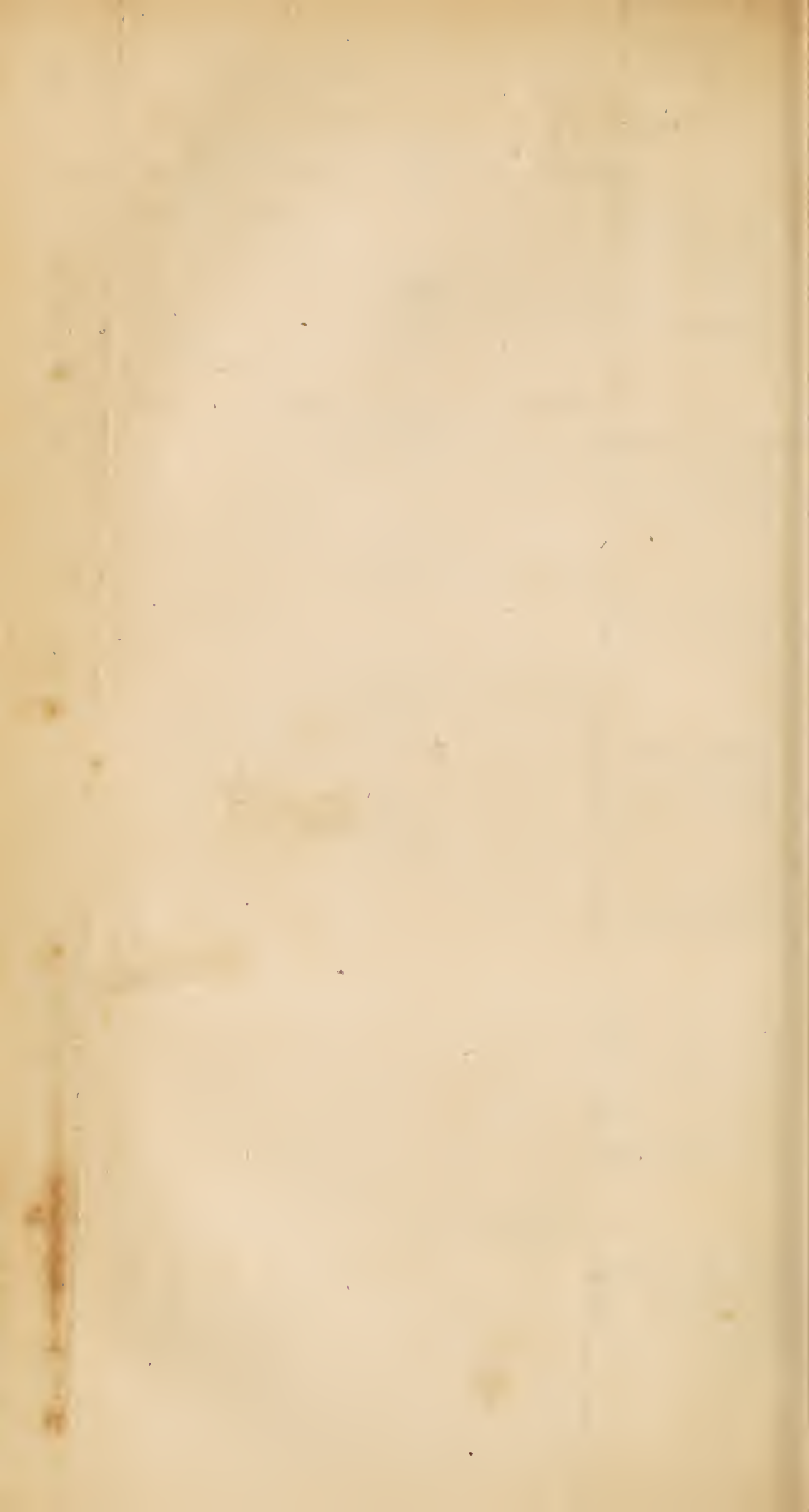
39. For a double and equally concave Lens, the Equation is $Id + Ir = Or$, and therefore $d + r : r :: O : I$. Theor. 26.

40. For a Plano-Concave it is $d + 2r : 2r :: O : I$. Theor. 27.

41. For a *Plain Lens*, or Glass, we have $\frac{Id}{O} = d + \frac{2}{3}t$, *per* Theor. 69; and therefore $Id = dO + \frac{2}{3}Ot$, and consequently $O : I :: d : d + \frac{2}{3}t$. Theor. 28.

42. It is in vain to raise Theorems for *Meniscus*, and other Sorts of Lens, or for any other





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other Matter besides Glas; for this would be to launch out into a boundless Ocean, that would afford more of Curiosity and Speculation than real Use; which shall be a Task therefore left for the young Optician's own Amusement, whilst I go on to reduce the foregoing Theorems to plain and practical Rules.



CHAP.

C H A P. III.

*The Theorems relating to GLOBES and
HEMISPHERES reduced to practical
Rules, and exemplified.*

I. **A**S in the first Part, so here I shall
make use of the *Tenths* of an Inch,
for the Measurements to be made in the en-
suing Examples. And first, for

A GLOBE.

2. CASE I. For *Diverging Rays*, Plate XIV.
Scheme I. Let there be given the Distance
of the Radiant D E, and the Radius of the
Globe C E, to find the Distance of the Fo-
cus I F.

R U L E.

Multiply the Distance of the Radiant by
the Radius, and to that Product add four
times the Square of the Radius; divide that
Sum by the Difference between twice the
Distance of the Radiant, and the Radius, the
Quotient will be the Distance of the Focus,
per Theor. 1. Art. 20. Chap. I.

3. E X-

3. E X A M P L E.

Let DE = 30 CE = 10 <hr style="width: 50px; margin-left: 0;"/> Product 300 Add 400 <hr style="width: 50px; margin-left: 0;"/>	CE = 10 CE = 10 <hr style="width: 50px; margin-left: 0;"/> Squ. 100 <hr style="width: 50px; margin-left: 0;"/> 4 <hr style="width: 50px; margin-left: 0;"/> 400 <hr style="width: 50px; margin-left: 0;"/>	2 DE = 60 CE = 10 <hr style="width: 50px; margin-left: 0;"/> Diff. 50 <hr style="width: 50px; margin-left: 0;"/>
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Then 50) 700 (14 = IF, the focal Distance required, *viz.* one Inch, and four Tenths.

50

200

200

...

4. As 2DE is greater, equal to, or less than CE; the Focus F will be affirmative, or behind the Globe, infinite, or negative, or before the Globe.

5. CASE II. Of *Parallel Rays*, Scheme 2. A Glass Globe exposed to Parallel Rays, will collect and converge them to a Point F, which will be just half the Radius of the Globe CI behind it, by Theor. 2. Such a Globe therefore, held in the Sun's Rays, will burn very intensely, if large.

6. CASE III. Of *Converging Rays*. The Focus F is found here by the same Rule as in CASE I; only the *Sum* and *Difference* there must

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there must be exchanged for the *Difference* and the *Sum* here. And according as the Product of the Distance of the Radiant Point, into the Radius CE, is greater, equal to, or less than 4 times the Square of the Radius, the Focus will be affirmative, within the Globe, or negative, *per* Theor. 3.

For an HEMISPHERE.

7. CASE I. Of *Diverging Rays*. If the *Convex Part* be towards the Radiant, the Focus will be found by this

R U L E.

To the Product of the Distance of the Radiant into the Radius, add the Square of Radius, multiply that Sum by 4: This divide by the Difference between 3 Times the Distance, and 6 Times the Radius, the Quotient will give the Distance of the Focus, *per* Theor. 4.

8. But if the *Plain Side* of the Hemisphere be turn'd towards the Object the Radiant, the Rule will be somewhat alter'd; thus,

R U L E.

R U L E.

To 6 times the Product of the Distance into the Radius, add 4 times the Square of Radius; divide that Sum by the Difference between 3 Times the Distance, and 4 Times the Radius; the Quotient is the Distance of the Focus required, *per* Theor. 7.

9. E X A M P L E

For the Convex Side towards the Radiant.

Let the Di-	}	30	3 Times the Di-	}	90
stance be —			stance is —		
The Radius		10	6 Times the Ra-	}	60
		—	dus is — —		
Product		300			
Add the Squ.	}	100			
of Radius					
		—			
Sum		400			
Multiply by 4		—			

Difference 30

Divide by 30)	1600	(53 = the Distance of
	150	the Focus, <i>viz.</i>
	—	5 $\frac{3}{10}$ Inches.
	100	
	90	
	—	
	10	

10. E X-

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10. E X A M P L E

Of the Plain Side towards the Radiant.

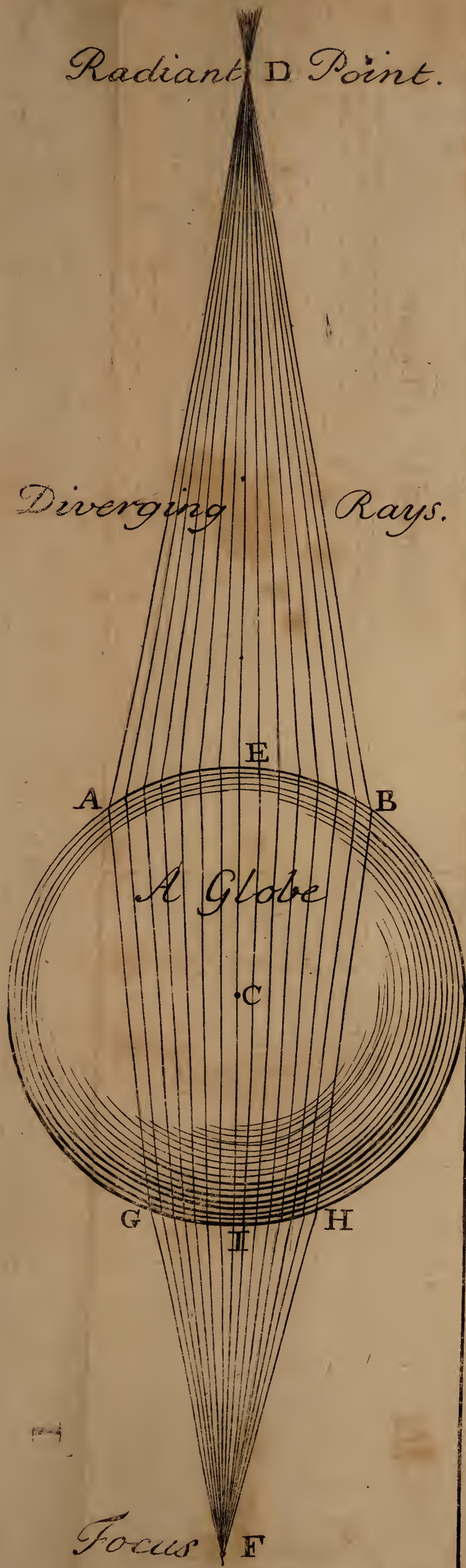
Let the Di-	}	30	3 Times the Di-	}	90
stance be			stance — —		
The Radius —	}	10	4 Times the	}	40
			Radius — —		
Product		300			
Multiply by 6					Difference 50
Product		1800			
Add 4 times	}	400			
the Sq. of R.					
Divide by 50		2200	(44 = the Distance of		
		200	the Focus, viz.		
		200	4 $\frac{4}{10}$ Inches.		
		200			
		...			

11. In this near Situation of the Radiant, the convex Side, exposed to it, gives the greatest focal Distance, as we see by the Examples; but if the Radiant be above 50, or 5 Inches distant from the same Hemisphere, then the convex Side will give the shortest focal Distance, as is easy to try.

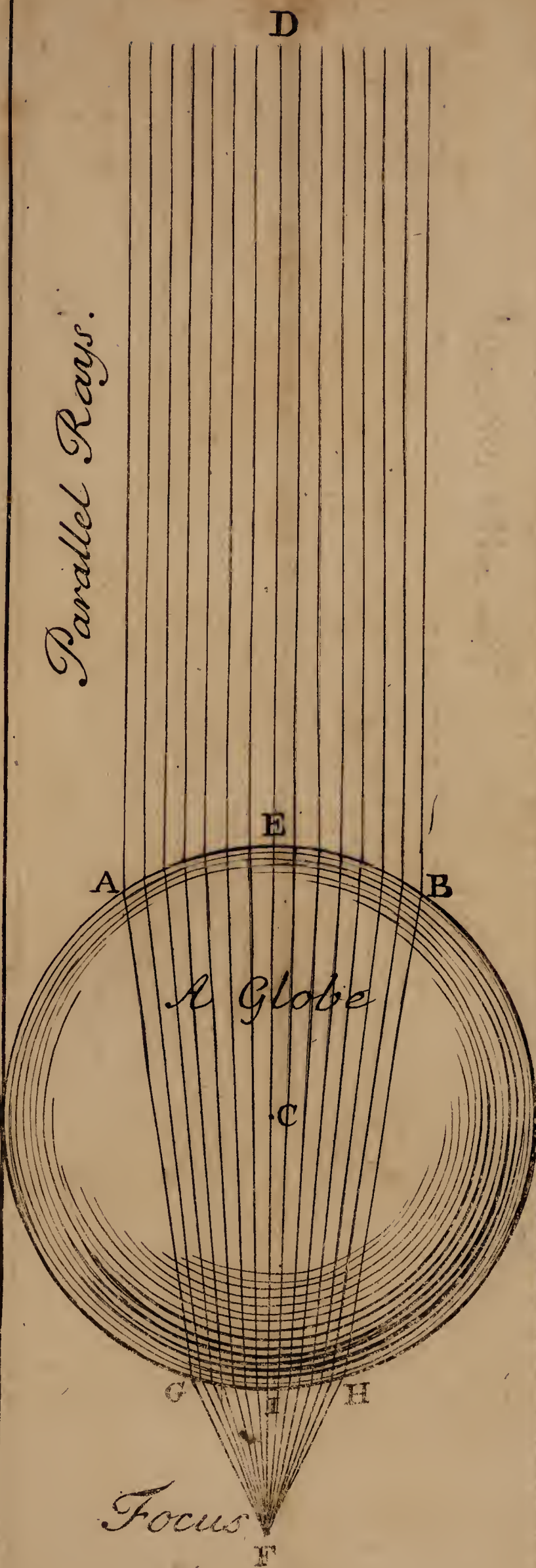
12. CASE II. Of *Parallel Rays*, Scheme 3. If the *Plain Side* be turn'd to the Rays of the Sun; they will be collected at F, at the Distance

Radiant D Point.

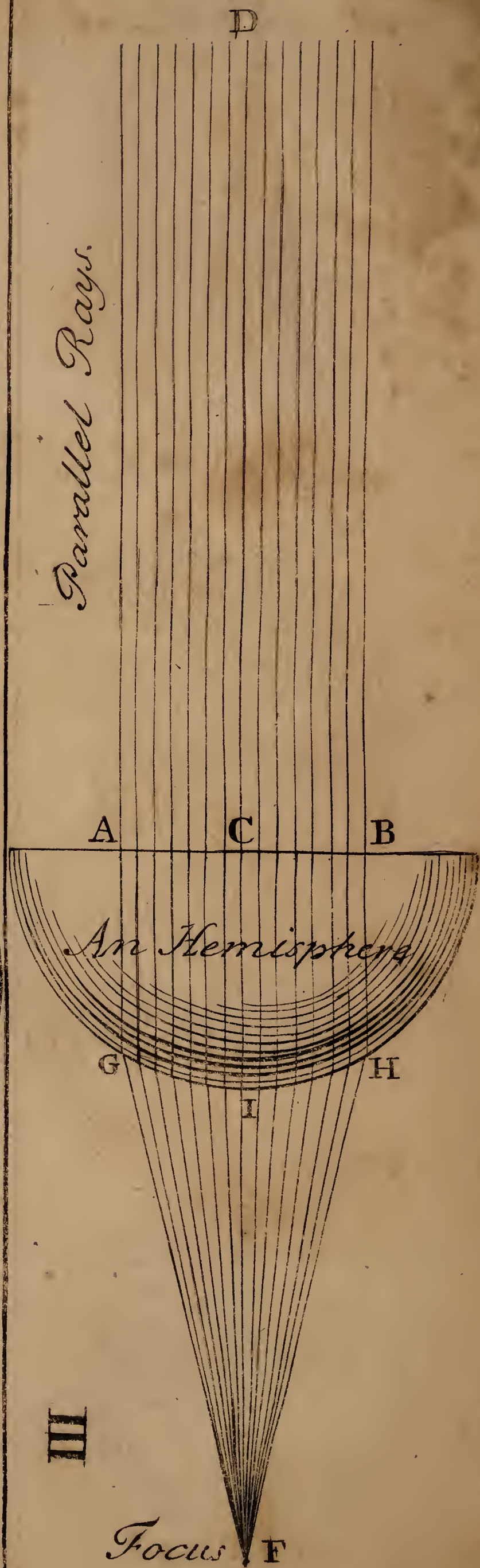
Diverging Rays.

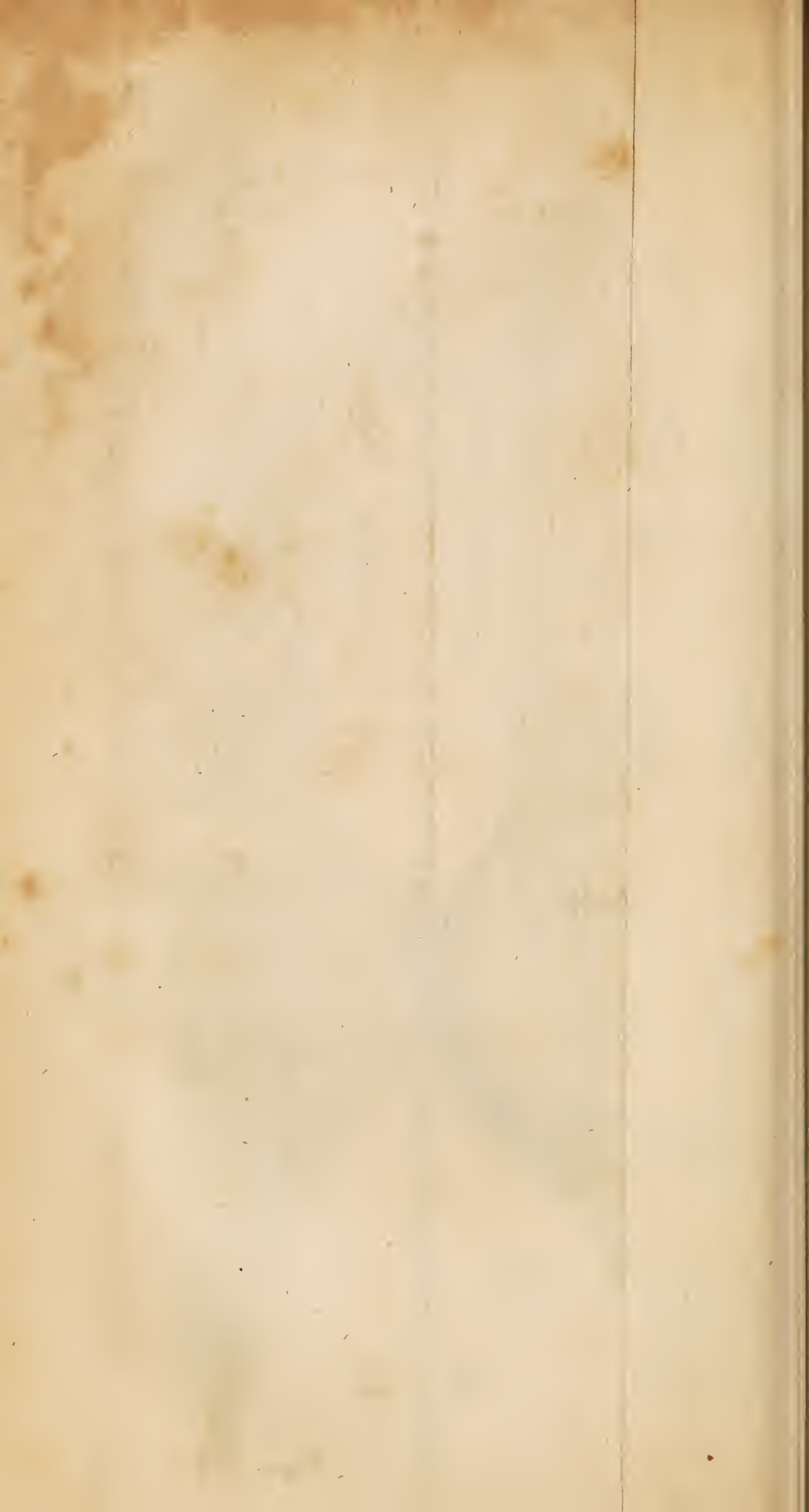


Parallel Rays.



Parallel Rays.





Distance IF, which will be just 2 IC , that is, twice the Radius of the Hemisphere, *per* Theor. 8. But if the convex Side be towards the Sun, the Distance IF will be but $1\frac{1}{3}$ of IC; that is, the focal Distance in the latter Case will be $\frac{2}{3}$ of IC, or the Radius, shorter than in the former, *per* Theor. 5.

13. Therefore the greatest Effect of *burning* by an Hemisphere, is with the convex Part towards the Sun; and also the greatest Effect of *magnifying* an Object seen through it, is with the same Part towards the Object. x

14. CASE III. Of *Converging Rays*. The Rule for finding the Focus here, is the same as for *Diverging Rays*, if for the *Sum* and *Difference* there, we take the *Difference* and the *Sum* of the same Quantities here, as was before observed of the Globe in the same Case.



CHAP. IV.

The Rules for finding the Focus of a DOUBLE CONVEX LENS.

1. **I**F the Lens be *unequally Convex*, and the Thickness not regarded, then if the Radius of each Surface be known, and the Distance of the Radiant from the Lens, the Distance of the Focus will be found for

2. CASE I. Of *Diverging Rays*, by the following

R U L E.

Multiply twice the Product of the Radii into each other by the Distance of the Radiant, this shall be the *Dividend*. Then take the Difference between the Product of the Sum of the Radii, multiplied by the Distance, and twice the Product of the Radii into each other; by this divide the Dividend above, the Quotient will be the Distance of the Focus required, *per Theor. 13.*

3. EX-

3. EXAMPLE.

Let the longest Radius be 50, and the shortest 30; and the Distance of the Radiant 300: Then

Longest Radius	—	—	50
Shortest	—	—	30
Product	—	—	1500
Multiply by	—	—	2
Twice Product	—	—	3000
Distance	—	—	300

$$\begin{array}{r}
 21000 \overline{) 900000} \quad (42.8 \\
 \underline{84000} \\
 60000 \\
 \underline{42000} \\
 180000 \\
 \underline{168000} \\
 12000
 \end{array}$$

Longest Radius	50
Shortest	30
Sum	80
Distance	300
Product	24000
Subduct	3000, twice Prod. of Radii
Difference	21000 Divisor.

Thus the focal Distance will be $42 \frac{8}{10}$.

G

4. CASE

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4. CASE II. Of *Parallel Rays*. The Focus of these is found thus.

R U L E.

Divide twice the Product of the unequal Radii into each other, by the Sum of the Radii, the Quotient will be the focal Distance required, *per* Theor. 14.

5. E X A M P L E.

Let the Radii be the same as before ;

Then the Longest Rad.	50	—	—	50
The Shortest	—	—	—	30
				30
Product	—	—	—	1500
				80
				Sum

80)	3000	(375 = the focal
	240	Distance re-
	600	quired, viz.
	560	37 $\frac{1}{2}$.
	400	
	400	
	...	

6. CASE III. Of *Converging Rays*. The Focus of these is found by the Rule of the first Case,

Case, if instead of the *Difference* we take the *Sum* of those Quantities mentioned in the Divisor, *per* Theor. 15.

7. If the Lens be *equally Convex*, as are most of those in common Use, and the Thickness neglected as inconsiderable, and causing no material Error, then will the Focus of this most useful Lens be most easily found by this short and plain Rule, for

8. CASE I. Of *Diverging Rays*, Plate XV. Scheme 1.

R U L E.

Multiply the Distance of the Radiant by the Radius of the Lens, divide that Product by the Difference between the said Distance and Radius, the Quotient is the focal Distance required, *per* Theor. 16.

9. E X A M P L E.

Let the Distance D E = 40 — — 40

The Radius — C E = 15 — — 15

The Product — — — 600 25 Diff.

Then 25) 600 (24 = F E, the focal Distance sought, *viz.*

50
—
100
100
—
...

2 $\frac{4}{5}$ Inches.

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10. And here it is to be observed, that according as the Distance of the Radiant DE is *greater, equal to, or less* than the Radius CE, so the Focus F will be *affirmative, infinite, or negative.*

11. CASE II. Of *Parallel Rays*, Scheme 2. In this Case the Focus F will be coincident with the Center C; that is, the Rays of the Sun will be collected into a Point F, whose Distance FE, from the Lens, will be equal to the Radius CE, *per Theor.* 17.

12. Hence a *Convex* Lens becomes a *Burning Glass*, whose Power is greater or less, as it's Surface is larger or smaller, if the focal Distance be the same; or as the said Distance is lesser or greater, if the Quantity of Surface be the same.

13. For if the Lenses be of different Bigness, and of the same focal Length, their Power of burning will be as the *Squares of their Diameters* directly, for it will be as the Quantity of Rays falling on their Surfaces directly; thus if the Diameter of one Lens be 1 Inch, and of the other 4; the latter will burn 16 times more intensely than the former.

14. Again, if the Diameters or Surfaces of two Lenses be the same, the Power of burning will be reciprocally, as the Surfaces of the *burning Spots*, which are the Images of the Sun, and these are as the Squares of their Diameters; but since these Diameters are as the

the Distances from the Glasses, the Power of burning will be reciprocally, as the Squares of the *focal Distances*. Thus suppose two Lenses of equal Diameters, and the focal Distance of one be 1 Inch, but that of the other 4; the Power of burning, in the former, will be 16 times greater than in the latter.

15. The absolute Power of burning, in these Lenses, is as the Number of Times the burning Spot is contained in the Surface of the Lens, or as the Square of the Diameter of the Spot is contained in the Square of the Diameter of the Lens. Thus suppose I have a Lens 4 Inches in Diameter, and the burning Spot made thereby I measure and find to be one Tenth, *viz.* $\frac{1}{10}$ of an Inch. Then in 4 Inches there being 40 Tenths of an Inch, the Diameter of the *Lens* to that of the *Spot* will be as 40 to 1; and the Squares of these Diameters will be as 1600 to 1; that is, the Spot is *one Thousand six hundred Times* less than the Surface of the Lens; and the Heat of the Sun's Rays will consequently be augmented in the same Proportion, which therefore will burn very strongly.

16. Note, if the Thickness of the Lens be at any Time considered, the *focal Distance* will be very nearly one *sixth Part* of the said Thickness less than the Radius, as appears from the Theory.

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17. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of these Rays is as follows.

R U L E.

Multiply the Distance of the Radiant by the Radius, divide that Product by the Sum of the said Distance and Radius; the Quotient will be the Distance of the Focus required, *per* Theor. 18.

18. E X A M P L E.

Let the Distance $dI = 30 \text{ --- } 30$

The Radius $\text{--- } CE = 15 \text{ --- } 15$

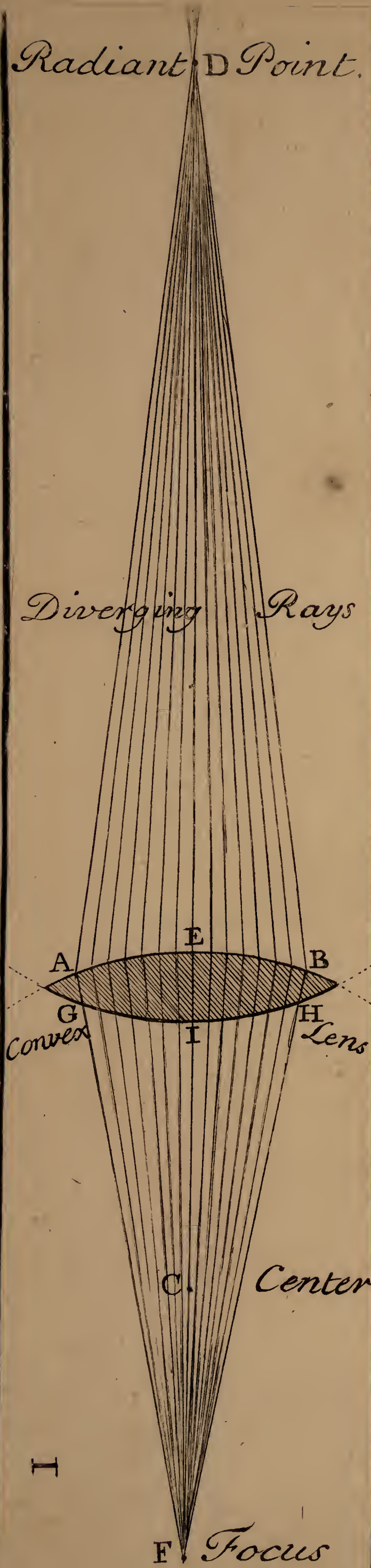
The Product $\text{--- } \text{--- } 450 \quad 45 \text{ Sum}$

Then $45) 450 (10 = FI$, the Distance
 $\begin{array}{r} 45 \\ \text{---} \\ 0 \end{array}$ of the Focus required, *viz.* 1 Inch.

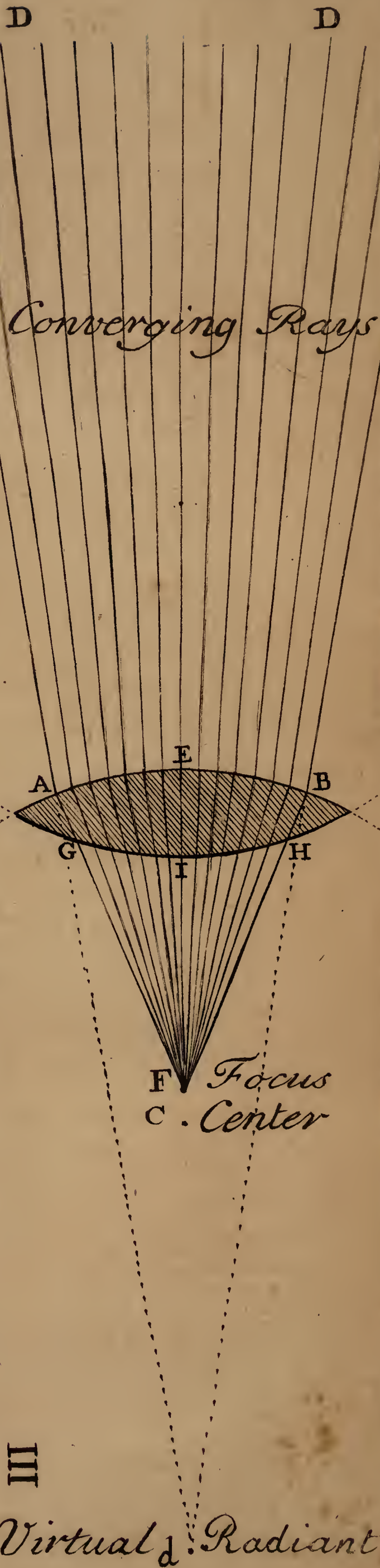
19. The Focus will in this Case be always affirmative, and it's Distance less than the Radius. As the double and equally Convex Lens is the most common and useful, so all it's Cases are of the last Importance for a due Understanding of the Nature, Construction, and Effects, of all Dioptric Machines, as will be shewn in the third Part.

Plate. XV.

Radiant D Point.



Parallel Rays





C H A P. V.

The Rules for finding the Focus of a
P L A N O - C O N V E X L E N S.

1. **I**F the Thickness of a *Plano-Convex Lens* be considered, and it be exposed on it's convex Side to Parallel Rays, as those of the Sun, the Focus will be at the Distance of *twice the Radius, wanting $\frac{2}{3}$ (two Thirds) of the Thickness of the Lens, per Theor. 20.*

2. But if the same Lens be exposed with it's plane Side to Parallel Rays, the Focus will then be precisely at the Distance of *twice the Radius from the Glass, per Theor. 26.*

3. If the *Thickness* of the Lens be neglected, the Rules for the Focus are the same for either of the two Sides towards the Radiant, as appears from the Theorems relating to these Cases.

4. CASE I. Of *Diverging Rays*, Plate XVI. Scheme 1. In this Case the Rule for finding the Focus is as follows, *viz.*

G 4

R U L E.

11. The Focus of *Converging Rays* will always be *affirmative*, or behind the Glass, and less than twice the Radius or Diameter of the Sphere. This Case of *Converging Rays* is of great Use in the Construction of Microscopes, or any Machine where a Composition of Lenses is necessary, as will be shewn in the third Part of this Treatise.



CHAP. VI.

The Rules for finding the virtual Focus of DOUBLE CONCAVE LENSES.

1. **I**F the Lens be unequally Concave, and the Thickness thereof neglected, (which is much more inconsiderable in these than in the Convex Lenses) then the Rule for finding the Focus for

2. CASE I. Of *Diverging Rays*, is as follows.

R U L E.

Multiply twice the Product of the Radii by the Distance of the Radiant; divide that Product by the Sum of the Radii multiplied by the Distance, and twice the Product of the Radii, the Quotient will be the Distance of the Focus, *per Theor. 34.*

3. EX-

11. The Focus of *Converging Rays* will always be *affirmative*, or behind the Glass, and less than twice the Radius or Diameter of the Sphere. This Case of *Converging Rays* is of great Use in the Construction of Microscopes, or any Machine where a Composition of Lenses is necessary, as will be shewn in the third Part of this Treatise.



C H A P. VI.

The Rules for finding the virtual Focus of DOUBLE CONCAVE LENSES.

1. **I**F the Lens be unequally Concave, and the Thickness thereof neglected, (which is much more inconsiderable in these than in the Convex Lenses) then the Rule for finding the Focus for

2. C A S E I. Of *Diverging Rays*, is as follows.

R U L E.

Multiply twice the Product of the Radii by the Distance of the Radiant; divide that Product by the Sum of the Radii multiplied by the Distance, and twice the Product of the Radii, the Quotient will be the Distance of the Focus, *per Theor. 34.*

3. E X-

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3. E X A M P L E.

Suppose the longest Radius be 50, the shortest 30, and the Distance of the Radiant 300.

Longest Radius — — 50

Shortest — — — 30

Product — — — 1500

Multiply by — — — 2

3000

Distance — — — 300

27000) 900000 (33.3

81000

90000

81000

90000

81000

9000

Longest Radius 50

Shortest — — 30

Sum — — 80

Distance — 300

Product — 24000

3000, twice Prod. of the Radii.

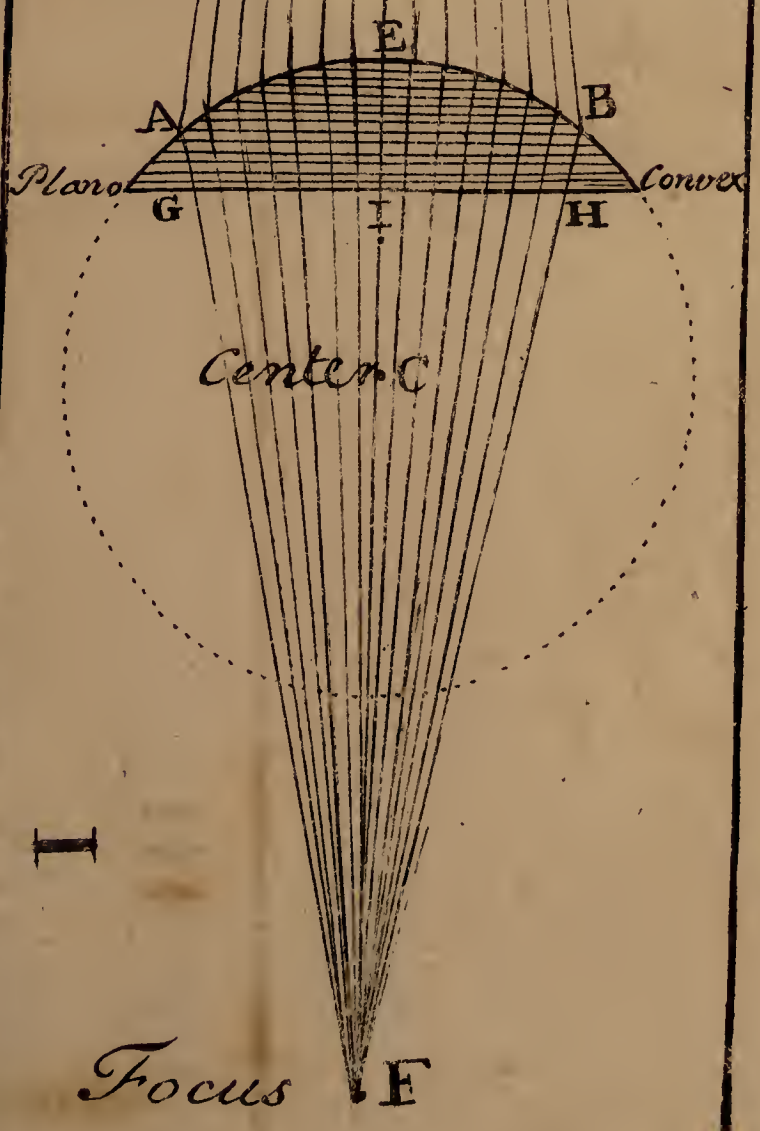
Sum — — 27000 Divisor.

Here the focal Distance is $33\frac{3}{10}$, and always negative, or before the Lens.

4. C A S E

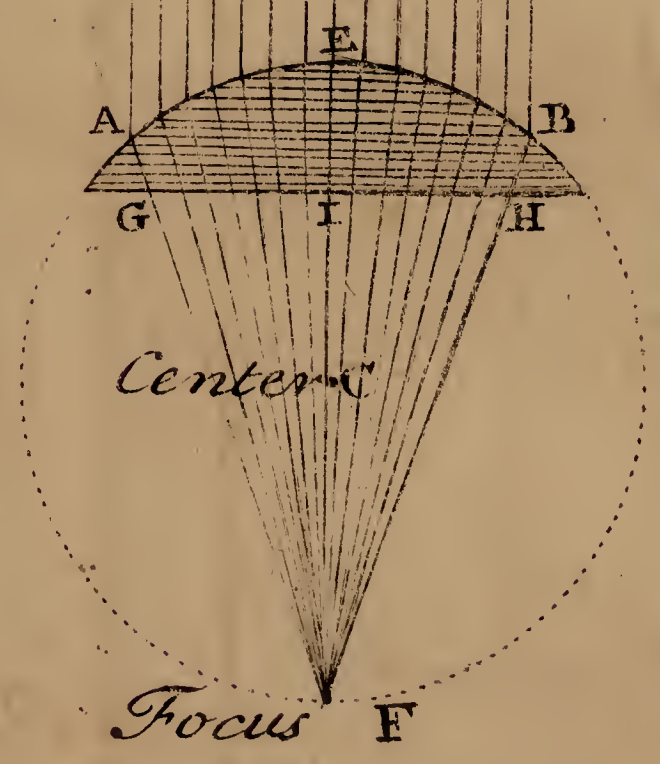
Radiant D Point

Diverging Rays



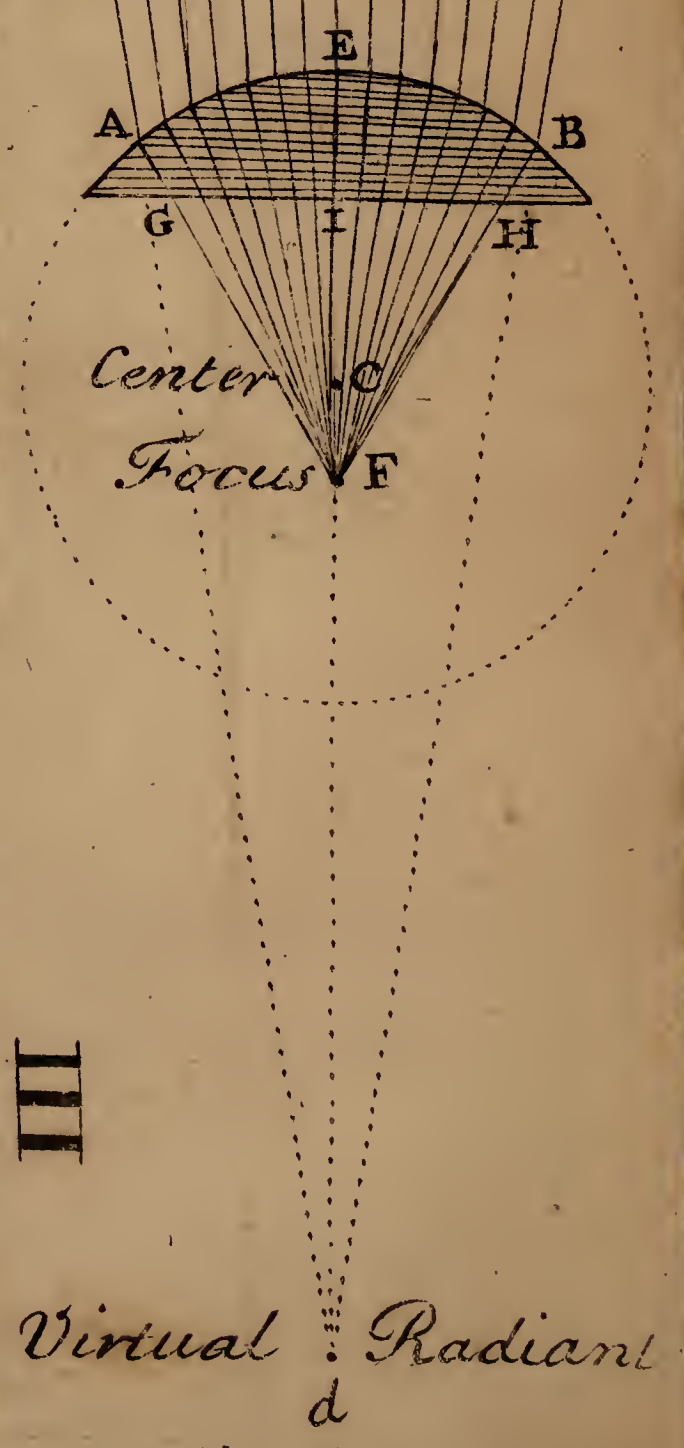
I

Parallel Rays



II

Converging Rays



III



4. CASE II. Of *Parallel Rays*. The Rule for finding the Focus here, is the same as for a Double unequally Convex Lens, *viz.* To divide twice the Product of the Radii by their Sum. See Chap. IV. Art. 4. But the Focus here is always *negative*, as it is there always *affirmative*. And consequently, since the Rule is the same, if the Radii of the Concavities and Convexities are respectively equal, and also the Distance of the Radiant; the Distance of the Focus will also be equal from either Lens.

5. CASE III. Of *Converging Rays*. The Rule for finding the Focus here is the very same as for *Diverging Rays*, in Chap. IV. Art. 2. Or that above in Art. 2. if instead of the *Sum* you take the Difference of the Quantities there mentioned, *per.* Theor. 36.

6. If the Lens be double and equally Concave, and the Thickness not considerable, the Rule for finding the Focus of *Diverging Rays* is as follows.

7. CASE I. Of *Diverging Rays*, Plate XVI. Scheme I.

R U L E.

Multiply the Distance of the Radiant by the Radius; divide that Product by the Sum
of

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Sum of the said Distance and Radius, the Quotient will be the Distance of the *virtual* Focus, always negative, *per* Theor. 37.

8. EXAMPLE.

Let the Distance DE = 40 — — 40

The Radius — CE = 10 — — 10

400 50 Sum

Then 50) 400 (8 = fE, the focal Distance

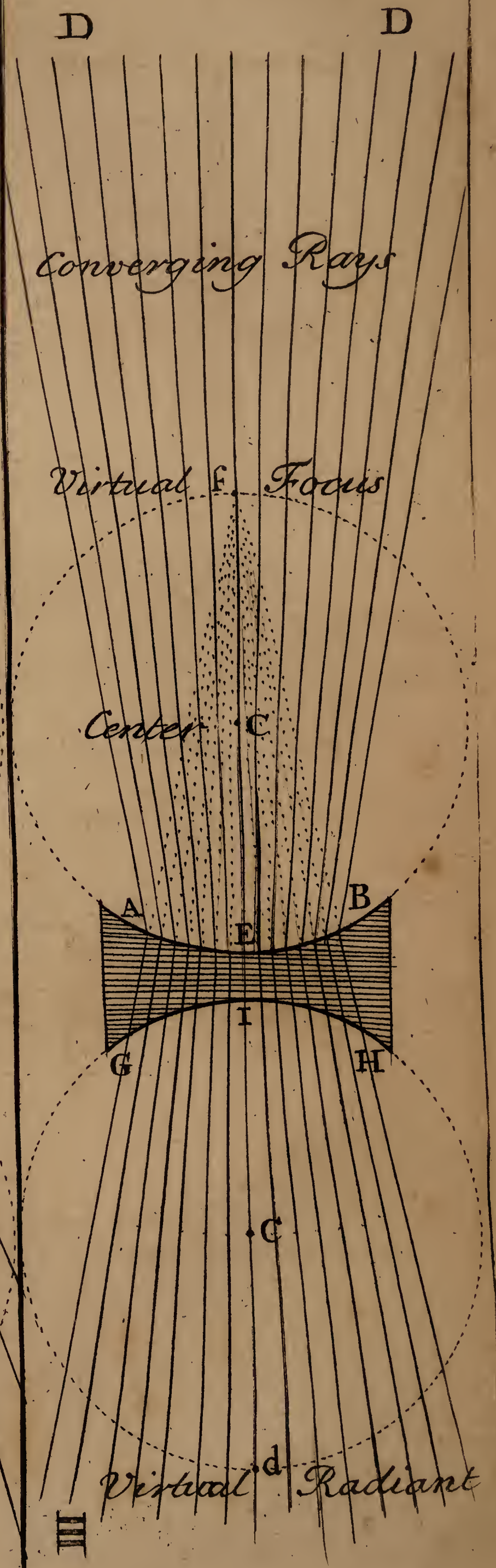
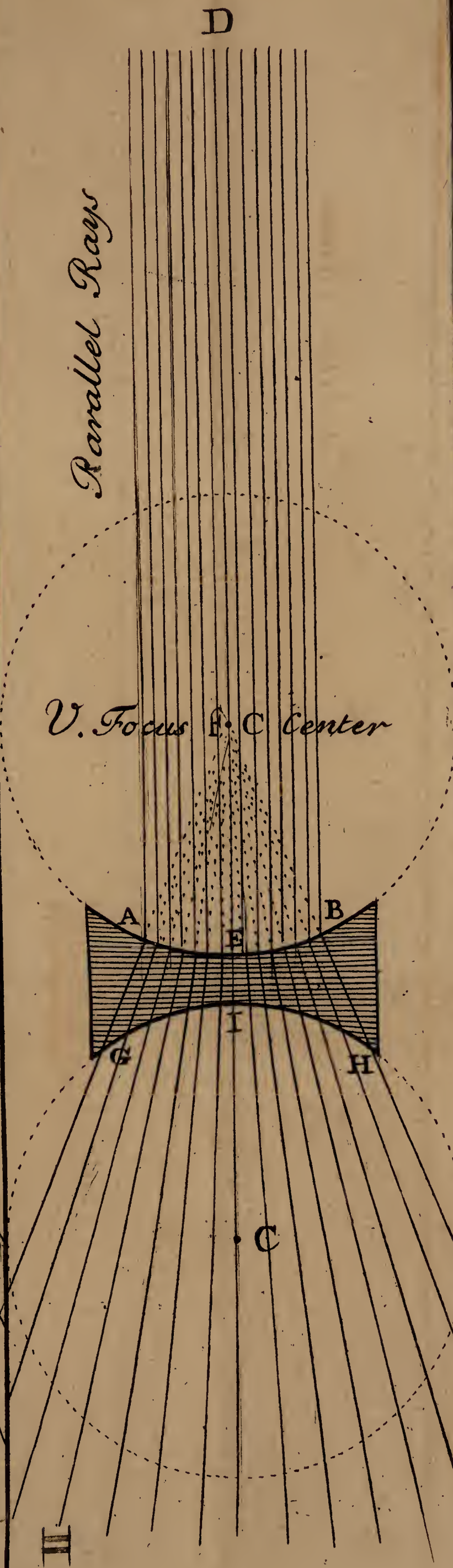
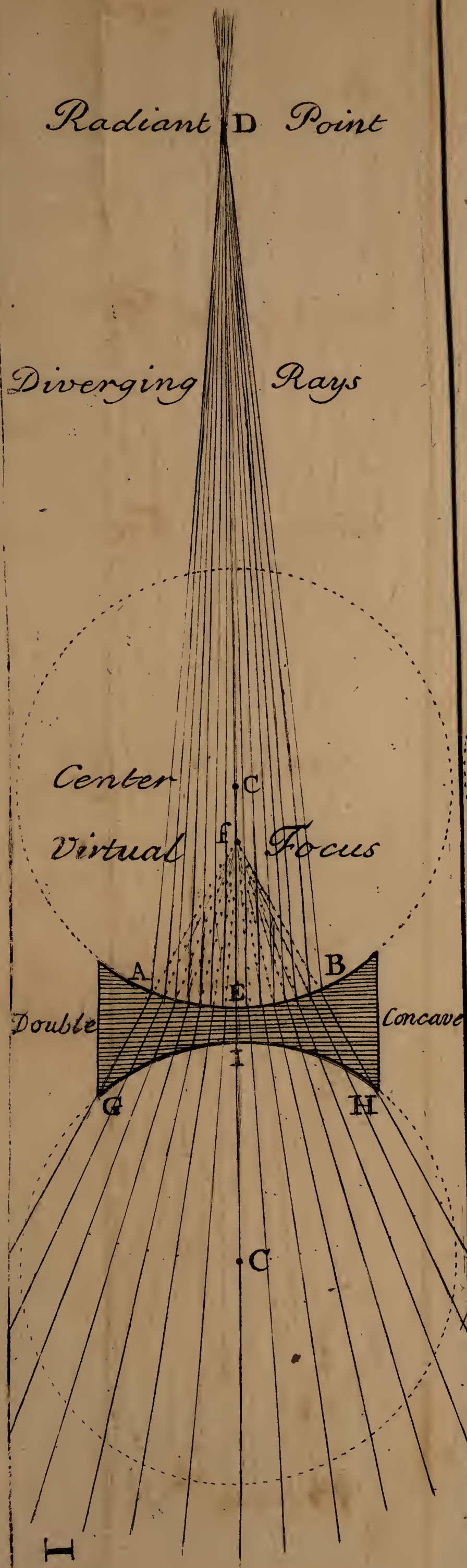
400

— — —
...

9. CASE II. Of *Parallel Rays*, Scheme 2. The virtual Focus of these Rays is always at the Distance of the Radius from the Lens, as in a Double and equally Convex Lens, *per* Theor. 38. But is here always *negative*, as there it was always *affirmative*.

10. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of these Rays is exactly the same as that for *Diverging Rays* in a *Double Convex*, Chap. IV. Art. 8. And according as the Distance of the Point d, towards which they tend, *viz.* dI is *greater*, *equal to*, or *less* than the Radius CI, the Focus will be *virtual* and *negative*, *infinite*, or *real* and *affirmative*, *per* Theor. 39.

11. That is, If Rays converge towards a Point d, as in Scheme 3, beyond the Center C, they





The Theory of DIOPTRICS. III

they will be so refracted at the Surface GIH , that they will proceed diverging as if it were from the Point f , which is their *virtual Focus* and *negative*. If dI be $= 2CI$, then will fE be $= 2CE$, *viz.* the Diameter of the Sphere.

12. If Rays fall converging on the Surface GIH , tending to a Point f , (as in Scheme 2) which is the Center of the Concavity, they will be so refracted at the Surface AEB , as to proceed *parallel*, whose Focus will therefore be at an *infinite* Distance.

13. If Rays fall on the Surface GIH , converging towards a Point f , whose Distance fE is less than the Radius CE , they will be so refracted at the Surface AEB , as to proceed converging towards a Point D , which will therefore be their Focus, real and affirmative.



C H A P. VII.

To find the Focus of a PLANO-CONCAVE LENS.

1. **I**F a Plano-Concave Lens be exposed with it's *Concave Side* to *Parallel Rays*, (*viz.* Rays of the Sun) and the Thickness of the Lens be considered, the virtual Focus will be at the Distance of *twice the Radius, or Diameter of the Sphere, lessened by $\frac{2}{3}$ of the Thickness of the Lens*, per Theor. 41.

2. The same Lens exposed on the *Plain Side* to *Parallel Rays*, will have their virtual Focus at just the Distance of *twice the Radius, or Diameter* of the whole Concavity, per Theor. 46.

3. If the Thickness of the Lens be neglected, as it always may in this Sort; then the Rules for determining the Foci of all Sorts of Rays are the same, let them fall on which Side of the Lens they will, as is evident from the Theory.

4. CASE I. Of *Diverging Rays*, Plate XVIII. Scheme 1. The Rule for finding the Focus of these Rays by a Plano-Concave is this, *viz.*

R U L E.

R U L E.

Multiply twice the Radius by the Distance of the Radiant, divide that Product by the Sum of the said Distance, and twice the Radius; the Quotient is the focal Distance, *per* Theor. 43.

5. E X A M P L E.

Let the Distance DE = 50 — — 50

Twice the Radius CE = 20 — — 20

1000 70 Sum

Then 70) 1000 (14 = fC, the focal Distance required,
viz. $1\frac{4}{15}$ Inch.

70
—
300
280
—
20

6. The Focus in this Case is always virtual and negative; for Rays which fall diverging on this Lens are always so refracted as to proceed still more diverging.

7. CASE II. Of *Parallel Rays*, Scheme 2. The Focus of these Rays will be at just twice the Length of the Radius from the Lens, as represented in the Scheme, *per* Theor. 44 or 49. That is, it will be distant from the Lens
H the

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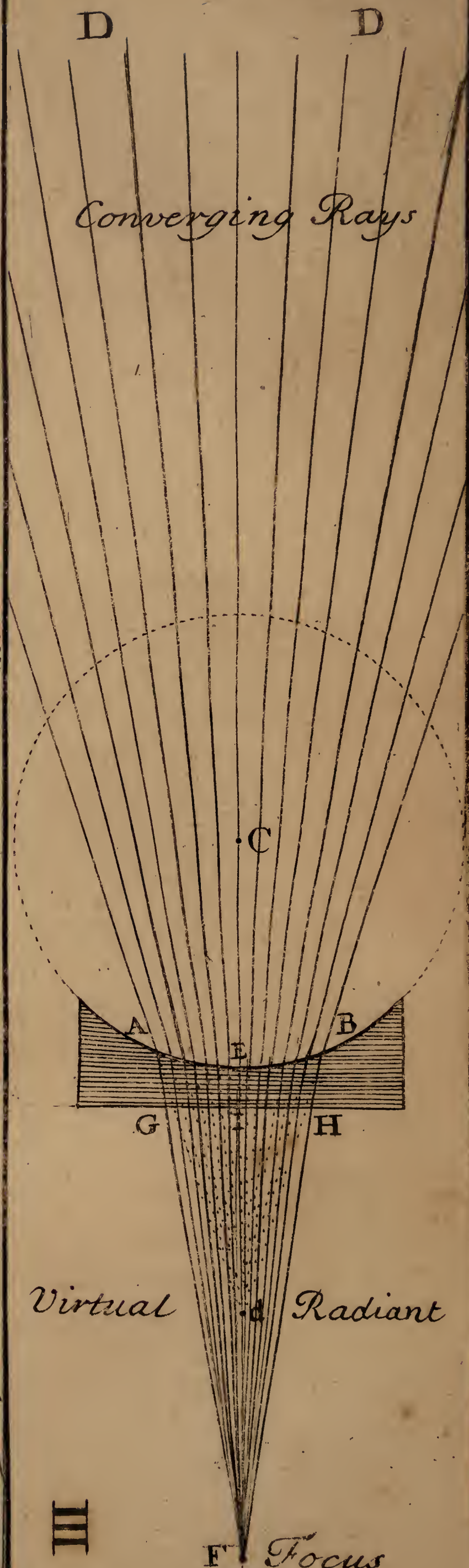
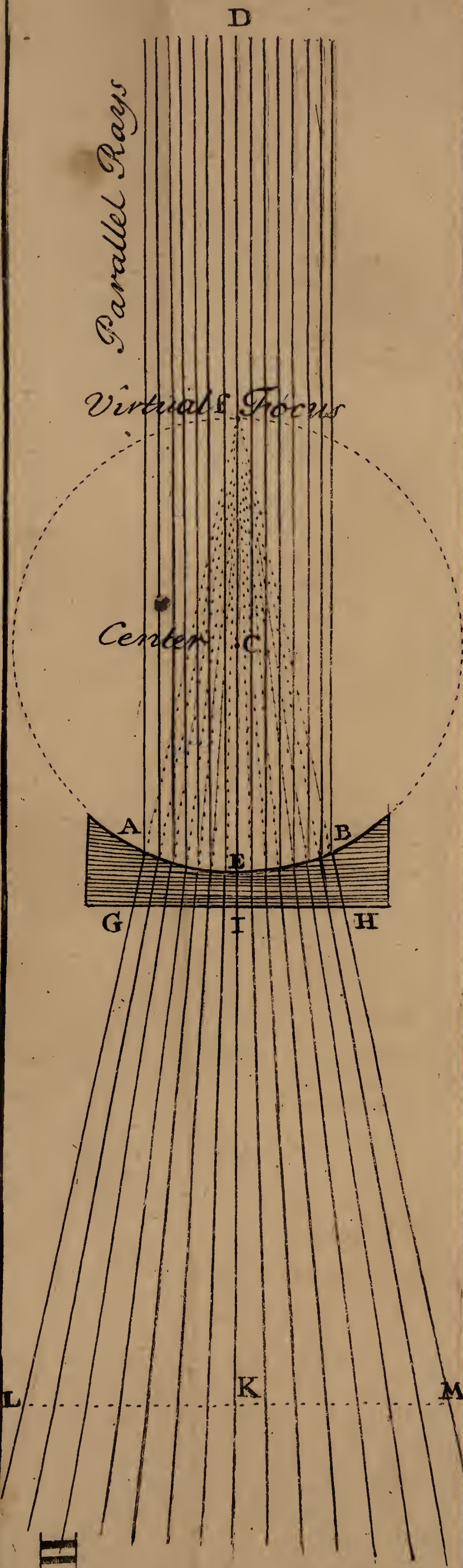
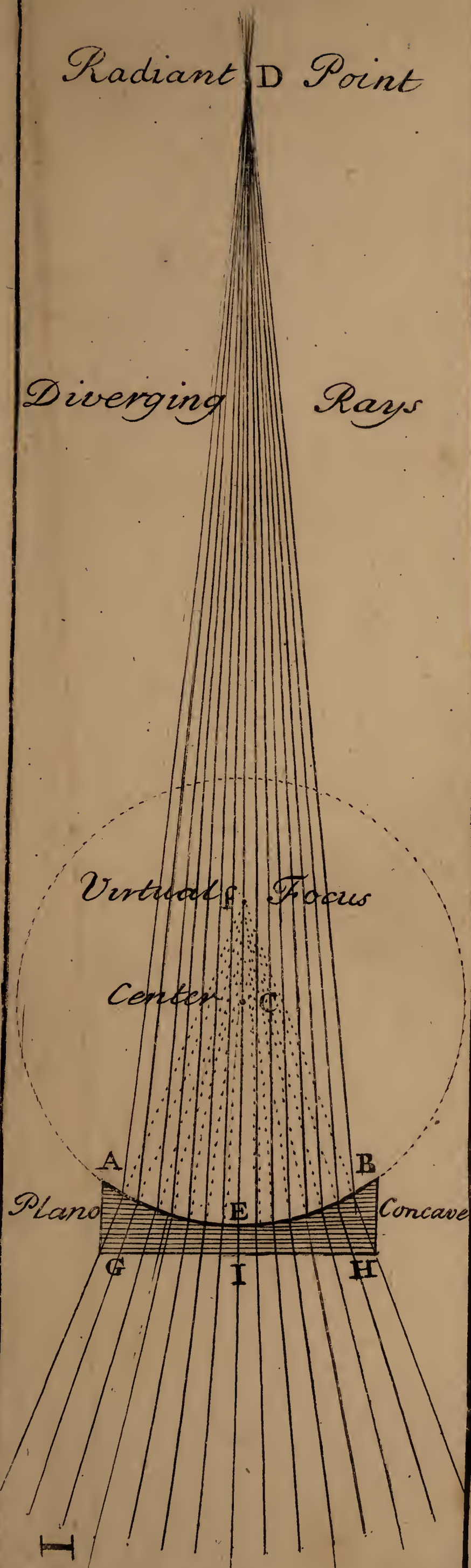
the Diameter of it's Concavity, and will be always virtual and negative. So that it is impossible any concave Lens should be a *burning Glass*; but on the contrary, the Sun's Rays may, by these Lenses, have their Light and Heat lessened in any Proportion whatsoever.

8. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of these Rays is Word for Word the same with that for CASE I, of *Diverging Rays*, in a *Plano-Convex* Lens. See Chap. V. Art. 4. This will be evident by comparing Theor. 45 or 50, with 22 or 28.

9. Here the Focus will be affirmative, infinite, or negative, according as twice the Radius is greater, equal to, or less than the Distance of the Radiant. In this Scheme, the Distance dE is equal to the Radius CE ; therefore the focal Distance FE is equal to twice the Radius CE .

10. It is easy by this time for the Reader to observe, that the same Rule which finds the Focus of *Diverging Rays* in a *Convex* Lens, finds the Focus of *Converging Rays* in a *Concave* one; and also that the Rules for finding the Foci of *Diverging* and *Converging Rays*, in any Lens, differ only in the *Sum* or *Difference* of the Quantities in the Divisor.

11. The principal Use of these concave Lenses is in that Sort of Telescopes, which we call





call *Perspective-Glass*, concerning which I shall treat largely in the next Part.

12. It often happens that we have a concave Lens, and know neither the Radius or Focus; to know the focal Distance therefore in such a Case, proceed thus: Take a Piece of Pastboard, &c. and cut in it a round Hole; and on another Piece of Pastboard, strike a Circle, whose Diameter is just double the Diameter of the said Hole; then apply the Piece with the Hole in it to the Lens, and hold them in the Sun-Beams, with the other Piece at such Distance behind, that the Light coming through the Hole may spread or diverge, so as to fill the Circle drawn there precisely; then is that Distance equal to the virtual Focus of the Lens; and also to the Radius, if a double Concave; or twice the Radius if a Plano-Concave. Let $GH =$ Diameter of the Hole, $LM =$ that of the Circle; then, since $LM = 2GH$, we have $fK = 2fE$; and therefore $IK = fI = 2CI$, the Radius of the Plano-Concave. Scheme 2. Plate XVIII.

C H A P. VIII.

The Rules for finding the Focus of a
MENISCUS LENS.

1. **I**F the Thickness of the Lens be not considered, the Radii of Convexity and Concavity unequal, and the convex Part exposed to *Diverging Rays*, we shall find the Focus by the following Rule.

2. CASE I. Of *Diverging Rays*, Plate XIX.
Scheme 1.

R U L E.

Multiply twice the Product of the Radius of Convexity into that of Concavity, by the Distance of the Radiant; this shall be the *Dividend*. Then multiply the Difference between the two Radii by the Distance of the Radiant; and to that Product add twice the Product of the Radii; the Sum shall be the *Divisor*, by which divide the Dividend above, the Quotient will be the focal Distance required, *per Theor. 54.*

3. E X-

3. EXAMPLE.

Let the Radius of Convexity $CE = 15$

The Radius of Concavity — $KI = 10$

Their Product — — — — — 150

Multiply by — — — — — 2

Multiply by the Distance — $DE = 40$

The Dividend — — — — — 12000

Then from Rad. of Convexity $CE = 15$

Take the Radius of Concavity $KI = 10$

The Difference is — — — — — 5

Multiply by — — — — — $DE = 40$

Product — — — — — 200

To that add twice the Prod. of Radii 300

The Sum is the Divisor, *viz.* — 500

Then $500 \overline{) 12000}$ ($24 = fE$, the Distance of the Focus required, *viz.* $2\frac{4}{10}$ Inches.

1000
—
2000
2000
—
.....

4. Here the Radius of Convexity *exceeds* that of Concavity ; but if they are *equal*, then
H will

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will the Distance of the Focus be equal to the Distance of the Radiant, and *negative* and *virtual*; for the Rays will proceed with the same Degree of Divergency as they had when they fell on the Glass; for in this Case, all that is effected by the convex Surface AEB, is destroyed by the equally concave one GIH. See Theor. 57.

5. And universally, in the Case of Converging Rays, as the Product of the Distance of the Radiant into the Radius of Concavity is *less, equal to, or greater* than the Product of the said Distance into the Radius of Convexity, added to twice the Product of the Radii into each other, the Focus will be negative, infinite, or affirmative.

6. CASE II. Of *Parallel Rays*, Scheme 2. Every thing besides remaining as before, the Focus of these Rays are found by this short

R U L E.

Divide twice the Product of the Radii by their Difference, the Quotient will be the focal Distance sought, *per* Theor. 55.

7. E X-

7. EXAMPLE.

$$\begin{array}{rcl}
 \text{Let the Radius of Con-} & \left. \vphantom{\begin{array}{l} \text{vexity be CE} \\ \text{The Radius of Con-} \\ \text{cavity be KI} \end{array}} \right\} & 15 \text{ --- } 15 \\
 \text{vexity be CE} = & & \\
 \text{The Radius of Con-} & \left. \vphantom{\begin{array}{l} \text{cavity be KI} \end{array}} \right\} & 6 \text{ --- } 6 \\
 \text{cavity be KI} = & & \\
 \hline
 \text{Product --- --- ---} & & 90 \qquad 9 \text{ Diff.} \\
 & & 2 \\
 & & \hline
 \end{array}$$

$$\text{Twice the Product --- } 180$$

$$\begin{array}{rcl}
 \text{Then } 9) 180 & (20 = fE, & \text{the focal Di-} \\
 18 & & \text{stance required,} \\
 \hline
 \dots 0 & & \text{viz. 2 Inches.}
 \end{array}$$

8. As the Radius of Convexity is *greater*, *equal to*, or *less* than the Radius of Concavity, the Focus of these Rays will be *negative*, *infinite*, or *affirmative*, as is plain from the Theory. In the last Case therefore, *viz.* when the *Convexity* is *less* than the *Concavity*, a *Meniscus* Lens will become a *burning Glass*.

9. CASE III. Of *Converging Rays*, Scheme 3. The Rule for finding the Focus of these Rays is the same as above for diverging ones; and as to the Nature of the Focus, it will be *negative*, *infinite*, or *affirmative*, according as the Product of the Distance and Radius of Convexity is *lesser*, *equal to*, or *greater* than the Product of the Distance into the Radius

H 4 of

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of Concavity, added to twice the Product of the Radii into each other, *per* Theor. 56.

10. E X A M P L E.

Let the Radius of	}	15 - - - - 15	
Concavity KI =			
The Radius of Con-	}	6 - - - - 6	
vexity CE =			Sub.
Product — — —		90	
Multiply by — —		2	9 Diff. 20 = dE
Twice Prod. of Radii		180 - - -	
Multiply by the Di-	}	20	180 } Add 180 } 360 Divisor
stance dE =			
The Dividend —		3600	
Then 360) 3600			(10 = FI, the focal Di- stance required, <i>viz.</i> 1 Inch, and the Fo- cus affirmative.
		360	
		...	

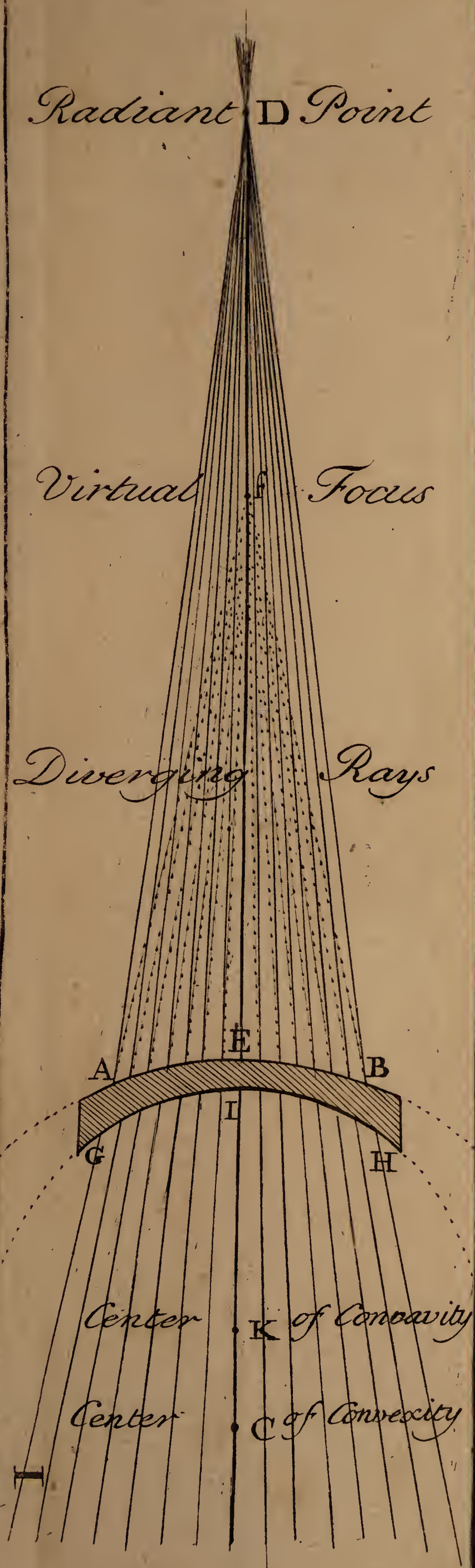
11. If the Radii of Convexity and Concavity are equal, the Focus of Converging Rays will be at the Distance of the Radiant and affirmative; that is, the Rays in this Case will pass on after Refraction with the same Degree of Convergency as they before had, *per* Theor. 59.

12. When

Radiant D Point

Virtual F Focus

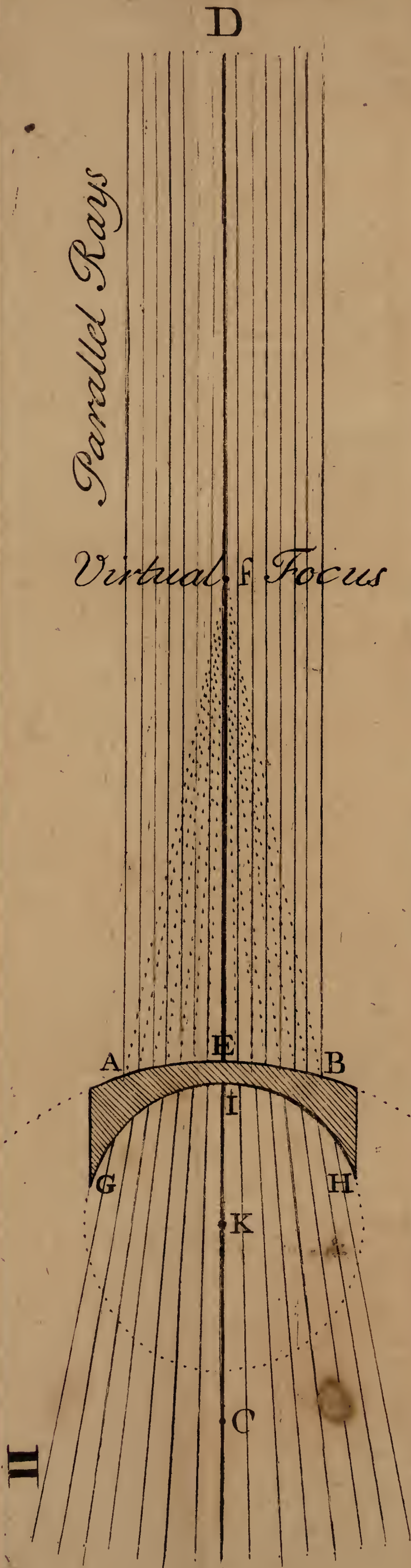
Diverging Rays



D

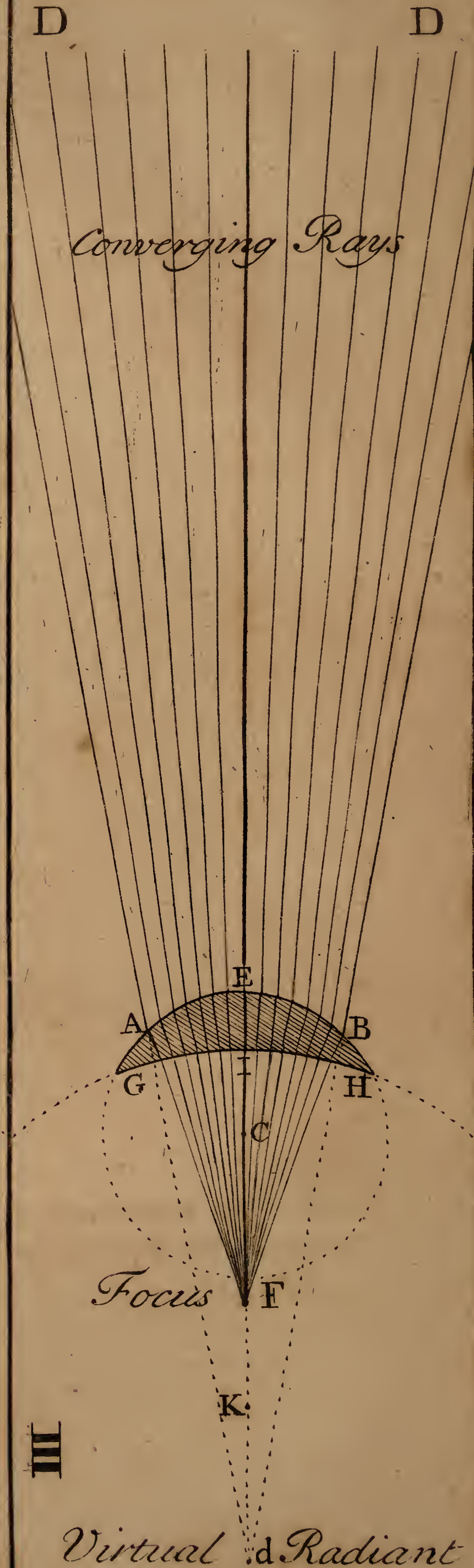
Parallel Rays

Virtual F Focus



D

Converging Rays



Virtual d Radiant



12. When the concave Part of the *Meniscus* is turned towards the Radiant, the Rule for the Focus of *Diverging Rays* is the same as for *Converging ones*, when the convex Side is towards it, but the Focus will here be negative, as it was there affirmative. The Converse of this holds good for *Converging Rays* in this Case. And as for *Parallel Rays*, the Rule is still the same as above in Art. 7, and the Focus will be affirmative or negative, as the Radius of Concavity is greater or less than that of the Convexity; and if the Radii are equal, the Focus will be infinite, or the Rays will still proceed on parallel.



C H A P. IX.

The Rules which determine the Distance of an Object that shall bear any assigned Proportion to it's Image formed by a CONVEX LENS.

1. **I**N Plate XX, FG is a double and equally convex Lens, C the Center, CE the Radius, CE A the Axis, O B an Object placed at right Angles therewith, and I M the Image formed by the said Lens. Suppose now it were required to know at what Distance the Object shall be placed, that it may bear the same Proportion to it's Image, as 40 to 10; this is the

R U L E.

Add together the Numbers expressing the Object and it's Image, multiply their Sum by the Radius of the Lens; divide that Product by the Image, the Quotient will be the Distance of the Object required, *per* Theor. 1. Chap. II.

2. E X A M P L E I.

The Object is — — 40

The Image — — 10

Their Sum is — — 50

Let the Radius CE = 10

Then 10) $\overline{500}$ (50 = E A, the
 50 Distance of the
 — Object, *viz.* 5
 10 Inches.

3. E X-

3. EXAMPLE II.

Suppose I have a double and equally convex Lens, the Radius being 35, and I would have an Image formed in Proportion to the Object, as 15 to 275; Query at what Distance the Object must be placed?

The Object is — 275

The Image is — 15

Their Sum is — 290

Mult. by the Radius 35

1450

870

Divide by 15) 10150 (676, the Distance
90 of the Object, viz.

115

105

100

90

10

67 $\frac{1}{2}$ Inches, and
a little more.

4. EXAMPLE III.

Suppose I would have an Image equal to the Object; Query the Distance of the Object? In this Case the Object must be placed at the Distance of twice the Radius of the Lens,

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Lens, *per* Theor. 17. And then will the Distance of the Image be the same also, *per* Art. 6, Chap. II. This Problem of *making an Image just as large as the Life*, may be found very useful to Painters, Designers, &c. on many Occasions, as will appear farther on.

5. EXAMPLE IV.

Suppose I would have an Object magnified in Proportion 15 to 150, that is, of 1 to 10, by a Lens of $3\frac{1}{2}$ Inches Radius. Then may the Distance of the Radiant or Object be found in the Manner as above. Thus.

The Object — — 15
The Image — — 150

Their Sum — — 165
Multiply by Radius 35

825
495

Divide by 150) 5775 (38.5, the Distance
450 of the Object,
1275 *viz.* $3\frac{85}{100}$ Inches
1200

750

750

...

6. EX-

6. EXAMPLE V.

Let there be a small double Convex, whose Radius is 5, or $\frac{1}{2}$ of an Inch, and let it be required to find at what Distance an Object must be placed, that it's Image may be 6 times larger than itself; proceed as above. Thus.

The Object is — 1

The Image is — 6

Their Sum is — — 7

Multiply by Radius 5

Divide by 6) 35 (5.8, the Distance re-
 30 quired, viz. $\frac{58}{10}$
 — of an Inch.

50

48

2

7. These Examples are sufficient to shew, that any Object may be magnified or diminished in any given Proportion by a convex Lens, with an affirmative Focus, viz. on the other Side of the Lens. That is, the Image will be *lesser, equal to, or greater* than the Object, according as the Distance of the Object is *greater, equal to, or lesser* than twice the Radius of the Lens.

8. The same Rule holds good for a *Plano-Convex* Lens, only with this Difference, that the

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the Distance of the Object will be always twice as great as for a *Double Convex* of the same Radius. Thus an Object is *diminished*, *made equal to the Life*, or *magnified*, according as it is placed at a Distance *greater, equal to, or less* than 4 times the Radius, or twice the Diameter of the Sphere, of which the Plano-Convex Lens is a Segment. All which is evident from Theor. 4 and 18.

9. If the Image be required on the same Side with the Object, the Theorem which finds the Distance of the Object for an assigned Proportion between it and it's Image, is thus expressed in Words.

R U L E.

From the Image take the Object, multiply the Difference by the Radius; divide that Product by the Image, the Quotient is the Distance required, *per* Theor. 16.

10. From the Rule it is evident the Image must, in this Case, be always greater than the Object, which therefore cannot be diminished at a negative Focus, nor ever equal to it's Image, but at the Vertex of the Lens.

II. EXAMPLE.

Let there be a double and equally convex Lens, whose Radius is 40, or 4 Inches; and let the Image be to the Object as 50 to 10; it is required to find the Distance of the Object for that Purpose. Proceed thus according to the Rule.

From the Image — 50

Take the Object — 10

The Difference is 40

Multiply by Radius 40

Divide by 50) 1600 (32 = the Distance
 150 of the Object re-
 — quired, viz. $3\frac{2}{10}$
 100 Inches.

100

...

12. The Distance of the Object will always be less than the Radius for a negative Focus; and when it becomes equal to the Radius, the Image is then at an infinite Distance, and infinitely larger than the Object.

13. In this Respect also, every thing is the same in a *Plano-Convex*, but the Distance of the Object; which is, for the same *Data*, always double to that for a double Convex of equal Radii.

14. The

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14. The Object hitherto considered is supposed to be a *Line*, or simple Length of any thing; if the Superficies of any Object be considered that will be altered in a *duplicate* Proportion of the Length or Breadth; that is, according to the Square thereof: Thus if the Length of an Object be magnified or diminished 2, 3, 4, 5, &c. times, the Superficies of that Object will be magnified or diminished 4, 9, 16, or 25 times, because these Numbers are the *Squares* of the others.

15. But I shall give a Rule that will find at what Distance an Object must be placed, that it's Surface shall bear any assigned Proportion to the Surface of the Image; and is as follows.

R U L E.

Divide the Object by the Image, and multiply the Quotient by the Square of the Radius; and to the square Root of that Product add the Radius; the Sum is the Distance of the Object required, *per* Theor. 2.

16. E X A M P L E.

Let there be a double and equally convex Lens, whose Radius is 30, or 3 Inches; and let the Surface of the Object be to that of the Image, as 1000 to 10; to find the Distance at which the Object must be placed for that Purpose. Proceed thus by the Rule.

Image

Image	Object	
10)	1000	(100 Quotient.
	100	900 = Radius Square.

	90000	(300 = Square Root.
		30

		330 = Dist. sought.

17. Hence it appears, that if an Object be placed 33 Inches from the Lens, the Surface of the Image will be an 100 times less than it's own. If this Object were a *Circle* or a *Square*, the *Diameter* of the Circle or *Side* of the Square would be 10 times less in the Image than in the Object at that Distance.

18. By Theor. 3. we have a Rule for the Distance of Objects for any assigned Proportion of the *Solidities* or *Bulks*; and is as follows.

R U L E.

Divide the Object by the Image, multiply the Quotient by the Cube of Radius; to the Square Root of that Product add the Radius, the Sum is the Distance of the Object required.

19. E X A M P L E.

Let the Radius of the Lens be 30, the Solidity or Bulk of the Object be to that of the Image 10000 to 10; the Distance of the Object is found as *per* Rule.

Image	Object	
10)	10000	(1000 Quotient.
		9000 = Cube of Radius.
		<hr/>
		9000000 (300 Cube Root.
		30 = Radius.
		<hr/>
		330 = the Dist.
		of the Object.

20. Here again the Distance is the same as before, *viz.* 33 Inches. At that Distance therefore the *Length* of an Object will be diminished 10 times, the *Superficies* an 100 times, and the *Solidity* 1000 times, which is according to the *Simple Square* and *Cubic Proportion*, as it should be by the geometrical Doctrine of Mensuration. But enough of these Matters, which are of more Speculation than Use.

21. When the Distance of an Object, and the Proportion thereof to the Image is given the Radius of a double and equally convex Lens is found by the following Rule.

R U L E

R U L E.

Multiply the Distance of the Object by the Image; divide that Product by the Sum of the Object and Image; the Quotient is the Radius sought, *per* Theor. 20.

22. E X A M P L E.

Let the Distance of the Object be 330, the Proportion of the Object to the Image that of 100 to 10, or 10 to 1.

Then multiply the Distance 330
By the Image — — — — 10

The Product — — — 3300

Divide by the Sum of } 110) 3300 (30 = the
the Object and Image } Radius
330 sought,
...0 viz. 3
Inches. See Art. 17.

23. For a *Plano-Convex* the Radius will be found just half as long, because you must then divide by twice the Sum of the Object and Image, *per* Theor. 21.

24. If the Distance of the Object, and Radius of the same Lens be known, the Proportion of the Object to the Image is then known by the following

I 2

A N A-

A N A L O G Y.

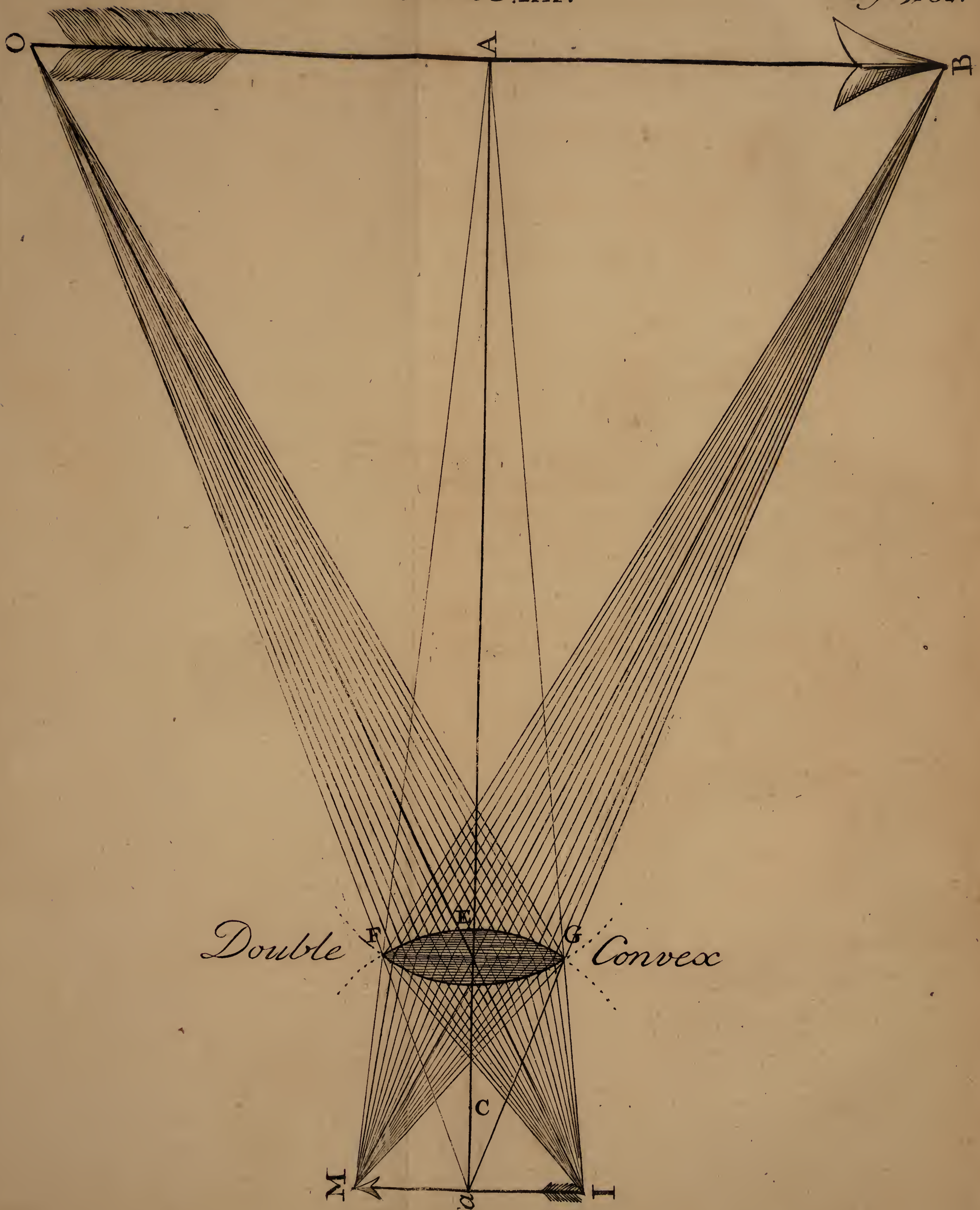
As the Distance lessened by the Radius, is
to the Radius ;

So is the Object to the Image, *per* Theor. 24.

25. E X A M P L E.

Let the Distance of the Object be 330, the Radius 30 ; then the Distance lessened by the Radius is 300 ; and therefore as 300 is to the Radius 30, so is the Object to the Image, but 300 is 10 times greater than 30, therefore the Object is 10 times larger than the Image. For a *Plano-Convex*, instead of *Radius* you must take the Diameter, or double Radius, *per* Theor. 21.

26. It is evident from the Scheme, that the Object seen at an affirmative Focus is *inverted*, but at a negative Focus it will appear in it's proper Position ; the Reason of which was made plain in the Theory, and these are the principal Properties of *Convex* Lenses.





C H A P. X.

The Rules which determine the Distance of an Object for any assigned Proportion between it and it's Image, formed by a Double-Concave, Plano-Concave, and a Meniscus Lens.

1. **F**ROM the Theory it appears, that Diverging and Parallel Rays can have no affirmative Focus in a Double or Plano-Concave Lens. Therefore all Objects will have their Images formed at a negative Focus, or on the same Side of the Lens with themselves.

2. The Rule therefore for finding the Distance of the Object that shall bear an assigned Proportion to it's Image formed by a double and equally concave Lens, as that in Plate XXI, is as follows.

R U L E.

From the Object subtract the Image, multiply the Remainder by the Radius; divide that Product by the Image, the Quotient is the Distance required, *per Theor. 7.*

3. E X A M P L E.

Let it be required to find the Distance of an Object that shall be in Proportion to it's
I 3
Image,

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Image, as 40 to 10, by an equal and double concave Lens, whose Radius is 10, or 1 Inch.

The Object is — — 40 = OB

Subtract the Image 10 = IM

Remainder — — 30

Multiply by Radius 10 = CE

300

Then 10) 300 (30 = A E the Distance
30 sought, viz. 3 Inches.

...0

4. In a Plano-Concave all other things remaining the same, the Distance will be twice as great, by Theor. 10.

5. If you would find the Superficies or Solidity of an Object directly, the Rules are the same as in Chap. X. Art. 15 and 18, only as there you *added* the Radius to the Square and Cube Root, here you must *subtract* it. See Theor. 8 and 9.

6. If the Distance of an Object A E, and the Radius CE of a double and equally concave Lens be known, the Proportion between the Object and Image is also known by this

A N A L O G Y.

As the Distance added to the Radius, is to the Radius ;

So is the Object to the Image.

7. EX-

7. E X A M P L E.

Suppose the Distance $AE = 30$, and the Radius $CE = 10$; then will the Object be to the Image, as 30 added to 10, *viz.* 40 to 10, or as 4 to 1; that is, it will be 4 times as large. In Plano-Concaves you must use *twice the Radius*, thus.

8. As the Distance added to twice the Radius, is to twice the Radius;

So is the Object to the Image. Theor. 27.

9. In Double and Plano-Concaves it appears, both from the *Theory* and the *Scheme*, that the *Image* will always be *less* than the Object, on the *same Side* with itself; *erect*, or in the *same Position*; and ever *between the Lens and it's Center C*; till the Distance of the Object become infinite, and then the Image will be formed in the very Center, as that of the Sun, and all vastly distant Objects, is seen to be; and these are the chief Properties of *Concave Lenses*.

10. A MENISCUS Lens, whose Radii of Convexity and Concavity are unequal, and being exposed on the convex Side to the Object, will magnify or diminish an Object in any assignable Degree, at an affirmative Focus; provided the Radius of Concavity exceeds the Radius of Convexity. This is evident from

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Theor. 13, which gives the following Rule for the Distance of the Object, *viz.*

R U L E.

Multiply the *Sum* of the Object and Image by twice the Product of the Radii; divide that Product by the Difference of the Radii multiplied by the Image, the Quotient is the Distance sought.

II. E X A M P L E I.

Let the Radius of Convexity be 10, the Radius of Concavity 25, and the Proportion of the Object to the Image, as 4 to 1.

Then	$\left\{ \begin{array}{cc} 25 & - & - & 25 \\ 10 & - & - & 10 \end{array} \right.$	Object 4	Image 1
Product	$\begin{array}{r} 250 \\ 2 \end{array}$	15 Diff.	Sum 5
	$\begin{array}{r} 500 \\ 5 \end{array}$		
	$\begin{array}{r} 15 \overline{) 2500} \\ 15 \\ \hline 100 \\ 90 \\ \hline 100 \end{array}$	(166 the Distance sought, <i>viz.</i> $16 \frac{6}{10}$ Inches.	

12. EXAMPLE II.

Let the Object be magnified in the Proportion of 4 to 1, by the same Lens.

Then	$\left\{ \begin{array}{cc} 25 & \text{---} & 25 \\ 10 & \text{---} & 10 \end{array} \right.$	<div style="display: flex; justify-content: space-between;"> Object 1 Image 4 </div>
Product	$\begin{array}{r} 250 \\ 2 \\ \hline 500 \\ 5 \\ \hline \end{array}$	<div style="display: flex; justify-content: space-between;"> <div> $15 = \text{Diff.}$ $4 = \text{Image}$ <hr style="width: 50px; margin: 0 auto;"/> 60 </div> <div> $\text{Sum } 5$ </div> </div>

$$\begin{array}{r} 60 \overline{) 2500} \\ \underline{240} \\ 100 \\ \underline{60} \\ 40 \end{array}$$

$(41 = \text{the Distance required,}$
 $\text{viz. } 4\frac{1}{10} \text{ Inches.}$

13. EXAMPLE III.

Let it be required to have the Object and Image equal by the same Lens. Then 4 times the Product of the Radii divided by their Difference, gives the Distance of the Object, thus ;

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$$\begin{array}{r}
 25 \quad \text{---} \quad \text{---} \quad 25 \\
 10 \quad \text{---} \quad \text{---} \quad 10 \\
 \hline
 \text{Product } 250 \qquad \qquad 15 \\
 \quad \quad \quad 4
 \end{array}$$

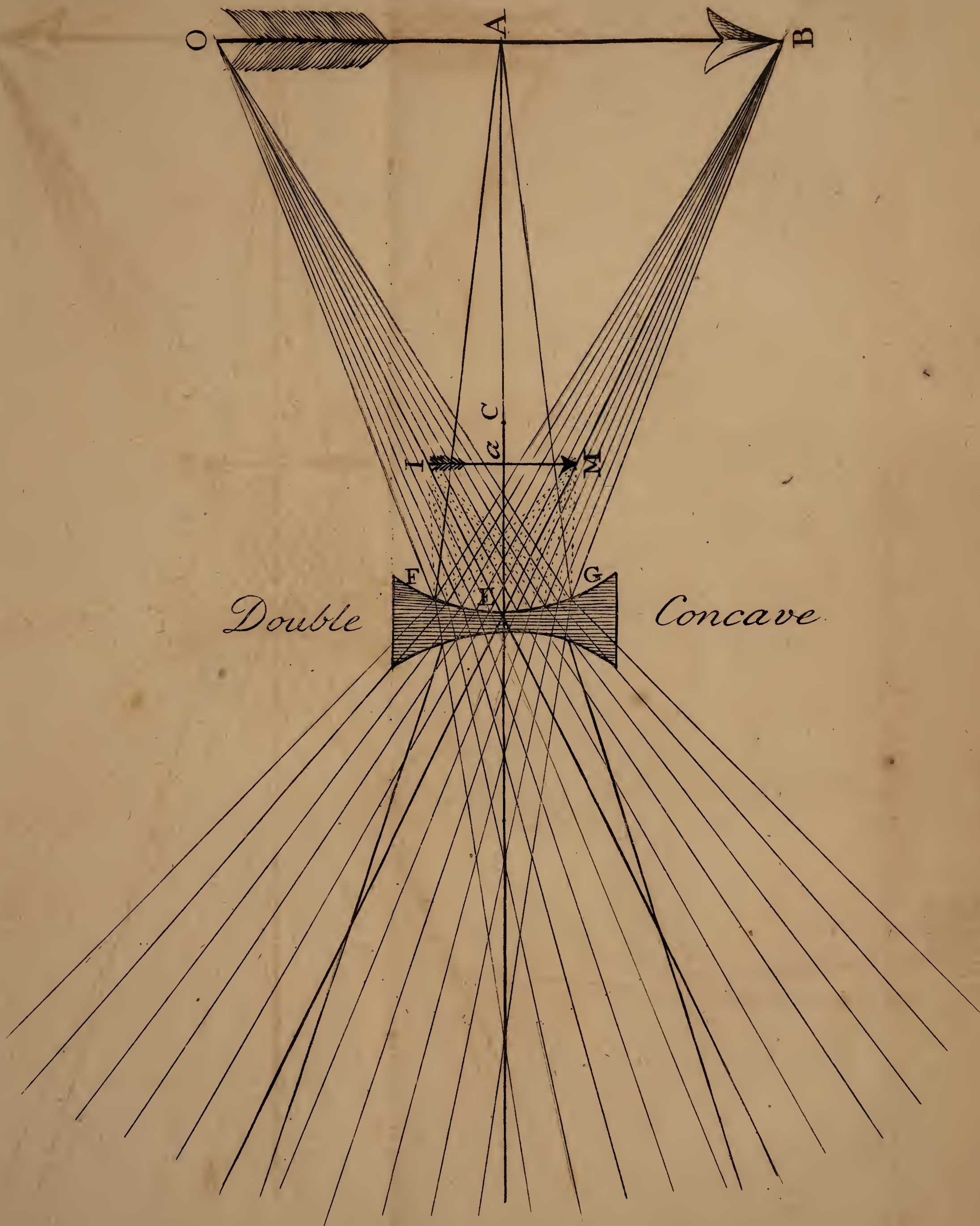
$$\begin{array}{r}
 15 \overline{) 1000} \quad (66 = \text{the Distance, } \textit{viz.} \quad 6 \frac{6}{10} \\
 \underline{90} \qquad \qquad \qquad \text{Inches.} \\
 100 \\
 \underline{90} \\
 10
 \end{array}$$

14. If the concave Side be turned to the Object, and the Image required at an affirmative Focus, the Rule is still the same, save only that in this Case, the Radius of Convexity must exceed the Radius of Concavity, as is plain from Theor. 14.

15. If the convex Side be towards the Object, and the Image required at a negative Focus; then you must multiply with the *Difference* of the Object and Image instead of their Sum; and if the Radius of Convexity be *greatest*, the Object cannot be *magnified*; if *least*, it can't be diminished.

16. But if the concave Side be towards the Object for a negative Focus, then if the Radius of Concavity be *greatest*, the Object cannot be *magnified*; nor *diminished*, if it be *least*.

17. If



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17. If the Radii of the Surfaces are equal, the Meniscus can neither magnify or diminish an Object, as appears by Theor. 15. Because, in that Case, all that is effected by one Surface is destroyed by the other.

18. An Object appears erect at a negative Focus, and *inverted* at an affirmative one. And these are the chief Properties of *Meniscus Lenses*.

19. A PLAIN LENS, or Piece of Glass, both whose Surfaces are perfectly plain, will magnify an Object in proportion to it's Thickness, according to the following

A N A L O G Y.

As the Distance of the Object, is to the said Distance increased by two Thirds of the Thickness;

So is the Object to it's Image.

20. E X A M P L E.

Suppose a Piece of Glass be $\frac{3}{10}$ of an Inch thick, then will an Object at one Inch Distance be to it's Image seen through the Glass, as 10 to 12, in Length, in Surfaces as 100 to 144, and in Bulk or Solidity as 1000 to 1728.

P A R T III.

Of D I O P T R I C Instruments *and*
Machines.

C H A P. I.

*Of the Structure of the Eye; and how
Vision is performed thereby.*

1. **T**HE Description of the Eye, I here intend, is rather Optical than Anatomical; as taking Notice of those Parts only which regard *Vision*, and assist more or less in effecting it.

2. In Plate XXII. Fig. 1. represents the Ball or Globe of the Eye, or rather a Section thereof through it's Axis. The Form of the Eye is too well known to need Description; it's Parts are of two Sorts, *viz.* Coats and Humours, of which in Order.

3. The Coats or Teguments of the Eye are the external Parts which contain the Humours, and are in Number 6, *viz.* The *Conjunctiva*, *Sclerotica*, *Cornea*, *Choroides*, *Uvea*, and *Retina*.

4. The

4. The *Tunica Conjunctiva* is common to the Eye-lid, and Ball of the Eye, which it firmly connects together, and makes what is called the *White of the Eye*.

5. The *Sclerotica* is the first or outmost of the *proper Coats* of the Eye represented by A E D B, Fig. 1. This Coat is thick, hard, and smooth, and on the fore-part is transparent where it forms

6. The *Cornea*, or *horny Coat*, because it resembles a thin Piece of transparent *Horn*, and is more protuberant than the rest; represented by A B. By this Part the Light first enters the Eye.

7. The *Choroides* lies under the *Sclerotica*, is much thinner than it, and on the fore-part thereof, between Q and Q is

8. The *Uvea*, sometimes called the *Iris*, which is of various Colours in different Persons; and in the middle has a round Hole, viz. I L, called *Pupilla*, or the Pupil of the Eye, by which the Light has admittance to the internal Substance of the Eye.

9. This *Iris* is of a round Form, is composed of two Orders of Fibres, one *Circular*, the other *Strait*, tending towards the Center of the *Pupil*, like Radii towards the Center of a Circle. By means of these Fibres, the Pupil is enlarged or contracted according as greater or lesser Light is required.

10. The *Retina* is not properly a Coat of the Eye, being only a fine Expansion of the
I Optic

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Optic Nerve over the Bottom of the Eye, nearly opposite to the Pupil. On this delicate Membrane are painted the Images of Objects formed by the *Crystalline Humour*. It's Representation is at S, S, S.

11. The *Humours* of the Eye are three; the *aqueous* or *watery Humour*, the *Crystalline*, and the *vitreous* or *glassy Humour*. The *Aqueous Humour* hath very much the Appearance of *Water*, it makes the Eye globular on the fore-part, lying immediately under the *Cornea*, and is denoted by QQQ.

12. The *Crystalline Humour* lies next the *Aqueous* behind the *Uvea*, opposite to the *Pupil*; represented by NOP. In Form it resembles a Double-Convex Lens, being somewhat more convex on the external, than on the internal Surface. It is connected to, and suspended by the *Ligamentum Ciliare* MN, MN, on the third Humour, called

13. The *vitreous* or *glassy Humour*; this is the largest in Quantity, making the hinder and far greatest Part of the Globe of the Eye, represented by RRR. It is contained within a fine Membrane, and over all the spherical Superficies thereof is spread the *Retina* in a most fine and curious Manner, quite to the *Ligamentum Ciliare*.

14. These are constituent Parts of the Eye; the Uses whereof, with regard to Vision, I shall now more particularly describe. But shall first premise, *that clear and distinct Vision is produced*

produced only by Parallel Rays, or such as are nearly so. For these Rays have but one determinate Focus, whereas the Focus of Diverging Rays is as various as the Distances of Objects, and therefore can't produce Vision in a Machine of a fixed and determinate Form and Position, as that of the Eye is well known to be.

15. Now in order that Rays, which proceed from near Objects, may be *nearly Parallel*, that they might render such Objects distinctly visible, it was necessary that the Aperture of the Eye, by which they were to be received, should be very small, that so the Base of a *Cone* of Rays, proceeding from any Point in such an Object, being very small in proportion to the Length of the said Cone, that part of the Cone at the Entrance of the Eye might differ very little from a *Cylinder* of Rays; or, which is the same thing, their *Divergency* should be so very small, that they might differ very little from *Parallel Rays*.

16. To illustrate this Matter, let CD, Fig. 2. be the Diameter of the Pupil or Aperture of the human Eye; and O a Point at the Distance of 6 Inches draw the Rays OC, OD; and let AC and BD be Parallel Rays. Now it is evident, that because CD is very small in respect of CO, the Divergency of the Rays OC, OD, is so very small also at the Pupil CD, that they almost coincide with the Parallel Rays AC, BD, at their Arrival;
and

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and therefore may be esteemed as such, and will produce distinct Vision.

17. But were the Point O nearer to the Pupil CD, or if the Pupil were larger, in either Case, the Rays would fall more diverging on the Eye, and the Images of Objects would be formed at a Point beyond the Bottom of the Eye; and so their Pictures on the *Retina* would be very imperfect, and consequently Vision would be very indistinct and confused.

18. Hence it appears why we can never see any thing distinctly at a less Distance than about *six Inches* by the bare Eye; and also why Objects at all considerable Distances, appear distinct and perfect: But all vastly distant Objects appear both *indistinct* and *obscure*. They appear *indistinct*, because their Images in the Bottom of the Eye are so *extremely small*, that the Distinction of Parts is imperceptible to the Mind it self, all the Parts taken together making as it were but one physical Point. Thus if a Man of 6 Foot Stature were to be viewed at the Distance of a Mile, his Image on the *Retina* would be but the *thousandth Part of an Inch* in Length; no wonder then if his Eyes, Nose, Mouth, &c. appear indiscernible in a Picture of such extreme *Miniature*.

19. They appear more or less *obscure*, according to their Distance or Degree of Light; for Objects at a great Distance are seen thro' a greater

a greater Quantity of the Medium, than those at a small one; and therefore the Rays, by a greater Refraction, will be much more effete, and produce a more obscure Vision in the former than in the latter Case.

20. Again, it is well known, that all Objects appear *bright* or *obscure*, according to the Degree or Quantity of Light with which they are illuminated; thus distant Objects, in a cloudy Day, appear dark and obscure, whereas, when the Sun shines full and strongly upon them they appear clear and bright.

21. I shall now observe the Method Nature takes in effecting Vision by the Eye; and how every Part is made subservient to so noble a Purpose and Service.

22. Suppose then CD were parallel Rays, falling on the Eye at AB, Fig. 3. The Aqueous Humour Q being about the same Density with Water, and of a Convex Surface, by means of the *Cornea* AB, the Rays CD would be made to converge towards a Point F, at the Distance of 4 times the Radius of the Convexity of the *Cornea*, if there were no other Medium to prevent it; this is evident from the Theorem in Chap. I. Art. 6. of Part II.

23. But the Point F being beyond the Bottom of the Eye, makes it necessary that some other Body, of greater Density, should be interposed in Form of a Convex Lens, to gather them to a Point nearer the Eye, and yet

a little beyond it; and this is effected by the *Crystalline Humour* NN.

24. I said *a little beyond the Eye*, because Nature determining the Eye to be of a globous Form, and for that Purpose having filled all the hinder Part with a Medium RR, of a less Density than the other Medium NN, and of a *Concave Surface* where it received the Rays, the Ray must necessarily be still *more converged* by this Medium, which is the *Vitreous or Glassy Humour*; and therefore had the *Crystalline Humour* thrown the Rays *just on the Bottom of the Eye*, the *Vitreous Humour* must have converged them to a Point behind it; and so would have made the Vision indistinct and confused.

25. The Density of the *Vitreous Humour* therefore is such, that the Rays are united at the Bottom of the Eye, on the Point S, where they paint the Image of the Object on the *Retina*, and in that Case only, produce *distinct Vision*.

26. Since with Regard to Objects at different Distances, those which are nearest will have their Foci, at a farther Distance from the *Crystalline Humour*, than those which are farther off; therefore a Power is given to the Eye to alter the Form of the *Crystalline Humour*, viz. To render it more or less Convex, by the Muscular Contraction and Relaxation of the *Ligamentum Ciliare*, to which it is connected, which also must occasion a
greater

greater or *lesser* Concavity in the *Vitreous Humour*; by which means the *focal Distance* is lessened for near Objects, and enlarged for those farther off, and is so nicely adjustable to the *Retina*, for all Distances of Objects, that their Images are all exactly painted thereon, in a natural or good Constitution of the Eye.

27. In the last Place, it has been observed, that there is a Power of dilating and contracting the Pupil of the Eye, by means of the Muscular Fibres of the *Iris*; on this Account, if Bodies be situated far distant, the Pupil being dilated receives Rays more diverging, and in a greater Quantity, and therefore such Objects appear more *distinct* and *enlightened*.

28. On the other hand, if Objects are situated *very near*, or are *extremely bright*; the Pupil is contracted, and takes in only the least divergent Rays, so that the Objects are more distinct; and, at the same Time, the *extreme Brightness* is diminished, so as not to be offensive to the Eyes.

C H A P. II.

Of the Position of the Image in the Eye; of the apparent Magnitude of Objects.

1. **H**AVING fully explained the Nature of Convex Lenses, and also largely shewn that Vision is performed in the Eye by Refraction through the *Crystalline Humour* principally, which bears the Form of such a Lens, it must needs be very easy to conceive, that the Images of all Objects are formed in an *inverted Position* in the Bottom of the Eye.

2. Thus in Fig. 4. Plate XXII. Suppose OAB an Object, in an erect Position before the Eye CD; then will the Pencil of Rays OEIF, paint the Extremity O in the Point I, and the Pencil BFME, will paint the Extremity B in the Point M; and since all the Points between O and B are represented between I and M; IM will be the Image of the Object OB. Again, since the Axis of the Pencils of Rays cross each other in the Pupil, which is their common Base, the Image must necessarily

necessarily be painted in an *inverse Position*, in the Bottom of the Eye.

3. Here constantly occurs the Question, *How it comes to pass, since the Images of all Objects are painted in the Eye invertedly, that we see them erect?* To this several Answers are given, but unsatisfactory; I can think of no better way to account for this Paradox, than as follows.

4. If we suppose an Eye viewing the Point I, in the Image in the Direction of the Axis of the Pencil of Rays which paints it there, it will refer it to the Point O in the Object, and there behold, and contemplate it: in like Manner, the Point M would be referred to the Point B, and by a successive Application of the Eye to every Point of the Image, the whole Image IM will be referred to, and considered in the Object OB. If therefore we conceive the *Mind* to be *all Eye*, capable at once of viewing every Point in the Image in it's requisite Directions, it must necessarily refer the Image in the Eye to the Object without it, which also will necessarily cause a Change of the Position; and therefore the Image, though *inverted* in the Eye, will be viewed and contemplated by the Mind in an *erect Position* in the Object.

5. We now proceed to consider the *apparent Magnitude* of Objects; which here shall be that of a *Line*, viz. it's Length. In Fig. 5. Let AB be an Object viewed directly by the

Eye QR: From each Extremity A and B, draw the Lines AN and BM, intersecting each other in the CrySTALLINE Humour in I, bisect AB in K, and draw IK; then is the Angle AIK, half the optic Angle AIB, which is the Measure of the *apparent Magnitude* or Length of the Object AB.

6. *Diverse Objects* AB, CD, EF, whose *real Magnitudes* are very unequal, may be situated at such Distances from the Eye, as to have their *apparent Magnitudes* all equal. For if they are so situated that the Rays AN, BM, shall touch the Extremities of each, they will then all appear under the same optic Angle AIK, which is equal to NIM, which determines the Magnitude of the Image MN, in the Fund of the Eye, the same for them all; and therefore they must all appear of an *equal Magnitude*.

7. *Objects situated at different Distances, direct to the Eye, whose apparent Magnitudes are equal, are to each other, as their Distances from the Eye directly.* Let the Objects be AB and CD, then because the Right-angled Triangles CIL, and AIK, are similar, it will be, as $IK : IL :: AK : CL$; but AK is half AB, and CL half CD; therefore it will be, as $IK : IL :: AB : CD$.

8. *Objects of equal Magnitude, situated directly before the Eye at unequal Distances, will appear unequal.* For let AB and GH be two Objects directly before the Eye at different Distances

Distances IK and IS; draw the Lines GP and HO crossing each other in I; then is the optic Angle GIH, manifestly greater than the Angle AIB, and the Image OP made by the former greater than the Image MN made by the latter. Therefore the Object GH, is *apparently* greater than the Object AH, tho' it is but equal to it.

9. *Equal Objects situated directly before the Eye at unequal Distances, have their apparent Magnitudes reciprocally proportional to their Distances.* For let AB, GH, be two equal Objects at unequal Distances IK, IS, from the Eye produce IG and IH till they intersect AB, each way produced in T and V. Then will TV be the apparent Magnitude of GH, at the Distance IK. Since the Triangles ISG and IK T are similar, we shall have $IS : IK :: SG : KT$; but SG is equal to AK; therefore it will be $IS : IK :: AK : KT$.

10. From what has been said it appears, that there is *no Standard of the true Magnitude of Things*. All that we can be sensible of is *the Proportion of Magnitude*. And yet, notwithstanding the sensible Magnitude of things is ever mutable, and varies in proportion to the Distance, we scarcely ever judge any thing to be so great or small as it appears to be, or that there is so great a Disparity in the visible Magnitude of two equal Bodies at different Distances from us.

11. Thus for Instance. Suppose two Men of six Feet Stature each, stand directly before a Person, one at the Distance of one Rod, the other at the Distance of an 100. We should, indeed, observe a Difference in their apparent Bigness, but should hardly think one appeared an 100 times greater or lesser than the other ; or that one appeared 6 Feet high, the other not $\frac{1}{4}$ of an Inch.

12. But this happens from our Prenotions, or Judgment, that we have beforehand formed of things. We know the Proportions of Magnitudes in general, and are apt to retain the Ideas of things we know to be true ; and to prefer them to others which are presented by the Senses, and exist there only, and not in the true Relations of things.



C H A P. III.

Of the Faults or Defects of Vision, and how they are remedied by Convex and Concave Lenses in Spectacles.

1. **I**T has been already shewn, that in order to effect perfect and distinct Vision, the Images of Objects are to be painted precisely on the *Retina*, in the Fund of the Eye, and that by Rays which are either parallel, or nearly so, in a natural and good Configuration of the Eye.

2. But as it often happens, (for Nature herself is not ever uniform to a Mathematical Nicety) that the Form of the Eye, but principally of the Crystalline Humour, is such, that it is either a *little less or more Convex than is just*; so of Consequence the Focus, or Place where the Images of Objects are formed, will be a little beyond or behither the Bottom of the Eye, which, in either Case, will prevent the Perfection of Vision, and render it indistinct and confused.

3. This Imperfection of Vision is, in a great Measure, remedied by the Succours of Art: For if, by a vicious Formation of the

Eye,

*In Order to clear & distinct perception of Objects
the Rays which they emit should be collected
the Retina. for it is the Focus. Part
of the Eye has naturally enough to give us*

Eye, the Focus be made to fall on this or on that Side the *Retina*, yet Glasses may be formed to such a Degree of Convexity or Concavity, that upon applying them to the Eye, the Focus shall be truly adjusted to the *Retina*, and thereby cause distinct Vision.

4. Thus suppose the *Cornea* CD, (See Fig. 6.) or the Crystalline EF, or both, should chance to be *too flat*, either from Nature, or (what is most common) from a Deficiency of the Aqueous Humours through Age, in *Presbyta*, or *Old Men*, so that the Rays which proceed from any Point A, are made to converge at a Point a, beyond the Eye, and thus cause a confused and imperfect Vision; I say, this Imperfection is cured, in a good Measure, by Convex Lenses in Spectacles.

5. For from what has been shewn of Convex Lenses, it is evident, (1.) That Rays coming from a distant Point A, and fall diverging on a Convex Lens GH, are thereby made to proceed *less diverging* than before. (2.) Those Rays which are less diverging, have their Focus nearer to the Lens, than those which are more so. Consequently the Rays, which proceed from the Point A, are by the Interposition of the Convex Lens GH, just before the Eye, made to fall *less diverging* on the Crystalline EF, and therefore will be converged to a Point b, nearer thereto than the Point a; and the Convexity of the Lens GH may be
such,

such, that the Point *b* shall be precisely on the *Retina*, and so cause *distinct Vision*.

6. For this Reason *Presbytæ*, or old People, always make use of Lenses more or less convex in their Spectacles, as their Eyes are flatter or rounder. And here it is to be observed, *that Objects appear brighter*, as well as more distinct, by means of Convex Lenses; for as much as they bring the Diverging Rays nearer together, some which otherwise would have fallen without the Pupil, will now be brought within the Compass thereof, and so a greater Quantity of Rays entering the Eye, the Object will appear *more bright*.

7. Again, we farther observe, *that Objects seen by Spectacles of Convex Lenses, appear to be more distant than they really are*. For the Rays which come from the Point *A*, being by the Lens *GH* made to proceed less diverging, they will appear to come from the Point *B*, which is farther off, because Rays, as they are more or less divergent at the Eye, come from Objects which are nearer or farther distant from the Eye.

8. Hence also the Reason is evident, why the older Men grow, the more they lose a distinct Vision of near Objects; so that very flat Eyes can see only distant Objects without Confusion: For when in Youth the Eye was so convex, as to form an Image of nigh Objects on the *Retina*, in Age that Convexity of the Eye diminishing, will cause the Images
of

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of the same Objects to be painted at a Focus farther from or behind the *Retina*; and those Objects only which are at a great Distance, will have their focal Distance short enough to fall on the *Retina*; and so these only can produce *distinct Vision*.

9. On the other hand, if the Cornea CD, or Crystalline EF, or both, (See Fig. 7.) be more convex than just, the Rays which proceed from a Point A, will be made to converge too soon; that is, to a Point a, which is between the Crystal Lens EF, and the *Retina*; and therefore must needs produce a *confused Vision*. A Person who hath these Eyes, is called *Myops*, *i. e.* Mause-ey'd; but he is vulgarly said to be *purblind*, or *short-sighted*.

10. He is with good Reason said to be *short-sighted*; for since all distant Objects have the shortest focal Distances, and so have their Images formed short of the *Retina*, they must needs appear indistinct and confused. Therefore only those Objects, which are at a short or near Distance, can produce *distinct Vision*; and that they do by having a longer focal Distance, and so reach the *Retina* before their Images are formed.

11. A *Myops* hath his Imperfection of Sight greatly relieved by *Concave Lenses*. For since Rays, which fall divergent on such a Lens, are made to diverge the more by Refraction through it; therefore the Rays which come from

It is only in the Focal Point,
that the Rays can sufficiently
converge to give distinct sensation
of the object. But is only given
the focal Point - Point - a distance
or focus can be found by
the distance of an object -

In eyes that on the corner
 the rays concentrate to
 seen — the is recorded
 eye — corner of glass —
 In eyes that on the corner
 the rays concentrate to
 seen — the is recorded
 eye — corner of glass —
 In eyes that on the corner
 the rays concentrate to
 seen — the is recorded
 eye — corner of glass —

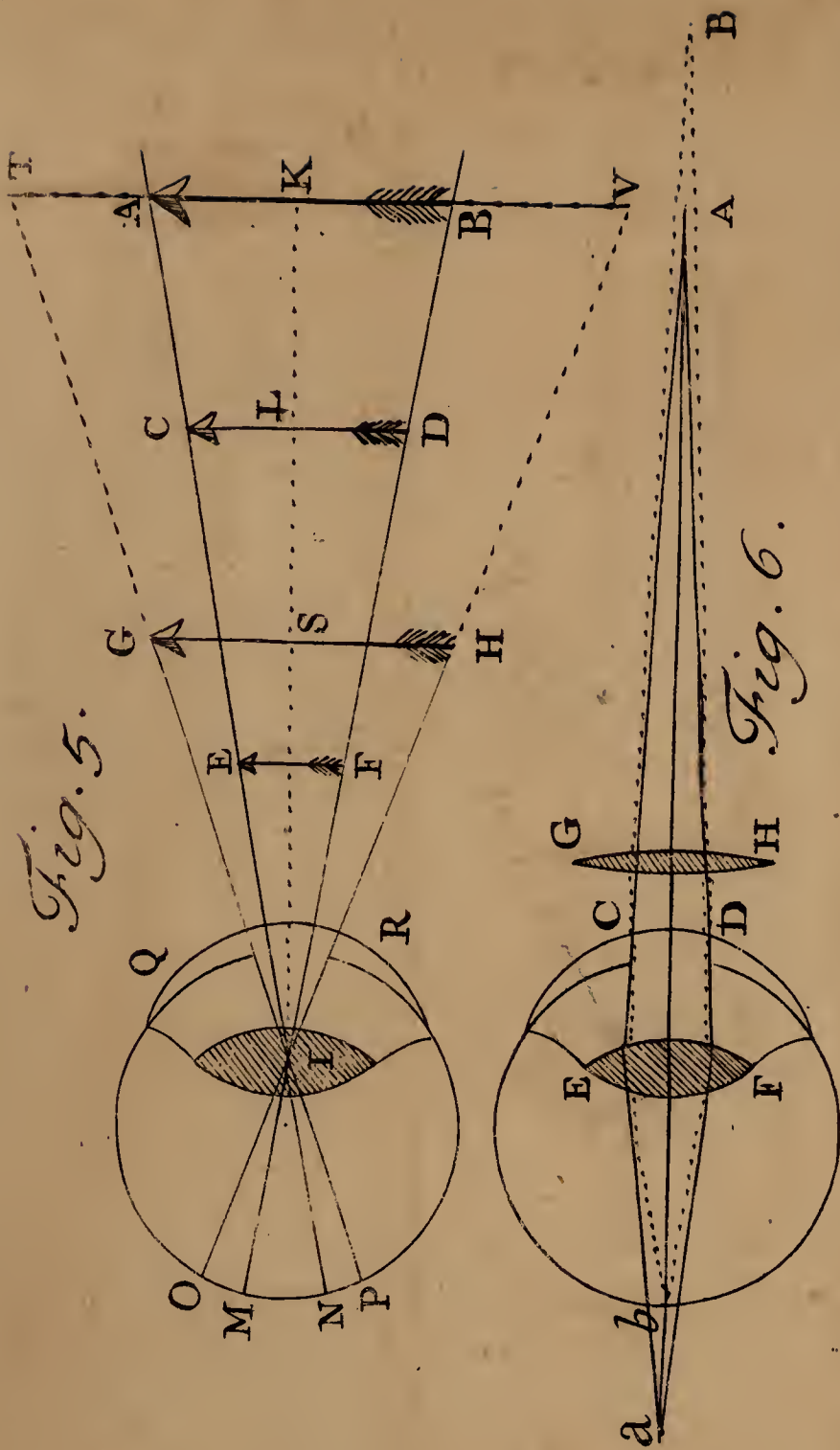


Fig. 5.

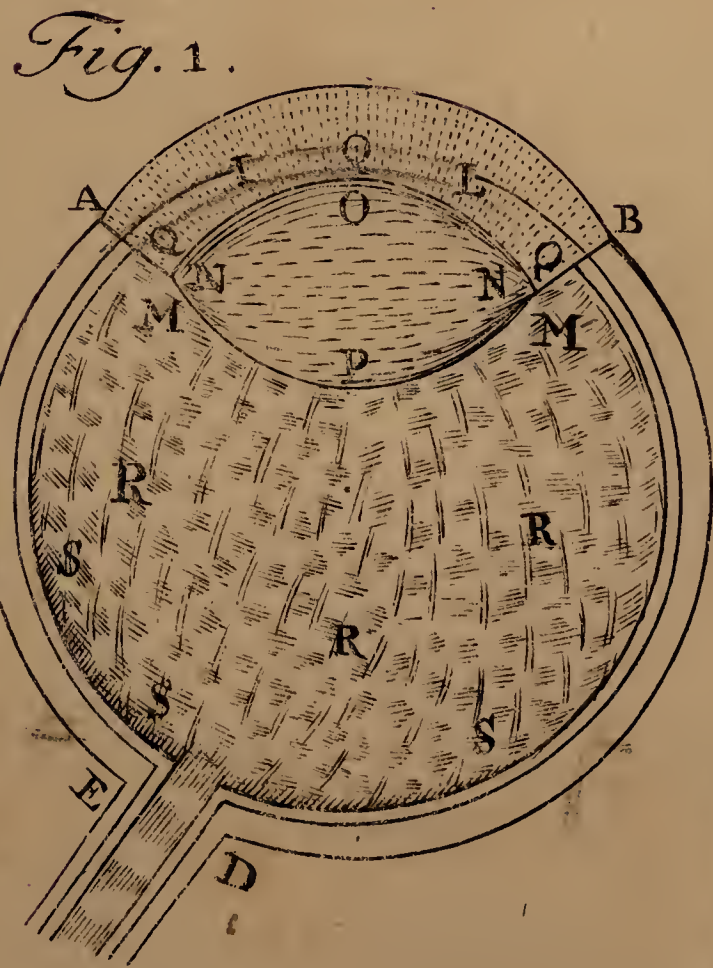


Fig. 1.

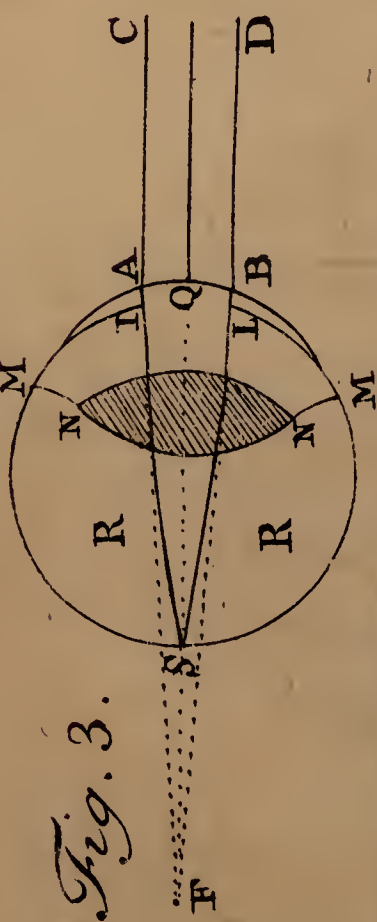


Fig. 3.



Fig. 2.

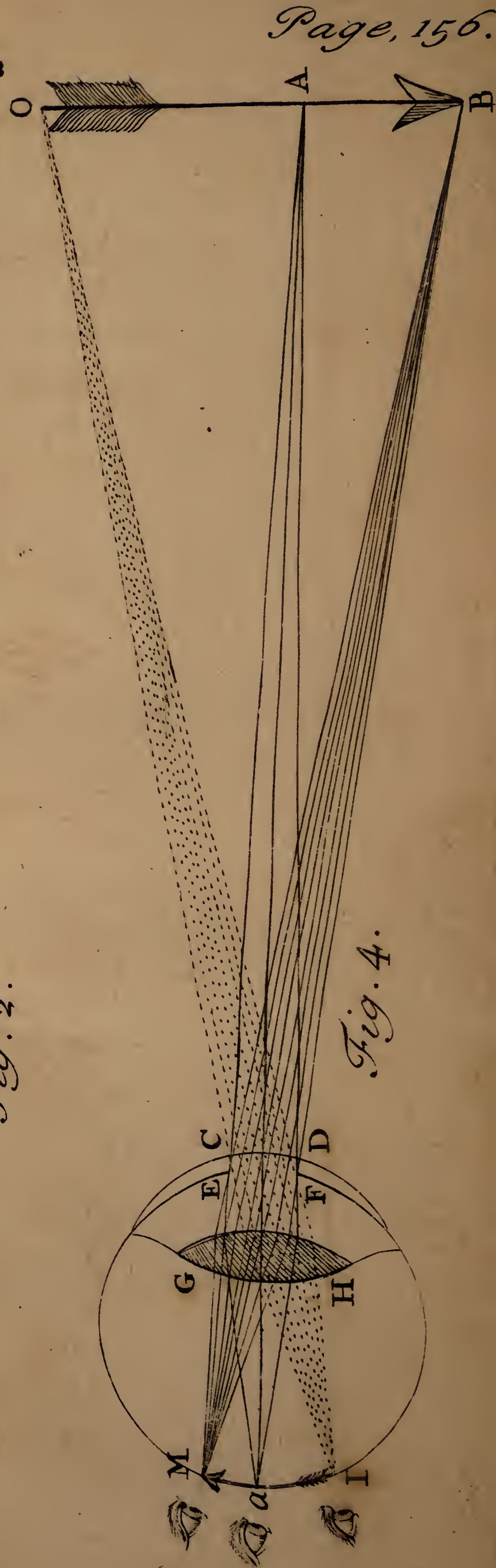


Fig. 4.

from any distant Object A, in passing through the Concave Lens GH, are made to fall more diverging on the Eye CD, and seem to proceed from a Point B, much nearer the Eye than the real Point A. Since therefore the Radiant Point is brought nearer to the Eye, it's Focus in the Eye will be removed farther from the Lens EF, and such Glasses may be chosen as shall exactly throw the Focus on the *Retina* at b, and there produce *distinct Vision*.

12. From hence we observe, that *Myops*, or purblind People, using Spectacles with Concave Lenses, (1.) Do behold things at *a nearer Distance than they are*. (2.) That Objects appear *less bright to them*; for the Rays being spread wider by the Lens, they cannot enter the Pupil in so great a Quantity as otherwise, and so the Object cannot be so much enlightened. (3.) Their *Eyes are amended, and made better by Age*, contrary to what happens to all other People. For as the Fault of their Eyes proceeds from a too great Protuberance of the *Cornea*, and *Crystalline Humour*; so this and the Aqueous Humour lessening by Age, diminishes the Convexity of the Eyes by Degrees, and renders them capable of viewing distant Objects better and better.

C H A P. IV.

Of the Camera Obscura, or darkened Chamber, and the Instruments pertaining thereto; by which Means the Images, or Pictures of external Objects, are curiously painted in their natural Colours and Motions.

1. **T**HE Words *Camera Obscura*, literally interpreted, is a *darkened Vault* or *Roof*; and from thence it came (with a little Difference) to signify a *Chamber*, or *Box*, or any Place made *dark* for *Optical Experiments*.

2. The *Camera Obscura* is, tho' a simple, yet a very curious and noble Contrivance; in as much as it most clearly and naturally explains the Nature and Manner of Vision in the Eye, and, at the same Time, entertains the Spectator with a most exquisite Picture of the Objects without, in their natural Proportions, Colours, and Motion, more vivid and beautiful than the Life it self; filling the Beholder with Delight and Surprise.

3. The Way of making a *dark Chamber* is very easy, and not expensive, and is as follows.

follows. The Chamber or Room proposed to be darkened, should have Window-Shuts to each Window, which being close shut, should be so true to the Frame, as to exclude all Light possible.

4. In one of the Window-Shuts there is to be a circular Hole cut, about 3 or 4 Inches Diameter, in such a Part thereof, as is judged most convenient, and capable of taking in a good View or Prospect of external Objects.

5. In this Hole is placed an Instrument called a *Scioptric Ball*, which hath three Parts, *viz.* A Frame, a Ball, and a Lens. The Frame consisteth of two circular Pieces of Wood, made spherically hollow through the middle, and screw into each other. In this hollow Part is placed a spherical *Ball of Wood*, contained by the two Parts of the Frame screwed together, and is voluble therein more or less easily, as the two Parts of the Frame are screwed less or more tight together. In this Ball is a circular Hole made thro' the middle, which hath a Screw at each End, in which is placed and fixed a Lens, either a *Double* or *Plano-Convex*.

6. The *Scioptric Ball* of this Structure, is a Sort of *Artificial Eye*, which very aptly represents the Natural Eye in Form and Office. For (1.) The *Frame* or *Socket* answers to the *Orbit* of the *Natural Eye*. (2.) The *Wooden Ball*, which turns every Way in the Frame, resembles the *Globe* of the Eye voluble every
Way

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Way in it's Orbit. (3.) The *Hole* through the *Wooden Ball*, represents the *Pupil* of the Eye. (4.) The *Convex Lens* in the Ball, corresponds to the *CrySTALLINE Humour* in the Eye. (5.) The *Dark Chamber* it self, is like to the internal Part of the Eye, which is lined all about, and under the *Retina*, with a Membrane, over all which is spread a Mucus of a very black Colour. (6.) The *White Wall*, or Frame of white Paper to receive the Picture of Objects on in a dark Chamber, is the true Representation of the *Retina* in the Eye.

7. So that the Structure of the *Scioptric Ball* and the *Eye* is perfectly similar; and the same Agreement will be found in the Offices of every Part of each. For (1.) The Frame of the *Scioptric Ball* is screwed or tacked upon the Hole in the Window-Shut, as the Eye is fastened within it's Orbit by Muscles. (2.) The *Ball* is voluble in the Frame every Way, to take in a View of Objects on every Side, as the Eye is in it's Orbit. (3.) The Hole in the Ball is, for the Admission of a competent Quantity of Rays, as is the Pupil in the Eye. (4.) The Lens in the Ball collects the Rays, and unites them at it's focal Distance, where it makes a Picture of the external Object placed before it; which is the Office of the *CrySTALLINE Humour* in the Eye. (5.) The white Wall or Paper, held in the Focus of the Lens in the Chamber, is to receive and shew the said Picture of Objects to Spectators; as the
Retina

Retina presents to the Mind a View of the Picture made on it by the Eye. (6.) The Chamber is made *dark* to render the Picture visible; as for the same Reason, the internal Part of the Eye is furnished with a *black lining*.

8. I shall give an Illustration of this Matter in a curious Scheme, which I have borrowed from Dr's *Gravesande*, (See Fig. 1. Plate XXIII.) where EF represents a darkened Room or Chamber; in one Side thereof IK is made the circular Hole V; in which, on the Inside, is fixed the *Scioptric Ball*; at some considerable Distance is exhibited a Prospect or Landskip of *Houses, Trees, &c.* ABCD. The Rays which pass from this Prospect to the Lens V in the Ball, are by it converged to their respective *Foci*, on the opposite Wall or Side of the Chamber GH, where they all together paint a most lively and beautiful Picture of all the Objects in the said Prospect.

9. This is Nature's *Art of Painting*, and it is with Ease observed, how infinitely superior this is to the finest Performance of the *Pencil*. For, (1.) You have here the *Perspective in Perfection*; that is, the just Diminution of Objects proportionate to the Distances, or the Proportion of the Images to the respective apparent Magnitudes of the Objects to an Eye at V. (2.) The *Colouring* is here perfectly *just and natural*; and not only that, but very much heightened, and rendered more
L beautiful;

beautiful; thus *green* Objects appear more *intensely green* in the Picture; *yellow, red, blue,* or *white* Flowers, appear incomparably more beautifully so in the Picture. (3.) The *Lights* and *Shadows* are not only perfectly just, but also greatly heightened, and make the Images appear extremely prominent and natural. (4.) The *Motions* of all the Objects are exactly expressed in the Picture; the Leaves *quiver*, the Boughs *wave*, the Birds *fly*, the People *walk*, the Cart is *drawn*, the Smoke *ascends*, the Clouds *soar*, the Ships *sail*, &c. and all as natural as the Life, and much quicker, as it is performed in a lesser Scene.

10. These are the inimitable Perfections of a Picture drawn by Nature's Hand; in Comparison of which, how *mean*, how *coarse*, how *imperfect*, yea, what sorry daubing is the finest artificial Painting! Select the peculiar Excellencies of the principal Artists, the *just Proportions* of Raphael, the *natural Tints* of Titian, the *pure Stile* of Corregio, the *Decorum* of Tibaldi, the *Terrour* of M. Angelo, the *Air* of Guido, the *Designing* of the Romans, the *Shadowing* of the Venetians, and the *Colouring* of the Lombards, all united would be unable to effect so finished a Piece, in any Branch of their Art, as Nature can exhibit with a *single Lens* only. The *Camera Obscura* is, at the same Time, the Painter's *Aid* and *Reproach*; from hence he receives the best Instructions; from hence he learns his

his Imperfections ; here he views what he should do, and knows it is what he cannot do.

11. There is one thing which may be thought an Imperfection in the Picture of a Dark Chamber, and that is, the *inverted Position of the Images* ; but, strictly speaking, this is not so, because Nature has furnished us with several Methods to make the Picture *erect*. For if it be a Sheet or Frame of Paper which receives the Picture, it is but holding it before you, and looking downwards on it, and every thing is right. Or if you stand before the Picture, take a *Looking-Glass*, and hold it against your Breast under an acute Angle, and looking therein you will see all the Images of the Picture restored to their natural or erect Position ; and not only so, but the Reflection of the Mirrour will give it such a Glare or Lustre, as makes it seem very surprizing and delightful. This may likewise be done by placing a large *Concave Mirrour* before the Picture at such Distance, that the Image of the Picture may appear before the Mirrour, which will then be erect and pendant in the Air.

12. There is another Method of erecting the Images on the Picture, and more direct than any of the foregoing, but yet is neither so easy to be done, nor so good if it be done, as it is by them ; and that is by placing another *Convex Lens* behind the Paper or Parti-

tion, which receives the first Picture at twice the focal Distance of the said Lens; if then a Hole be made in the Partition to permit the Rays to pass on to the second Lens, (which must be placed in the Axis of the first) there will be a Picture formed thereby, wherein the Images will be *erect*, and as large as in the first, but not so bright, nor will the Field or Extent of the Picture be considerable; and therefore as this Method is seldom practicable, so it is as little worth while.

13. In making a Dark Chamber, the Glass should not have too small nor yet too large a focal Distance. For if it be too short, the Images will be very small, and not distinct and discernable, and will be so near the Window, that there will not be Room for a Person to stand to view it. The Lens should have it's Focus at *three Foot distance* at least.

14. On the other hand, the *focal Distance* of the Lens should not be *too large*; for if it be larger than the Distance of the opposite Wall, your Design will be frustrated, as you can have nothing to receive the Picture upon. And if your Room or Chamber be very large, yet will the Picture be faint, and the Images less pleasing on a double Account. For, (1.) The focal Distance being very great, the Images will be proportionally large, and therefore the more faint and obscure. (2.) The larger the focal Distance, the larger the Diameter of the Lens, or the Hole in the *Scioptric Ball*,

Ball, which therefore will admit too much Light, for the Chamber to be sufficiently dark for viewing the Picture; and if you make the Hole smaller, there will not enter Light enough to make the Images visible at so great a Distance.

15. The Focus then should not be at a less Distance than 3 Feet, nor at a greater than 15 or 20 at farthest; and those from 6 to 12 are by much the best of any. If it happens that your Lens be too short a focal Distance, you may magnify the Images to almost what Degree you please, by viewing the Picture with a large Convex Lens in your Hand; by this means you may supply the Want of Lenfes of distant Foci, which are very scarce and dear; and thus a Lens of 5 Feet may be made to answer the End of one of 15 Feet.

16. A Dark Chamber ought never to be attempted *but when the Sun shines*; for it is necessary the Objects, which are intended to make your Landskip, should be strongly illuminated by the Sun-Beams; otherwise the Picture, which ought to be vivid, bright, and beautiful, will appear obscure, dull, and of a dirty Hue; as the Objects themselves would appear by Twilight.

17. Whence it follows, that a *South Window* is never to be used for this Purpose; because the Sun can never enlighten the North Side of the Object, which alone can be taken

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in by a South Window. Besides, the Sun in this Case would be apt to shine on the Glass, which would make the Picture appear with a false or confused Lustre, and therefore you ought to be very careful always to avoid that thing's happening.

18. An *East Window* will do very well in the *Afternoon*, as a *Western* will for the *Morning*; but none is so good, or will make so noble and glorious a Picture as a *North Window* about *Noon*; for then the Sun being in his Meridian Height, and shining with the greatest Strength and Splendor possible, the Picture made in such a Case will far exceed all others in *Vivacity*, *Beauty*, and *Lustre*.

19. This noble Experiment is not only admirably pleasing and delightful in it self, but also very useful for many Purposes of Business, principally with Respect to *Perspective*, *Painting*, *Designing*, &c. For whatsoever is to be *drawn* or *painted*, if it be first exposed to the *Scioptric Ball*, the Perspective thereof will be truly formed, and the Lights and Shadows for every Position, and Action of the Objects, will be represented just as they ought to be in the Images of the Picture. In short, the *Camera Obscura* is the School in which every *Designer* and *Painter* ought to learn the first Rudiments of his Art. He can never, without this dark Education, turn out a bright Proficient. No Instructions can come up to those of Nature; her Lessons are all perfect

fect Patterns and Ensamples, and every one excels in Painting and Drawing so much the more, by how much the truer he copies after them.

20. And another great Convenience is, that the Picture of Objects may be made of any Size you please, either less or greater than the Life, if the Objects be moveable. For if you place the Object farther from the Ball than twice the focal Length of the Lens, the Images will be less than the Object; if they are placed at just twice the focal Length of the Lens, the Images will be just as big as the Life; if they are placed nearer, the Images will be greater than the Life; all which is manifest from the *Theory*, and which I have before observed in the practical Part.

21. Also for immoveable Objects, as those of Houses, Gardens, Fields, Trees, &c. if you have different Lenses, you may form the Picture of so many different Sizes, the shorter Focus making the lesser Picture, and the larger Focus the largest. But these Matters are so obvious, that I need not farther insist on them.

22. But the Use of the *Camera Obscura* does not rest in *Drawing* and *Painting*; but the *Optician* himself is greatly interested therein. By this grand Experiment he demonstrates ocularly the Principles of his Art. For by admitting the Sun-Beams thro' the Hole of the Window-Shut into the darkened Chamber, he can actually shew the Focus of *Parallel Rays* by Reflection from *Concave*

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Mirroures, and by Refraction through *Convex Lenses*, to be just as the Theory defines them, by holding those Glasses in the said Beams. Thus also he shews, that the Sun's Rays, after Reflection from *Convex Mirroures*, and after Refraction through *Concave Lenses*, are ever made to diverge agreeable to the Theory of *Parallel Rays*.

23. Again, by holding a large *Convex Lens* in the Sun's Rays, he can by that means produce *Diverging* and *Converging Rays* in any Degree, and so can prove the Truth of all those Properties of *Convex*, *Concave*, and *Meniscus* Glasses, with respect to these Rays, as the Theory teaches. Also the Ratio or Proportion of the *Sines of the Angles of Incidence*, and Refraction in Water, Oil, Glass, &c. is easily proved to be as it is stated.

24. The Reason and Nature of *Telescopes* and *Microscopes* is demonstrated to the Senses hereby; for if the Glasses in either be taken out of the Tubes, and fixed at their proper Distances, on a strait Piece of Wood, so that the Sun Beams fall directly and fully on the Object-Glass, you will then see the Forms they take in their Course thro' the Glasses fixed at their proper Distances, to be such as the Theory points out, and are necessary to answer the Purposes of these Instruments.

25. But in making Experiments with the Sun-Beams in a Dark Chamber, since those Rays fall with great Obliquity, especially in the

the *Summer-Time*, and so the more inconvenient for Use, the best Way is to fix a small Plain Mirrour to the under Part of the Frame of the *Scioptric Ball*, immediately next the Ball itself, by a Hinge, that by moving up and down it may receive and reflect the Sun-Beams in any Direction whatsoever; and as by this Contrivance they may be made *parallel*, it will be found extremely useful and expedient on many Occasions.

26. If the Mirrour be fixed to the Ball itself, then may the Cylinder of Rays be not only cast in any Direction, but also to any Part of the Room, than which nothing can be more useful, as I have found in numberless Cases, having made this Addition to one of my *Scioptric Balls*.

27. One great Use of the *Camera Obscura*, is the easy Method it supplies of *measuring the focal Length of any Lens or Mirrour*; which otherwise is many Times very difficult to be determined. This Matter is extremely easy and certain for *Convex Lenses* and *Concave Mirrours*, because they have a real and very visible Focus; and therefore setting one End of a Rule on the Lens, the Distance of the Focus is seen on the Rule in Feet, or Inches, and Decimal Parts of an Inch.

28. But with respect to a *Convex Mirrour* or *Concave Lens*, which have no real Focus, it will be easy to find the Distance of the virtual Focus thus: Describe a Circle, suppose
of

of 3 Inches Diameter, on a Piece of clean white Paper; hold this Paper directly behind the Concave Lens in the Rays, and move it to and from the Lens till the circular bright Spot of the Diverging Rays exactly fill the Circle on the Paper, then measure the Distance of the Paper from the Lens, and also the Diameter of so much of the Lens as is concave; then *multiply the Distance of the Paper by the Diameter of the Concavity, and divide that Product by the Difference between the Diameter of the Circle on the Paper and Concavity of the Lens, the Quotient will be the Distance of the virtual Focus of the Lens.* In the same Manner the said Focus is found for a Convex Mirrour.

x 29. Another grand Experiment of the Dark Chamber, is *shewing the Spots on the Sun's Disk.* This is easily done, by putting the *Object Glass* of a 10 or 12 Foot Telescope into the *Scioptric Ball*, which turn about till it be filled with the Sun's Rays, then holding a Sheet of white Paper in the Focus of the Lens, you will see a most exceeding fair and bright Image of the Sun formed on the Paper of about an Inch Diameter, in which the Spots on the Sun's Surface will be very distinctly seen, and will afford a very pleasing Spectacle. This Image is rather too bright to be viewed without Offence to the Eyes; and therefore should be viewed through a large Lens of about 6 or 8 Inches focal Distance, which

which will magnify both the Image and it's Spots to a very great Advantage.

30. The Dark Chamber might also be made to answer the End of a *nocturnal or aërial Telescope*; for if an Object Lens, whose focal Distance is 12, 15, or 20 Feet, were fixed in the Ball, it would give a large Image of the Moon, and a small one of *Venus, Jupiter, and Saturn*; which may be so far magnified by one or more small Lenses, such as are the Eye-Glasses, that the *Phases* of the one, and the *Satellites, Belts, and Rings* of the other may, in all likelihood, be rendered visible and distinct; as I have Reason to think from some Experiments I have made with Lenses of a shorter focal Distance, not having Room to try the larger.

31. Divers *Microscopical Experiments* are also to be exhibited in the *Dark Chamber*, both by *Convex Lenses* and *Concave Mirrours*. Thus if you fit a Tin or Pastboard Tube into the Hole of the *Scioptric Ball*, and make an Aperture on each Side to slide a Piece of Glass through freely, and then either in or upon this Tube you put another, containing two Lenses properly disposed; this may be moved backward or forward till such times as it gives clear and very large Images on a Sheet of Paper, of small Objects that are stuck upon the Slip of Glass aforesaid, which are strongly illuminated by the Sun-Beams passing thro' the Ball for that Purpose.

32. The

x 32. The same thing is to be done with a *Concave Mirrour*, thus: Let a Cylinder of Rays fall on a large Concave Mirrour, fixed in a Frame for that Purpose; then take the Slip or thin Plate of Glass, and having put any small Objects thereon, hold it in the incident Rays a little more than the focal Distance from the Mirrour; then will you see on the opposite Wall the Images of those small Objects very large, and exceeding clear and bright among the reflected Rays. This is a most easy and delightful Experiment, which I have often tried with great Pleasure to my self and others.

33. The *new and noble Doctrine of Colours*, and the *different Refrangibility of the Sun's Rays*, were the Result of Experiments by the *Camera Obscura*; if you admit a Cylinder of Rays, about $\frac{1}{4}$ of an Inch Diameter, into the darkened Chamber, and therein hold a triangular *Glass Prism*, you will find, by turning it a little upon it's Axis, the Place where the Rays in passing through it will be refracted in a different Degree, some more, some less; and by that means will paint an oblong *Spectrum* or Image of various Colours most exceeding strong and vivid; and in the following Order, *Violet, Indigo Blue, Green, Yellow, Orange, and Red*. This is a most surprizingly fine and agreeable *Phænomenon*, which, together with divers other Experiments relating thereto,

thereto, is the Subject of Sir *Isaac Newton's* *Book of Optics*.

34. The Experiment of the *Rain-bow* in the *Dark Chamber* is very pleasant, and is thus made: Take a hollow Glass Globe and fill it with Water, and then hold it before you in the Cylinder of Rays, moving it a little about, till you see it paint a fair, vivid, and large *Iris* or Bow on the Cieling above your Head, or on the Wall before you; the Colours of this Bow are just the same as of that in the Clouds; it is also a double Bow, as that is, the upper one being larger and fainter than the minor one. In short, this is a good Experiment towards explaining the Nature of the Celestial Bow.



C H A P. V.

Of Microscopes in general; of single Microscopes in particular, made with a small Lens, Spherule, or Mirrour.

1. **T**HE Word *Microscope* imports an Instrument for viewing very small Objects. I have before observed, that Nature has so formed the *human Eye*, that we can't distinctly view an Object at a nearer Distance than six Inches; and since there is an Infinity of Objects, which at that Distance appear either as *Points*, or are wholly imperceptible, whatsoever Instrument or Contrivance will render such minute Objects *visible and distinct*, we properly call a *Microscope*.

2. It is usual to say that the *Microscope magnifies Objects seen through it*; but this is true only with Regard to the apparent, not the *real* Magnitude of Objects; they indeed appear to be larger with than without the Microscope, but, in Truth, they are not; and the Reason why they appear to be magnified will be easy to apprehend, by any Person who understands what has been delivered concerning the *apparent Magnitude* of Objects in Chap. II, hereof.

3. For

3. For there it was shewn, that the *apparent Magnitude* of Objects is measured by the Angle which they are seen under by the Eye; and farther, that those Angles are reciprocally as the Distances from the Eye. If therefore, at the Distance of 6 Inches, I can but just discern an Object, and then by interposing a Lens, or other Body, I can come to view that very Object at a nearer Distance, the Object will appear to be as much larger through the Lens, than before to the ~~naked~~ Eye, as it's Distance from the Lens is less than it's Distance from the Eye.

4. That this, is the Case, is evident from Fig. 2. Plate XXIII; where A is a Point in an Object not clearly visible to the ~~naked~~ Eye, at a less Distance than A B, because the Rays which proceed from it are too divergent to admit of distinct Vision till they have passed that Distance; but if the same Object be placed in the Focus C of the Lens D, the Rays which proceed from it will become parallel, by passing through the said Lens, and therefore the Object is distinctly visible to the Eye E, placed any where before the Lens D. Consequently it will appear as much larger through the Lens than to the ~~naked~~ Eye, as C D is less than A B.

5. If an Object A B be placed in one Focus C of a Lens D E, and the Eye in the other Focus F; (See Fig. 3.) the Eye will see just so much of the Object as is equal to the Diameter

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Diameter of the Lens; for the Rays AD and BE, which go from the Object to the Extremities of the Lens D and E, and are united at the Focus F, must necessarily proceed from the Object to the Lens parallel to the Axis FC, and therefore parallel to each other; consequently the Part of the Object AB, seen by the Rays DF, EF, will be equal to the Diameter DE of the said Lens. All which is evident from the Theory.

6. If only the Part de of the Lens be open, then only so much of the Object ab, as is equal thereto, will be perceived by the Eye. Now since AB is equal to DE, or ab to de, therefore the Angle DFE, or dFe, is the Optic Angle under which the Part of the Object AB or ab appears to the Eye at F; and since GF is but $\frac{1}{2}$ FC, therefore the Angle DFE, or dFe, is double to that under which the Part AB or ab would appear to the naked Eye at the Distance FC. That is, the *Eye sees the Object, situated as above, twice as large with the Lens as it would do without it.*

7. *If you would see a Portion of an Object larger than the Lens, your Eye must be placed nearer the Lens than it's Focus.* Let the Lens be DE, (Fig. 4.) it's two Foci F and C; in the Focus C let there be an Object AB larger than the Lens; suppose the Rays AD, BE, proceed from the Extremities of the Object to those of the Lens, it is evident from the Figure they will be *convergent*, and therefore

therefore will by the Lens be united in a Point K, between the Lens D E, and it's Focus F: If then the Eye be placed at K, it will take into it's View an Object, or Portion of an Object, greater than the Lens D E.

8. Again, let G H be a Portion of an Object A B, leffer than the Lens D E; draw G D, H E, which will be *Diverging Rays*, and therefore will be united at a Point I, farther distant from the Lens than the Focus F: *Hence if an Eye be placed farther from the Lens than it's focal Distance, it can never see any Object, or Part of an Object, at one View, so large as the Lens, but always smaller. And universally, the visible Part of an Object will be to the Lens, as the focal Distance of the Lens, to the Distance of the Eye. All which may be easily deduced from the foregoing Theory.*

9. Since then it is evident, the Nature of a Convex Lens is such as will render an Object distinctly visible to the Eye, at the Distance of it's Focus, the Reason why they are used as *Microscopes* is exceeding plain. For suppose the Distance A B (Fig. 2) be 6 Inches, where the ~~naked~~ Eye B, can but just perceive the Object A distinctly, and let the focal Distance C D of the Lens D be $\frac{1}{4}$ an Inch; then since C D is but $\frac{1}{12}$ of A B, the *Length* of the Object at C, will appear 12 times as large as at A; if it were a *Surface*, it would be

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144 times as great; and the *Solidity* or *Bulk* would be magnified 1728 times.

10. If CD , the focal Distance of the Lens D , be but $\frac{1}{4}$ of an Inch, then will that be but $\frac{1}{24}$ of $AB = 6$ Inches, and so the *Length* of Objects will be magnified 24 times; the *Surface* 576 times, and the *Solidity* 13824 times, for those Numbers are the *Square* and *Cube* of 24. From whence it appears, that *single Glass Lenses* make very good Microscopes, which have these Advantages, that the Object appears most clear, they lie in little Room, may be carried about any where, are to be had for a small Price, and are most easy to be used.

11. The Form of such a Microscope, which I think most convenient, is that in Fig. 5. where AB is a circular Piece of Wood, Ivory, &c. in the middle of which is a small Hole, $\frac{1}{20}$ of an Inch Diameter; upon this Hole is fixed, with a Wire, a small Lens C , whose focal Distance is CD . At that Distance is a Pair of Pliers DE , made of a Watch Spring, and open'd by means of the two little Studs a, e ; with these you take up any small Object O , and view it with the Eye placed in the other Focus of the Lens at F . And according to the focal Length of the Lens, the Object O will appear more or less magnified, as represented at IM . If the focal Length be $\frac{1}{2}$ or $\frac{1}{4}$ of an Inch, the *Length*, *Surface*, and *Bulk* of the Object will be magnified

nified, as expressed in Article 9 and 10 hereof. This small Instrument may be put into a Case, and carried about in the Pocket without any Incumbrance. I have made Trial of various Lenses, and find those whose focal Lengths are $\frac{3}{10}$, $\frac{4}{10}$, and $\frac{5}{10}$ of an Inch, the best for common Use. x

12. Since the nearer the Eye can approach to an Object, the larger it appears, it is plain a *double and equally Convex Lens* is far preferable to a *Plano-convex Lens*; because if the Sphere or Convexity be the same, the focal Length of the former, is but half as long as of the latter: And since the Double-Convex consist of two Segments of a Sphere, the more an Object is to be magnified, the greater must be the Convexity, and therefore the smaller the Sphere; till at last the *utmost Degree of magnifying* will require that these Segments become *Hemispheres*, and consequently the *Lens* will be reduced to a perfect *Spherule*, or very *small Sphere*.

13. With these small *Spherules* extraordinary Degrees of magnifying may be arrived at; for as I have shewn in the Theory, the Focus of *Parallel Rays* is but at $\frac{1}{2}$ the Radius distant from the *Spherule*; therefore if the Radius of the *Spherule* be $\frac{1}{10}$ of an Inch, the Eye will have distinct Vision of an Object by means thereof, at the Distance of a *Radius and half*, i. e. $\frac{3}{20}$ of an Inch, which, as it is but the 40th Part of 6 Inches, shews that the

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Length of an Object will be magnified 40 times, the Surface 1600 times, and the *Solidity* 64000 times, by such a small Sphere.

14. If the Radius of a *Spherule* be but $\frac{1}{20}$ of an Inch, then will the Eye have distinct Vision of an Object at the Distance of $\frac{3}{40}$ of an Inch, which, as it is but the 80th Part of 6 Inches, shews the *Length* of Objects will appear 80 times greater, the *Surface* 6400 times, and the Bulk 512000 times greater than to the naked Eye at 6 Inches Distance.

15. Again, if the Diameter of a *Spherule* be $\frac{1}{20}$ of an Inch, or the Radius $\frac{1}{40}$, then will the Eye approach the Object so near as $\frac{3}{80}$ of an Inch, which is but the 160th Part of 6 Inches; and therefore the Length of Objects will be magnified 160 times, the *Surface* 25600 times, and the Solidity 4096000 times by this *Spherule*; which is so great a Power of magnifying, as surpasses all human Imagination and Comprehension.

16. And yet there are Methods of making *Spherules* as small, and smaller than any above-mentioned. I shall mention only two Ways of doing this: The first is by breaking a Piece of clear white Glass into very small Particles, which are to be taken up by the Point of a fine Needle, and held in the blue Part of the Flame of a Candle, or rather of a Lamp burning with Spirits of Wine; which by means of a Blow-Pipe, will immediately melt the glass Particles on the Point of the Needle;

Needle; and being melted they will naturally run into a roundish Form, and, by a proper Motion of the Needle, which a little Practice will teach, they may be brought to a true spherical Form. And as there are more or less Particles on the Needle, the Globule will be greater or smaller.

17. The other Way is by melting a Piece of fine Glass in a small Crucible, or Bowl of a Tobacco-Pipe, and then by dipping therein the End of a Wire, you may draw out very fine and long Threads of Glass, which, when cold, are to be broken into proper Lengths; and one End of such a Thread put into the Flame of a Candle, will immediately melt, and run into a round or globular Form, which, when you think is big enough, is to be taken out, and broke off the Thread.

18. In either of these Ways, great Care must be used not to hold them long in the Flame after they are melted, lest they are burnt, and thereby rendered opake, and unfit for Use. The few that are good, among the many you make, are to be well cleansed, and let into a very small Hole in a Piece of Brass, *in order* for Use.

19. In using these *Spherule Microscopes*, the Objects are to be placed in one Focus, and the Eye in the other; and since the Focus is so exceeding near the Glass, it is impossible to view any but pellucid Bodies; for if any opake Object were to be applied, the Eye

being as it were just on the Spherule, would entirely prevent any Light falling on it, and it would be too obscure to be viewed.

20. It was with these Sort of Microscopes, that the famous *Dutch* Philosopher Mr *Leeuwenhoek* made such wonderful Discoveries; and it must be with these, if with any, that the *Corpuscles* or *Atoms*, of which Bodies consist, are to be discovered; which the great Sir *Isaac Newton* thought was possible. But the great Difficulty of making very small, and, at the same Time, very good ones; their Prejudice to the Eyes in poring very hard and near, the Trouble of positing Objects at a due Distance, and the very small Part which can be seen of any, makes this Sort of Microscopes very little known or used.

21. In Fig. 6. let *ABH* represent a small Globe or Spherule, whose Center is *C*, and Radius *CB*; also let *GB* be $= \frac{1}{2} CB$, then will the Point *G* be the Focus of Parallel Rays passing thro' and refracted by the said Spherule; and therefore, if an Object *EF* be placed in the said Focus *G*, it may be distinctly seen by an Eye in the other Focus of the Spherule. Suppose now the Spherule be removed, and in the Place of it's Center be placed the Lens *LN*, whose Radius is *CG*; then will *G* be also the Focus of the said Lens, and the Object *EF* will be distinctly such a Lens also. And since the Angle *ECF*,
under

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under which the Object appears at the Center of the Spherule and Lens, is the same, the Object will be equally magnified by them both. But CG, it's Distance from the Lens, is three times greater than BG, it's Distance from the Spherule; and therefore much more Light will fall on it when viewed by the Lens, than can when viewed by the Spherule; and consequently in all Cases, where the Power of magnifying is not required to an extreme, the Use of a Lens is much preferable to that of a Spherule.

22. The next Sort of single Microscopes are those made of *Concave Mirrours*; and the larger the Mirrour, the fitter for the Purpose. One 10 Inches Diameter, and 12 Inches focal Length, will do very well. By such a Mirrour, a small Object, may be made to appear very large either behind or before it.

23. The Distance at which an Object should be placed, in order to magnify it any proposed Number of Times, at a Focus behind the Glass, is found by the Rule delivered in Art. 11. Chap. VI. Part I. Thus suppose I would magnify the Diameter of a small Object 10 times, I find by that Rule, that it must be placed $10\frac{8}{10}$ Inches before the Mirrour; and when the Diameter, or Length of an Object, is magnified 10 times, the Surface will be magnified an 100 times, and the whole Bulk a 1000 times. This is a very considerable Effect, and is attended with this

peculiar Advantage, that not only a small Part of one Object, but the whole of diverse Objects is here exhibited to the View, which makes it pleasant and delightful to behold.

24. The other Way of magnifying Objects at a Focus before the Mirrour, is that already described in Chap. IV. Art. 32, as being performed in a *dark Chamber*. There remains one other Sort of single Microscope, which consists of a Glass Globe filled with Water, which serves for magnifying the small Animalculæ therein very well; this was the Discovery of Mr *Stephen Gray*, and is as follows.

25. Let KL be a Globe filled with Water, and AB two parallel Rays falling thereon, of which let BH be the Axis of the Globe, which passing through the Center C , will suffer no Refraction; the Ray A will be refracted to a Focus F , distant from the Globe H , the Diameter thereof, *i. e.* $HF = BH$, as may be deduced from the Theorem in Art. 16. Chap. I. of *Dioptrics*, by putting I to R , as 4 to 3; which is the Proportion of the Sine of Incidence, to the Sine of Refraction out of Air into Water.

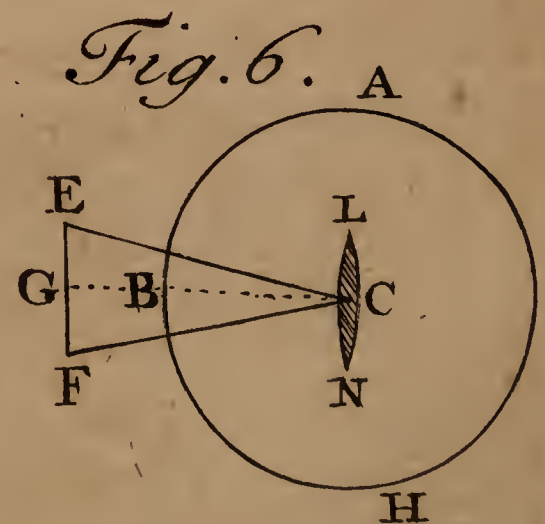
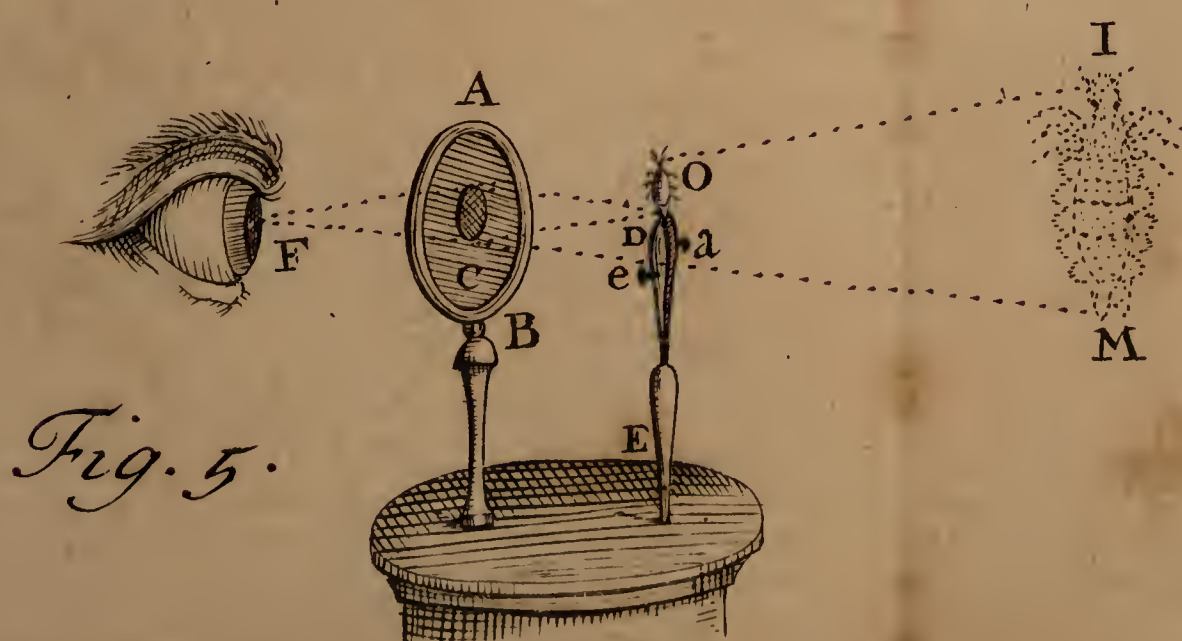
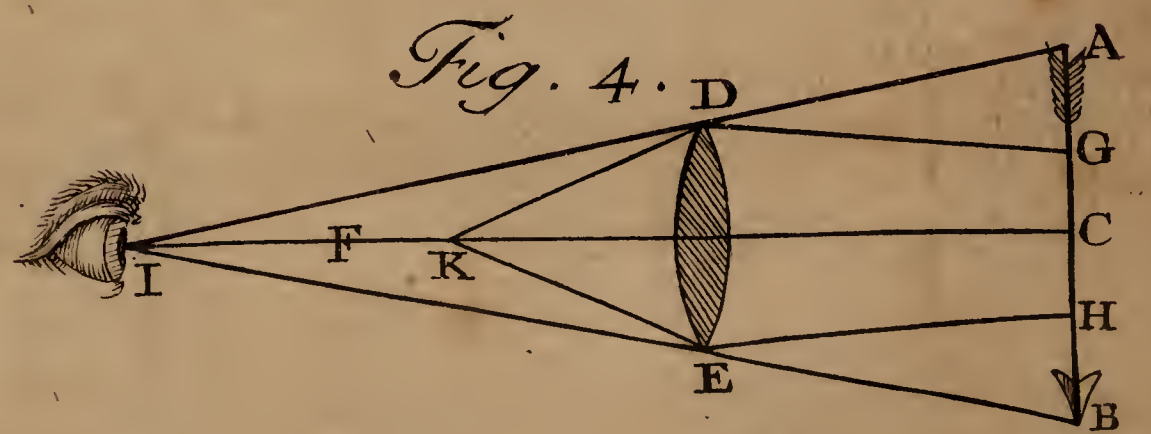
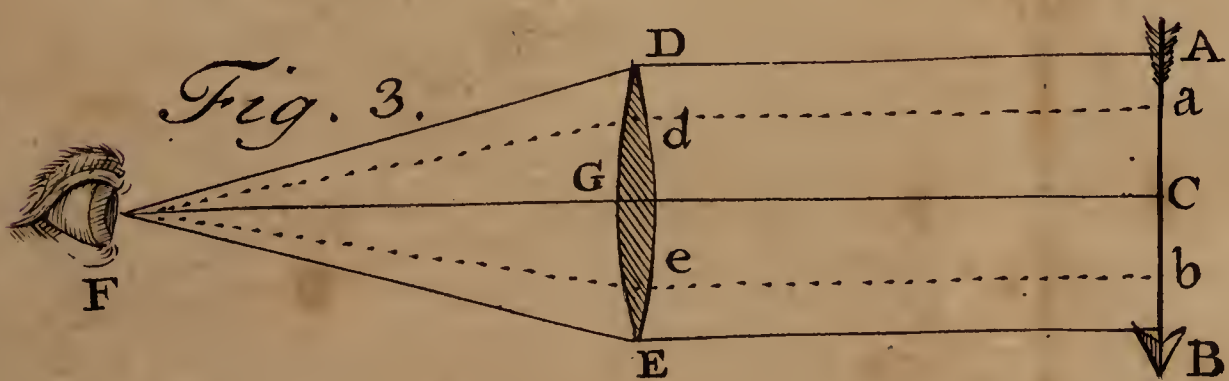
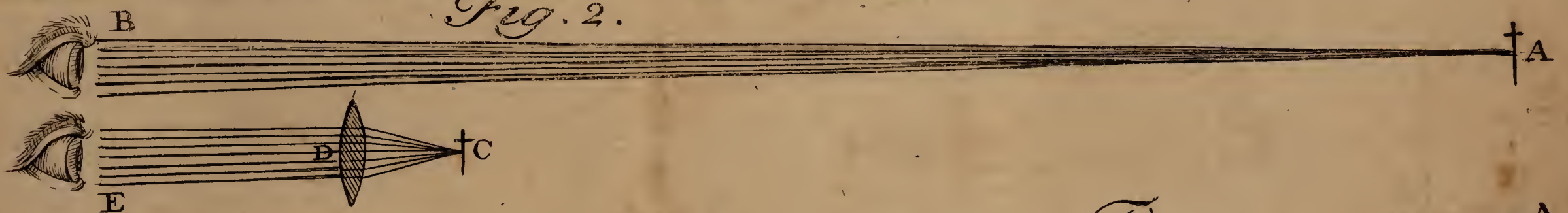
26. Let us now consider the refracted Ray AG , as reflected from the Concave Surface of the Globe at G , to the Focus E in the Axis. Then will an Object placed at E be seen by Reflection in the same Manner, as an Object at F will be by Refraction. Through the
Center

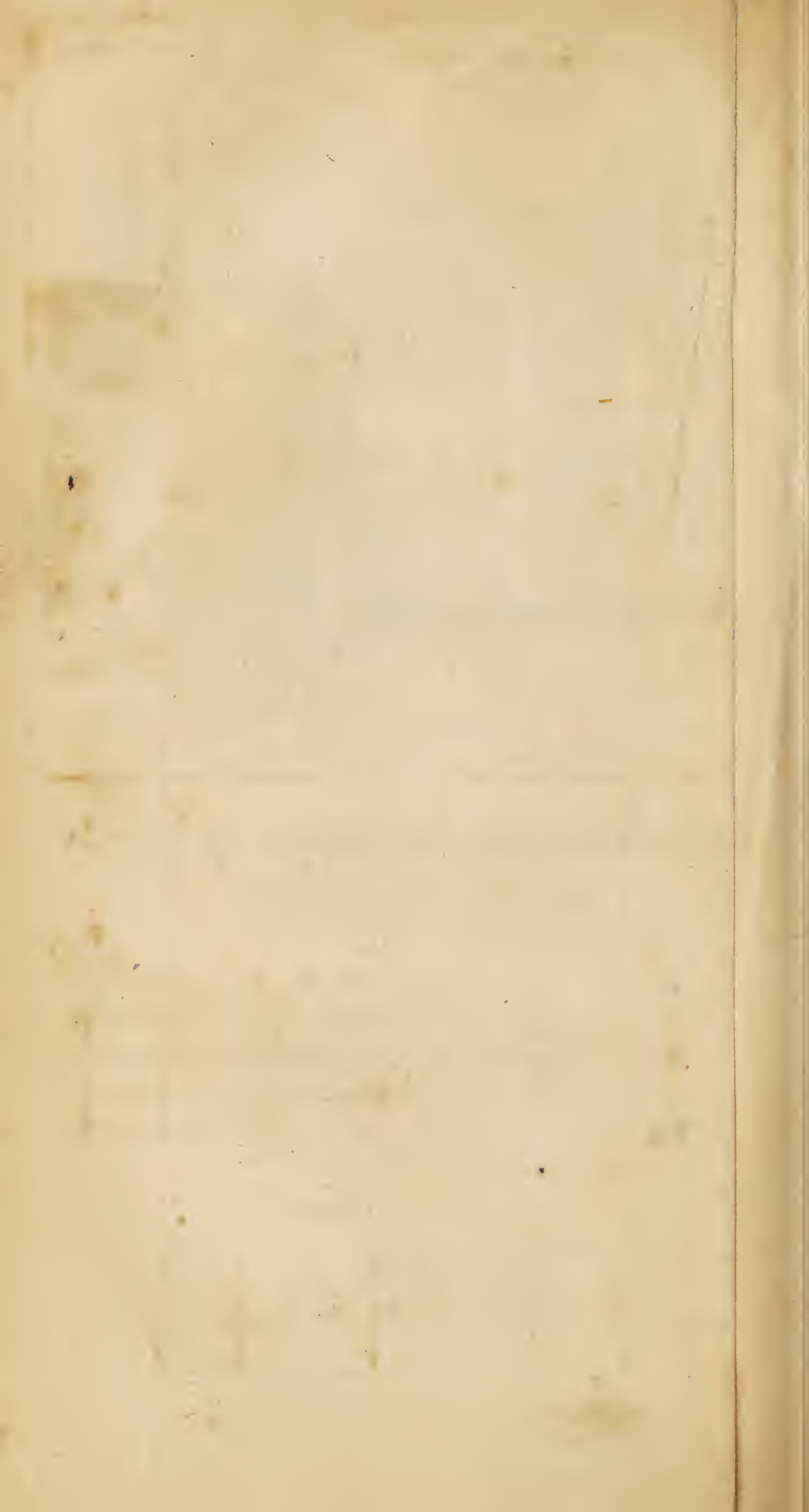
therefore the Theorem becomes $\frac{dr}{-2d-r}$
 $= \frac{2rr}{-5r} = \frac{2}{5}r = f = HE$; and so CE
 $= \frac{3}{5}r$: but $CF = 2r$, therefore $CF : CE ::$
 $2r : \frac{3}{5}r :: 2 : \frac{3}{5} :: 10 : 3 :: 3\frac{1}{3} : 1$. And con-
 sequently an Object, in the Focus of Re-
 flection E within the Globe, will be magni-
 fied $3\frac{1}{3}$ times more than it would be, if
 placed in the Focus of Refraction F, without
 the Globe.

28. Now suppose the Globule KL were $\frac{1}{5}$
 of an Inch Diameter, then would the appa-
 rent Magnitude of Objects at F, by Refrac-
 tion through the Water, be to the apparent
 Magnitude at 6 Inches from the Eye, as 6
 to $\frac{1}{5}$, or as 30 to 1; and therefore the appa-
 rent Magnitude at E, by Reflection, will be
 as $3\frac{1}{3}$ times 30, that is, 100 times magni-
 fied.



Fig. 2.





C H A P. VI.

Of double or compound Microscopes, viz. those which consist of two Lenses; and the Manner of computing their Power of magnifying.

1. **I**F a Machine be constructed with more than one Lens or Mirrour, proper for magnifying small Objects, it is called a *double or compound Microscope*; such as is that represented in Fig. 1. Plate XXIV.

2. In that Figure $d e f$ is a small Lens, called the Object-Glass, as being next the Object $a c b$ placed a little below it to be view'd. Suppose the Focus of this small Lens be at O . Then, as is plain from the Theory, if any Object $a c b$ were placed in the Focus O , the Rays which came from any one Point divergent on the Lens, would by it be so refracted, that they would afterwards proceed parallel to one another; or their Focus would have been at an infinite Distance.

3. It has also been shewn, that if an Object $a b$ be placed at any Distance from $e C$, greater than the focal Distance $e o$, the Rays $d c$, $f c$, which come from any Point c , and
fall

fall divergent on the Lens df , will be so refracted, that they will afterwards proceed converging towards, and at last unite in a Point C , which will be the Focus of all the Rays proceeding from the Object ab , for the Distance ce .

4. Therefore the Rays which proceed from the Points a, c, b , in the Object ab , will be respectively represented at A, C, B , in the aforesaid Focus; and consequently ACB will be the Image of the Object acb ; in an *inverted Position*, and is so much greater than the Object, as the focal Distance eC is greater than the Distance of the Object ec . And the lesser ec is, the greater will Ce be, and the more will the Image ACB be enlarged, or the Object magnified: But, as before observed, the Distance ec must always be greater than the focal Distance eo . All which things are likewise demonstrated and exemplified in the Theory.

5. In single Microscopes, or those of one Lens, we view the Image, or rather the Object itself, at an affirmative Focus, or behind the Lens; but in compound Microscopes it is otherwise: For here AB , which is absolutely the Image of the Object, far distant from it at a negative Focus, or on that Side the Lens next the Eye, is the Thing we view, and consider it now as a new Object.

6. If then any Lens DF be placed just at it's focal Distance EC , from any Object AB ,
the

the Rays which come from any Point C, will after Refraction through the Lens, proceed parallel to the Eye at I, and there cause distinct Vision: Thus the Object AB, that is, the Image of the Object ab greatly magnified, will be clearly and distinctly seen thro' a second Lens DF; and if the Eye's Distance be less, equal to, or greater, than the focal Distance EC, then such a Portion of the Image AB will be seen as is greater, equal to, or less than that Part of the Lens DF, which is open. Note, this Lens DF is called the *Eye-Glass*, as being next to the Eye in viewing Objects.

7. Thus is Vision performed in the Double Microscope, composed of two Glasses. I shall now shew the *Power of magnifying* in such a Microscope, which will be easily apprehended in the following Manner. I have before observed, that no Object can be clearly and distinctly viewed at a less Distance than six Inches, and by many Eyes not so near. But six Inches is the Number I shall found my Computations upon, because I shall then be sure to be within Compass.

8. First, let us suppose df the *Object-Glass* to be a double and equally Convex Lens, whose focal Distance $eo = \frac{8}{10}$ of an Inch. Then if an Object ab be placed at the Distance $ec = 1$ Inch, the Image AB will be formed at the Distance $ec = 4$ Inches, as is manifest

manifest from the Theory, and the Rule exemplified in Chap. X, Art. 1 and 2, Part II.

9. Since then the Distance ec is but one Inch, the Angle aeb , which the Object subtends at the Lens de , is 6 times greater than the Angle under which it would appear to an Eye, at 6 Inches Distance; and therefore the *Length* of the Object ab , will be magnified 6 times by the Object-Lens only.

10. And since the Angles aeb , and AeB , under which the Object and it's Image appear at the Lens df , are equal, the Image AB to an Eye at e , will appear equal to the Object ab , *i. e.* *six times* larger than the said Object at the Distance of *six Inches* from the Eye.

11. But since the Image AB is rendered visible, by means of the Eye-Glass DF , and is seen under the Angle AEB ; the apparent Magnitude thereof seen by an Eye at E , will be to the same to an Eye at e , as the Angle AEB , is to the Angle AeB , or as the Distance Ce , to the Distance CE : But Ce is 4 Inches; if then we suppose $CE = 1$ Inch. The Image will appear 4 times greater at the Eye-Glass, than at the Object-Glass; and therefore 4 times 6, that is, 24 times greater than the Object itself ab .

12. The *Length* being magnified 24 times, the *Surface* of Objects will be magnified 576 times, and the Solidity or Bulk 13824 times. And the Effect of this double Microscope is
equal

equal to that of a single one of a Lens, whose focal Distance is but $\frac{1}{4}$ of an Inch, as is evident from Art. 10, of the foregoing Chapter.

13. Again, if the Distance of the Object ec be $\frac{3}{4}$ of an Inch, and the Distance Ee between the Eye-Glasses remaining the same; then will the Object ab appear as much greater by the Eye-Glass df , than to the naked Eye at 6 Inches Distance, as 6 is greater than $\frac{3}{4}$, *viz.* 8 times. Then also, since $Ce = 4$ Inches, and $CE = 1$, the Object will appear 4 times greater through the Eye-Glass DF , than by the Object-Glass df ; and therefore 4 times 8, that is, 32 times greater by both than to the naked Eye. The *Length* being magnified 32 times; the *Surface* will be magnified 1024 times; and the *Bulk* 32768 times, by such an Object-Lens.

14. If you use an Object-Lens that shall give the Distance $ec = \frac{1}{2}$ an Inch, then will it magnify the Object ab 12 times, and the Eye-Glass DF magnifying that Image AB 4 times more; the Length of the Object will by both be magnified 4 times 12, *viz.* 48 times; and therefore the *Surface* will be magnified 2304 times, and the Solidity 110,592 times.

15. In the last place, if you use an Object-Lens, which gives the Distance of the Object $ec = \frac{1}{4}$ of an Inch; then will the Object be magnified by that 24 times, and 4 times more by the Eye-Glass, and therefore
by

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by both 96 times; and thus the *Surface* will be magnified 9216 times; and the *Solidity* 884736 times, which is to a very great Degree indeed.

16. If you are desirous of knowing what focal Distances eo the 4 several Object-Glasses must have to give the Distance of the Object $ec = 1$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, as above-stated for Calculation, you find them according to the Rules of the Theory, to be $\frac{8}{10}$, $\frac{63}{100}$, $\frac{44}{100}$, $\frac{23}{100}$; supposing those Lenses are double and equally Convex.

17. The *Power of magnifying* is to be varied also by varying the focal Length of the Eye-Glass DF , for the same Object-Glass df ; for as now, the focal Length EC being 1 Inch, the Lens DF magnifies the Image AB 4 times, if the said focal Length were but $\frac{3}{4}$ of an Inch, it would magnify the Image AB $5\frac{3}{10}$ times, for then $Ce : CE :: 5\frac{3}{10} : 1$. And if CE be but $\frac{1}{2}$ an Inch, then would $Ce : CE :: 8 : 1$; and so the Lens DE would magnify the Image AB , 8 times.

18. But we can't vary the focal Length of the Eye-Glass DF at pleasure, or to that advantage as we can the Focus of the Object-Glass df , for Reasons hereafter to be given.

19. There is yet another Way, by which the *Power of magnifying* is to be varied or augmented in this Microscope; and that is, by varying the Distance between the two Glasses DF and df , by means of two Tubes sliding one within the other; in the uppermost is fixed

fixed the Eye-Glass DF , and to the lower is fixed the *Object-Glass* $d f$. By this Contrivance the Distance of the Image Ce is considerably lengthened, and bears a less Proportion to CE than before; and consequently the *magnifying Power* will be so much the greater.

20. These *double Microscopes* are much preferable to single ones, as they have or may be contrived with as great a Power of magnifying, and at the same Time opens a much larger, and more spacious *Field of View*; and hath divers other Advantages to be mentioned hereafter.



C H A P. VII.

Of Microscopes compounded with three Lenses, viz. an Object-Lens, and two ocular ones.

1. **W**E have seen the Nature and Effects of a Microscope, constructed with *one* and *two* Lenses, let us now contemplate the Nature and Construction of one of *three*, viz. An Object-Lens *df*, and two Eye-Glasses, one a larger *GK*, and the other lesser *DF*. This broad Lens *GK* is added (not to increase the *magnifying Power*, or *Distinctness of Vision*, for that is better effected by one Lens *DF*, but) to increase or *amplify the optic Angle*, or to enlarge the *visible Area*, or *Circle of Vision* in the Microscope. Which it does very considerably, as is thus shewn. See Fig. 2.

2. Let *ab* be an Object so posited, that it's Image *AB* may be formed at the Distance of *eC*, from the Lens *df*; at *H* suppose the Lens *GK* intercept the Rays *eK* and *eG*, which it will refract towards some Point in the Axis, as *O*; and consequently there will be a new and lesser Image *AB* formed than before, the Place of which is found by drawing

ing the Lines AH , BH , from the Extremities of the first Image AB , to the Center of the Lens; for they will intersect the Rays KF , GD , in A and B , the Place of the Image AB . For if the Image AB be considered as an Object with regard to the Lens GK , and FK , DG two incident Rays, they will emerge from the Lens in the Direction Ke and Ge , and since CH is less than the focal Distance of the Lens GK , the Rays of every Pencil going from the Object AB , will emerge from the Lens diverging, and tend towards a *virtual Focus* C , where they form the imaginary Image ACB . But the Object AB , and it's Image AB , are seen under equal Angles AHB , from the Center of the Lens H ; and therefore the Lines AH and BH determine the Places and Proportions of the Images AB and AB . All which is plain from the Theory.

3. Let DF be another Lens placed at it's focal Distance EC , from the Image AB , which then will be distinctly seen through it by an Eye placed any where in the Axis EO . Let Hc be the focal Distance of the Lens GK , draw ab , and join aH , then will the Angle aHc be the optic Angle under which the half of the Object ac is seen through the Lens GK ; and is to the Angle $aec = aec$, (under which it appears by the Lens df) as ec is to Hc .

4. Since $Ha = Hc$, the Point a will be the Focus of a Pencil of Rays falling obliquely on the Lens GK , of which aH is the Axis, and aK the extreme Ray; and since all the Rays of the Pencil emerge parallel from the Lens in this Case, if KO be drawn parallel to aH , O will be the Point in which the Ray eK , after Refraction through the Lens GK , will cut the Axis HO , making the Angle $HOK = aHc$.

5. But the Ray KO , by the Interposition of the Lens DF , will suffer a second Refraction through it at F ; and to find what that will be with the focal Distance EC of the Eye-Glass DF , sweep the Arch Ce , and join Ee , then will e be the Focus of an oblique Pencil of Rays falling on the Lens DF , of which eE is the Axis, and eF an extreme Ray. If then you draw FI parallel to eE , it will be the refracted Ray, and I the Point in which it cuts the Axis HO ; making the Angle $EIf = CEe$.

6. From D and F let fall the perpendiculars Dd , Ff , on the Image AB ; and if there were no other Lens but DF , the Part df in the Image is all the Eye could view in the proper Focus L , for $df = DF$. See Art. 5. Chap. V, hereof. But now by means of the Lens GK , the Rays KF and GD , which terminate the Extremities of the Image, are made to fall within the same Aperture DF of the Eye-Glass, and consequently the whole
Image

Image A B is seen by the Eye at the Point I, under the Angle E I F.

7. But this Angle E I F is to the proper Angle E L F, as L E is to I E, by Art. 9. Chap. II, of this. Now in order to express this Proportion in Numbers, we must first find H O, for which the Distance e H must be given, which suppose 5 Inches. Then esteeming e as a Radiant Point from whence proceeds the Pencil of Rays G e K, to the Lens G K, whose focal Distance, or Radius of Convexity H c = 3 Inches. We shall find the Focus O of this Pencil after Refraction,

by the Theorem $\frac{dr}{d-r} = y$. See Art. 36.

Chap. I, of the Theory of *Dioptrics*.

8. For here $d = e H = 5$, $r = H c = 3$, and $y = H O$, to be found thus; $\frac{dr}{d-r} =$

$$\frac{5 \times 3}{5 - 3} = \frac{15}{2} = 7.5 = y. \quad \text{Or thus by Ana-}$$

logy, $d - r (e c) : d (e H) :: r (H c) : y (H O)$
 $= 7 \frac{1}{2}$ Inches. Having thus found H O, suppose the Distance between the two Glasses H E = 2 Inches, then will E O = $5 \frac{1}{2}$ Inches; and if the Radius of Convexity E C, of the Eye-Glass D F be 1 Inch; we shall find E I, by the Theorem in Art. 38. Chap. I,

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of *Catoptrics*, viz. $\frac{dr}{d+r} = y$; for here $d =$
 $EO = 5.5$, $r = EC = 1$, and $y = EI$;
 therefore $\frac{dr}{d+r} = \frac{5.5}{6.5} = 0.85$ very nearly.

Or by Analogy, as $d - r (= CO = 6.5) : d$
 $(= EO = 5.5) :: r (= EL = 1) : y (= EI$
 $= 0.85$, fere).

9. Therefore the optic Angle DLF proper
 to the Lens DF alone, is to the amplified
 Angle DIF, as 0.85 to 1, or as 85 to 100;
 the visible Areas therefore will be as the
 Squares of these Numbers, viz. as 7225 to
 10000, or as 7 to 10 nearly; which Aug-
 mentation of the optic Angle by the Addition
 of the Lens GK is very considerable, and
 renders the Pleasure of viewing Objects pro-
 portionably greater. Now this Angle DIF
 is greater or smaller, as the Distance between
 the Lenfes HE is greater or smaller; for as
 the Lens DF approaches to the Lens GK, the
 Distance EO approaches nearer to an Equa-
 lity with CO, or EI with EL, that is, the
 Angles I and L become nearer equal; and the
 contrary, as DF is placed farther from GK.

10. The Degree or *Power of magnifying* in
 this given Combination of Lenfes, is compu-
 table in the following Manner. Suppose I
 can see the Line ac distinctly at the Distance
 of 6 Inches, but by means of the Lens df
 I can see it at the Distance eC, under the
 Angle

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Angle $a e C$. Again, since $a e C = a e c$, and $H c$ is the focal Length of the Lens $G K$, the Object $a C$ will appear to an Eye at H , under the Angle $a H c$, which is to the Angle $a e c$, (or $a e C$) as ec to $c H$. Lastly, the Angle under which it appears by the Eye-Glass $D F$ is $E I F = C E e$, but this Angle $C E e$ is to the Angle $a H c = C O e$, as $C O$ to $C E$. Therefore the first Ratio of magnifying, *i. e.*

by the Lens $d f$ is $6 : C e$, or $\frac{6}{C e}$; the second

Ratio, by the Lens $G K$ is $ec : c H$, or $\frac{ec}{c H}$;

therefore the Appearance of $a C$ at H , is to

that at the naked Eye, as $\frac{6}{ec} \times \frac{ec}{c H}$. The

third Ratio is by the Lens $D F$, in regard of

the last, as $C O : C E$, or $\frac{C O}{C E}$; and conse-

quently an Object $a c$ will appear under an Angle $D I F$, which is to that under which it appears to the naked Eye at 6 Inches Di-

stance as $\frac{C O}{C E} \times \frac{ec}{c H} \times \frac{6}{e C}$ to 1.

11. To give an Example, let $C e = \frac{1}{2}$ an Inch; $c H = 3$, $C E = 1$; and the Distance $e H = 5$; then will $ce = 2$, and $C O = 6.5$, things remaining as in Art. 9. Then

$$\frac{CO}{CE} \times \frac{ec}{cH} \times \frac{6}{eC} = \frac{6.5}{1} \times \frac{2}{3} \times \frac{6}{0.5} = \frac{78}{1.5} =$$

52. Therefore the apparent Magnitude thro' the Microscope, is to that by the naked Eye, as 52 to 1, for a *Line*, as 2704 to 1, for a *Surface*; and as 14068 to 1, for a *Solid*.

12. The Lens GK being removed, all other things remaining the same, the magnifying Power will be greater than before; for

$$\text{it will be as } \frac{Ce}{CE} \times \frac{6}{ce} = \frac{6}{1} \times \frac{6}{0.5} = \frac{36}{0.5}$$

= 72, above a third greater than 52. And by removing GK we are obliged to change the Object-Lens df to keep the Distances the same; yet if df were retained, the Distance ce would be so very little enlarged to bring the Image AB, to the Situation AB, that if the Lens DF be the same, and at the same Distance from df, the magnifying Power will still be greater much by itself, than in Combination with the Lens GK.

13. If instead of Hc = 3, we take it = 4 Inches; then the optic Angle DIF will be diminished, all other things remaining the same, as in Art. 11. For now r = Hc = 4, and d = He = 5, and d - r = ce = 1;

$$\text{therefore } \frac{dr}{d-r} = \frac{5 \times 4}{1} = 20 = y, \text{ or HO;}$$

and so CO = 19, and EO = 18. Therefore to find EI we have d = EO = 18, and

and $r = EL = 1$, and $d + r = CO = 19$;

and then $\frac{dr}{d+r} = \frac{18 \times 1}{19} = 0.95$ nearly,

which in this Case is the Length of EI, whereas before it was but 0.85; (Art. 8.) and consequently the Angle DIF is now diminished in the Ratio of 95 to 85, or nearly as 9 to 8.

14. But the magnifying Power will be increased; for $\frac{CO}{CE} \times \frac{ce}{cH} \times \frac{6}{eC} = \frac{19}{1} \times \frac{1}{4} \times$

$\frac{6}{0.5} = \frac{114}{2} = 57$; whereas before (Art. 11.)

it was but 52; but since this Increase of the magnifying Power is small, and the Diminution of the optic Angle very considerable: (Art. 13.) a Lens GK, whose focal Distance is 3 Inches, is preferable to one of 4 Inches.

15. Again, if we make the focal Distance of the Lens GK equal to 2 Inches, other

things being the same; we shall have $\frac{dr}{d-r}$

$= \frac{5 \times 2}{3} = \frac{10}{3} = 3.33 = HO$; and then

$EO = 1.33 = d$; and so $\frac{dr}{d+r} = \frac{1.33}{2.33} =$

0.56 = EI; and therefore the Angle DIF : DLF :: 1 : 0.56 :: 100 : 56, that is, the
optic

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optic Angle at I, is almost as big again as the proper Angle at L. But notwithstanding this Advantage, since the magnifying Power is di-

minished, (it being only $\frac{CO}{CE} \times \frac{ce}{cH} \times \frac{6}{eC} =$

$\frac{2.3}{1} \times \frac{3}{2} \times \frac{6}{0.5} = 42$.) and the great Convexi-

ty of the Lens GK, distorting and colouring the Object about the Extremities, the focal Length of three Inches is preferable to this of 2 Inches; and consequently to any other focal Length of GK, in this Construction of the Microscope.

16. In this Combination of Eye-Glasses, the Effect of the Microscope is $\frac{CO}{CE} \times \frac{ce}{cH} \times$

$\frac{6}{Ce}$ to 1; but in a Microscope with a single

Eye-Glass DF, whose focal Distance is CE, and the Distance of the Image from the Object-Lens df, is Ce, hath it's Effect in the

Ratio of $\frac{Ce}{CE} \times \frac{6}{ec}$ to 1. If now we suppose

an Object ab, equally distant from the Object-Lens df, when distinctly seen in both,

then according as $\frac{CO}{CE} \times \frac{ce}{cH}$ is *lesser, equal*

to,

to, or greater than $\frac{Ce}{CE}$, the Power of magnifying, in the former Microscope, will be *lesser, equal to, or greater,* than the same in the latter, or that with a single Eye-Glass.

17. If the focal Distance of the Eye-Glass DF, be the same in both Microscopes, their magnifying Effects will be in the Ratio of

$CO \times \frac{ce}{cH}$ to Ce . And if $ce = cH$, then

will their Effects be in the Ratio of $\frac{CO}{CE}$ to

$\frac{Ce}{CE}$. If then we would have them magnify

equally in this Case, $CE : EC :: CO : Ce$, is the Analogy.

18. If the Microscope hath the Dimensions of Art. 11, the Aperture of the Lens GK being given, and the Distance between the Lenses HE, the Aperture of the Lens DF is given also; for $HO : EO :: GK : DF$. Suppose $GK = 1.5$, then $HO = 7.5 : EO = 5.5 :: GK = 1.5 : DF = 1.07$. The half of which $EF = 0.53$; whence we may also find the Quantity of the optic Angle DIF: for $EI = 0.85$, by Art. 13. Wherefore we have this Analogy, As $IE = 0.85$ is to $EF = 0.53$, so is the Tangent of Radius

45°.

45° , to the Tangent of $27^{\circ} : 45'$ ferè, the double of which, viz. $55^{\circ} : 30'$, is the Quantity of the Angle DIF, in this Constitution of the Microscope.

C H A P. VIII.

Of the apparent Position of the Objects, and the visible Area, or Field of View, in a Microscope of Convex Eye-Glasses.

P L A T E XXIV. Fig. 3.

1. **W**E have hitherto been considering the Structure or Disposition of Glasses, and the Power of magnifying in Microscopes; the next thing to be taken Notice of is the *apparent Position* of the Object, and the *Area, or Field of View*, therein.

2. In order thereto, let N, O, P, be Cones of Rays coming from the Object-Lens, and painting in their Focus the Image ACB. This Image will be in a Position *inverted*, or contrary to that of the Object, as is evident from Fig. 1 and 2, of this Plate; the Reason whereof has been fully explained before.

3. The

3. The Image ACB we are now to consider as an Object viewed through a Convex Lens DF , by the Eye $ABQR$. This Object, as being seen in the Microscope, must be also conceived to be in the Focus of the Lens DF , which is the Eye-Glass; for in this Case only, the Rays AD , Ag , Ab , also Cn , Co , Cp , and the Rays BF , Bm , Bl , which come from the single Points A , C , B , will fall parallel on the Pupil QR , and if they do not come parallel, those Points of the Object will not be distinctly seen, as has been shewn in Chap. I, of this Part.

4. Now from the Points A , G , H , I , C , K , L , M , B , suppose parallel Rays proceed to the Glass DF , in the Points D , g , b , i , n , o , p , k , l , m , F , then shall these be all united in the Focus at E , in which we will suppose the Pupil of the Eye to be placed where it will then view as much of the Object AB , as is equal to the Diameter of the Eye-Glass DF ; as was demonstrated in Chap. V, Art. 5.

5. The Eye being posited in the Focus of the Glass, will by it's Humours converge or unite the Rays which enter parallel on the *Retina*, in the Bottom thereof; and therefore the Rays AD , Ag , Ab , proceeding from the Lens parallel to the Eye, will by it be united on the *Retina* at A , and there represent the Point A of the Object ACB . After the same Manner we conceive the Points B and C in the Object, represented at B and C
in

in the Bottom of the Eye ; and all the intermediate Points G, H, I, K, L, M, after having passed the Lens, will traverse in the Point E, and proceed to g, h, i, k, l, m, where they will be represented on the *Retina*.

6. Consequently *ACB* on the *Retina*, will be the Image of the Object *ABC*, that is, it will be the second Image of the small Object *ab*, under the Microscope, of which *AB* is the first enlarged Image. And since the Image *ACB*, is contrarily posited to the Image *ACB*, it will be rightly posited with the Object itself, or the Object *ab*, and it's secondary Image *AB*, have both the same Position, that is, their similar Extremities *a*, *A*, and *b*, *B*, lie towards the same Parts.

7. Now it has been shewn, that when an Object appears *right* or *erect*, it's Image in the Bottom of the Eye will be *inverted*, or have a Position *contrary* to that of the Object ; (See Chap. II, Art. 1, 2, 3, 4.) and therefore when the Image of an Object in the Eye is the same with that of the Object itself, the said Object cannot appear *right* or *erect*, but *inverted* or *contrarily posited* to what it truly is. And such therefore is the Position of an Object, as it appears through a *double Microscope*.

8. The Position of the Object, in a single Microscope, is not altered ; for let *DF* be the magnifying Lens, and *AB* the Object in one Focus, and *QR* the Pupil of the Eye in
the

the other; then, as hath been shewn, the Position of the Image ACB , in the Bottom of the Eye is *inverted*, or contrary to that of the Object; in which Case it will appear *erect*, or in the same Position with the Object.

9. The Object AB remaining in the Focus of the Lens DF , if another Lens be interposed between the Eye and the said Lens DF , it will not alter the apparent Position of the Object, from which it is by the single Lens DF alone. For all the Effect this will have, is only making the Converging Rays FE , DE , converge the sooner, and so shorten the focal Distance OE ; but there will still be but one Point E , in which Parallel Rays, which come from one Part of the Object cross those which come from the contrary Part; and consequently there can be but one Inversion of the Object AB , and therefore the apparent Position of the Object must be the same as before, whether in the double or single Microscope.

10. It is very easy however to rectify this inverted Position of the Object in the double Microscope, by the Addition of two more Lenses, as will be shewn when we come to treat of Telescopes; and then it may be asked, Why is not this Addition commonly made to the Microscope, as it is to the Telescope? I answer, first, It is not material in small Objects how they appear situated; for instance, there can be no more Advantage in seeing the
Head

Head of a Mite lie towards the Right-Hand, than towards the Left. Secondly, the Object seen through 4 Glasses will not be so bright and lively as through only 2 or 3 Glasses. Thirdly, the Disposition of three Eye-Glasses, that shall erect the Object, will occasion the Microscope to be enlarged, and will very much alter the present commodious and beautiful Form thereof for the worse. Not to mention, that four Glasses will come dearer than two or there.

11. The inverted Image may indeed be made to appear right by two Eye-Glasses only: for if the lowermost DF be removed a little farther from the Image AB, than it's focal Distance, it will form a second Image on the Side, which may be viewed by a third Lens, placed at it's focal Distance from that secondary Image. But unless the Lens DF be extremely convex, the secondary Focus will be at a greater Distance than will be proper for this Machine; besides that, the Object will be very much coloured and distorted; and, in short, any Method of rectifying the Object will prove a Remedy much worse than the Disease.

12. Concerning the *visible Area*, or what is commonly called the *Field of View* in a Microscope, you are to understand the following Particulars relating thereto. If the Image AB be in one Focus of the Eye-Glass DF, and the Eye QR in the other, the Eye will

will see just so much of the said Image, as is equal to the Glass DF, either in Diameter or Surface; that is, in such a Case the *Area*, or *Field of View*, is equal to the Area of the Glass DF, or that Part of it which is open. This is evident from Chap. V, Art. 5, of this Part.

13. But since, in this Case, the Pupil of the Eye is posited in that Point E, where Rays coming from the Extremities of the open Part of the Lens as DE, FE, intersect each other, it is evident the Eye takes in then the greatest Area possible; and therefore an Object or Image AB, which is greater than the open Part of the Lens can never be all seen at once, but one that is less than the Aperture of the Lens may.

14. If the Eye be moved from the Focus E, either nearer to or farther from the Glass DF, the visible Area will be diminished; and you will not discern so much of the Image as before at E: For if it be moved nearer to the Glass in the Axis EC, it will soon arrive to a Part of the Cone DEF, whose Diameter is greater than that of the Pupil, and consequently the external Rays DE, and FE, will fall without the Pupil, and so the Points of the Image A and B (whence they proceed) cannot be seen.

15. If it approach still nigher, it will leave the Rays gE, mE, and then the Parts AG
O and

and BM in the Image, will not be seen. In like manner if it approach so near, that the Rays bE , lE , are excluded, the Parts AH and BL will disappear; and so the Image will vanish more and more from the Sight, till the Eye comes to be upon the Glass, and then it will perceive no more of the Image, than is equal to the Pupil of the Eye. And it is easy to apprehend, that the same things will happen if the Eye be removed farther off in the Cone AEB .

16. The Eye by being moved transversely through the Cone of Rays DEF or AEB , will successively view all the Parts of the Image AB ; for suppose it enter the Cone DEF , on the Side DE , it will successively take in the Rays DE , gE , bE , &c. which, at the same Time, render visible the Points D , g , b , &c. in a direct Order. But if the Eye enters the Cone BEA , on the Side BE , in passing through it, it will indeed successively view all the Parts of the Image AB , but in an inverse Order, *viz.* from B towards A .

17. I would add one thing for the Sake of those who make Microscopes, and that is, that the Focus E , or Vertex of the Cone DEF , should arise so far above the Top of the Microscope, that when it is applied to view, it may be capable of receiving and adjusting the said Focus E to the Pupil, for
 then

Fig. 1

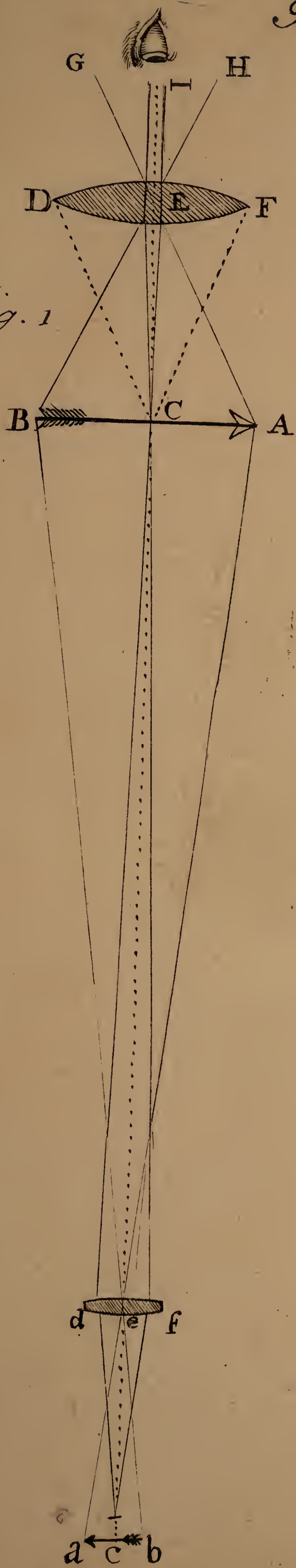


Fig. 2.

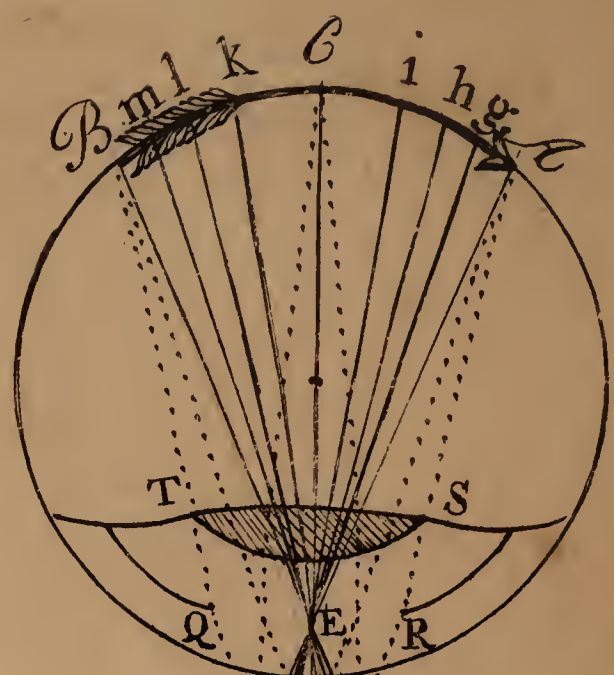
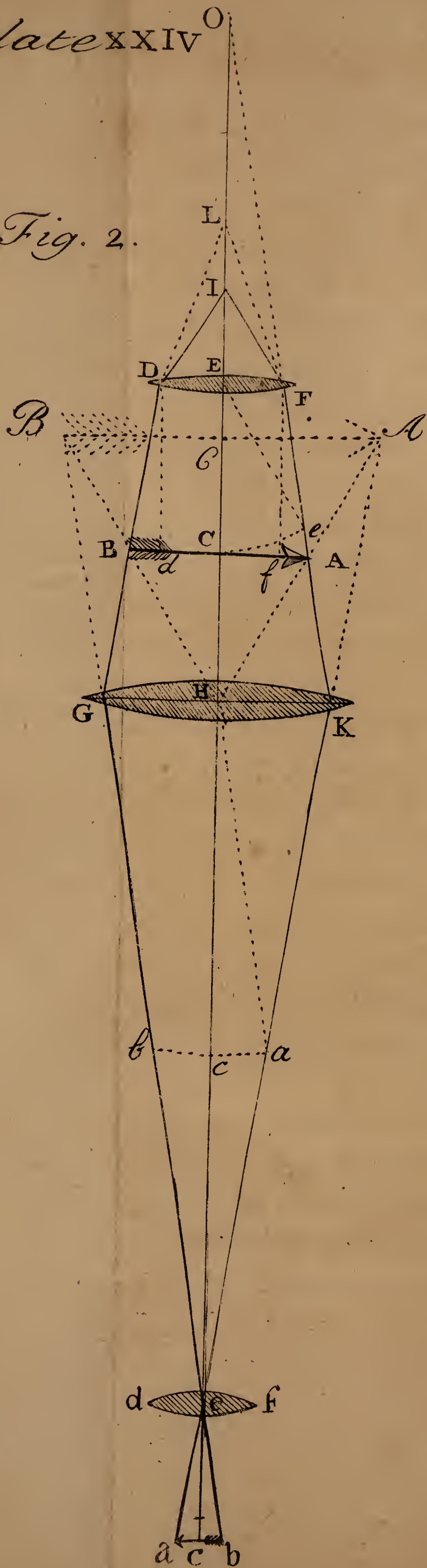
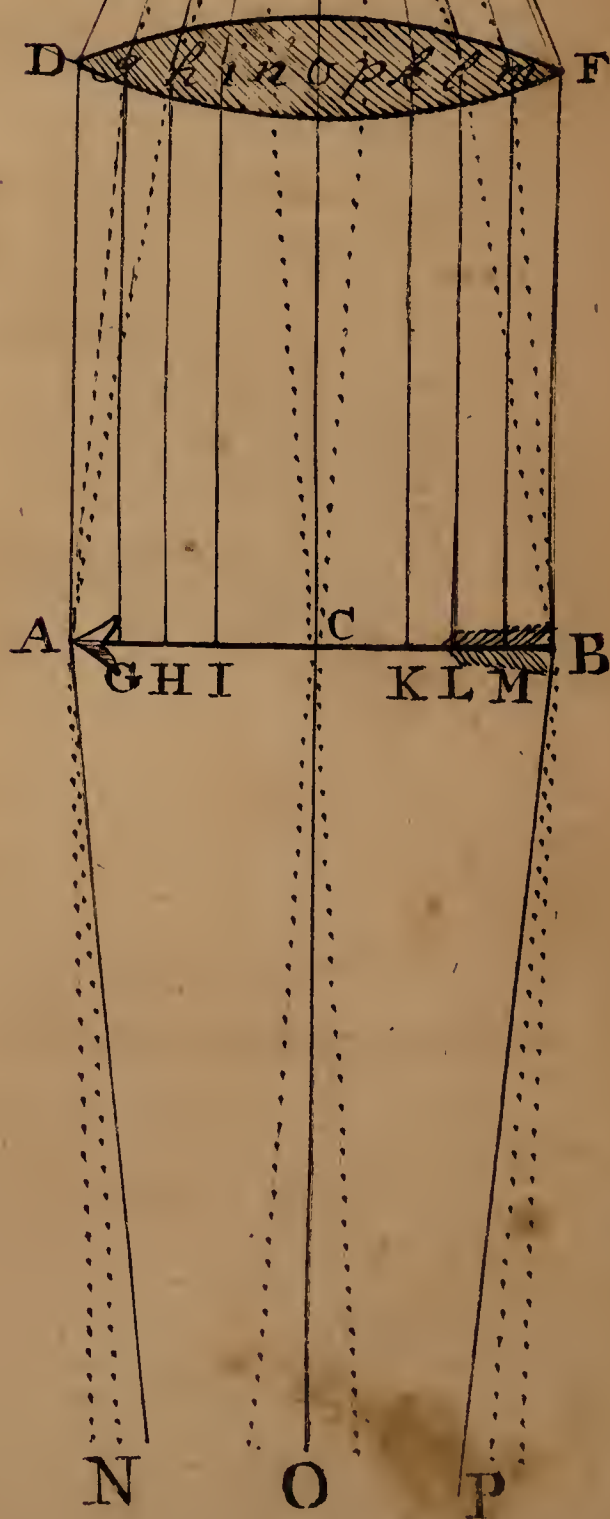
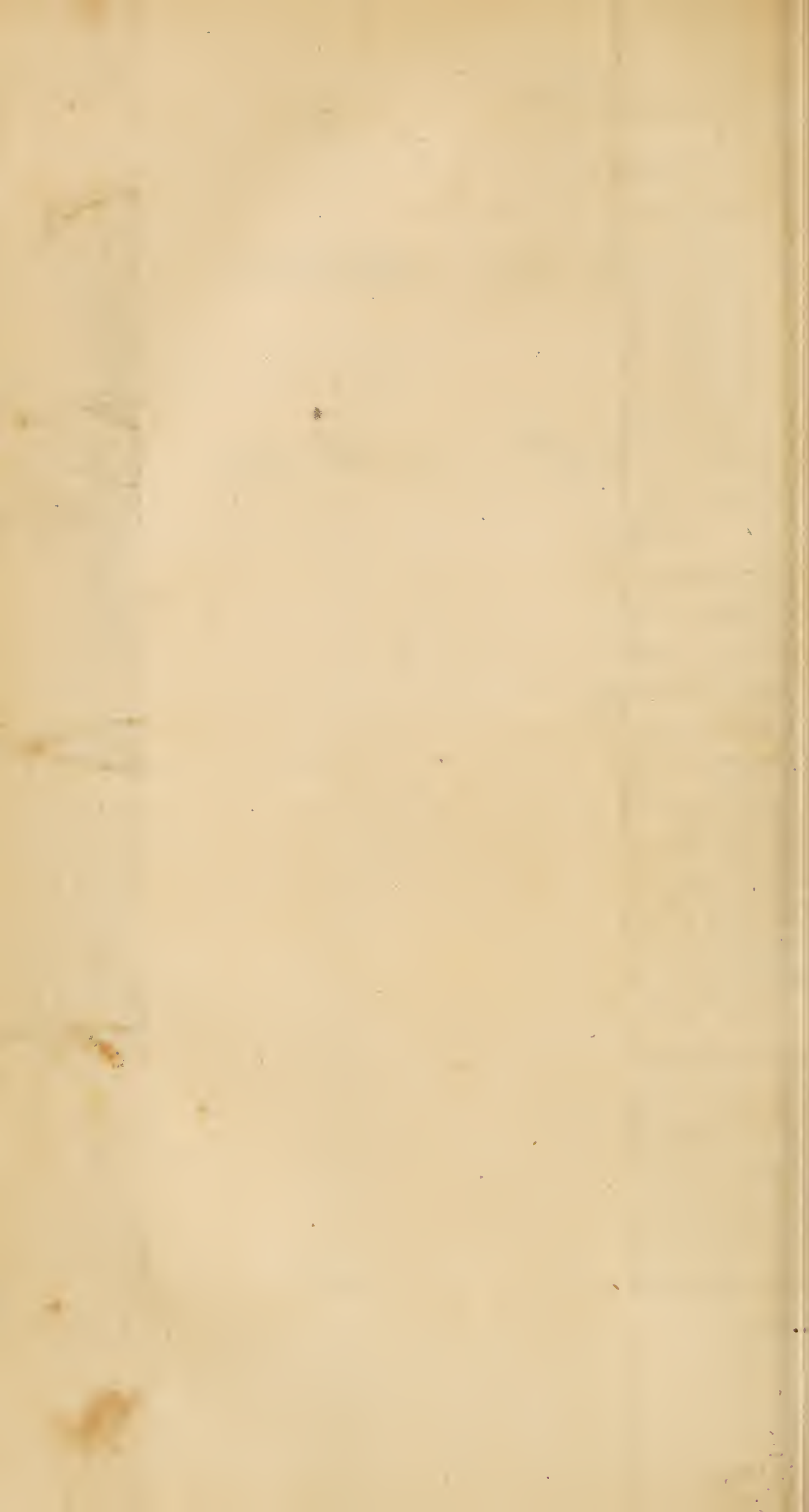


Fig. 3.





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then only (as has been shewn) it takes in the *greatest Area*, or *Field of View*.

18. What I have said with Respect to the *visible Area*, is for a Microscope with a single Eye-Glass D F ; but what relates to the greatest visible Area in a Combination of two Eye-Glasses, or how the optic Angle is vastly amplified thereby, is taught in the last Chapter.



C H A P. IX.

Of the Nature and Effects of a compound Microscope, with a double Concave Eye-Glass.

1. **T**HE Construction of this Microscope is represented in Fig. 1, of Plate XXV. Where ab is the Object, and df the Object-Lens, as before. But the several Pencils of Rays $a.d.Afa$, $b.d.Bfb$, &c. which, in the Microscope with a Convex Eye-Glass, formed the Image ACB , are in this Case, by the Interposition of the double Concave DEF , made to diverge still more towards the Parts K and L ; according to what has been taught of a double Concave Lens in the Theory.

2. It has been also shewn, that if any Rays dEf fall converging on a double Concave DEF , placed at the Distance of the Radius from the Focus C of those Rays or Point, towards which they converge; then those Rays will be so refracted by the said Concave, as to become *parallel* afterwards, and so fit to produce *distinct Vision*. The like is to be understood of the Rays dDf , and dFf ; and therefore,

3. A

3. A double Concave DEF , placed at the Distance of the Radius below the Focus ACB , of the Rays proceeding from the Lens df , will equally cause a Parallelism of those Rays by Refraction, with a double Convex GHI , placed at the Distance of the Radius above it. And consequently AO , the distinct Vision of the Object ab , whence these Rays proceed, will be produced by a double Concave thus posited, as by the double Convex Eye-Glass.

4. Let us suppose the Rays ad , bf , after they are refracted by the Lens df , pass parallel to the Concave DF ; then because dD , and fF , are parallel, they will be so refracted into DK and FL , that those Rays DK , FL , being produced below the Concave, shall meet in a Point B in the Axis, which is the Center of Concavity; from what has been shewn in the Doctrine of Concaves. This Point B therefore will be the Focus in which the Image ABC , of the Object acb , will be formed, and rendered visible to the Eye.

5. The same things supposed, let AC , AB , be drawn thro' the Vertex of the Concave E , from each Extremity of the Image AC ; and because the Distance EB , EC , are equal, therefore the *visible Image* ABC , will be equal to the *virtual Image* ACB ; or it will appear under an Angle AEC , equal to the Angle AEB . Wherefore,

6. Since the virtual Image AB , subtends the Angle AeB , under which the Object ab

appears by the Lens df ; and the Angle AEB , under which it appears by the Concave DF , the Object will be magnified in the Proportion of those Angles; that is, the Object ab will be to the Image AB , (or AC) as the Distance EC (or EB) to the Distance Ee . See Chap. II. Art. 9, of this Part.

7. Hence it follows, that if the Radius of an equally Concave Eye-Glass DF , be equal to the Radius of an equally Convex Eye-Glass GI , the Power of magnifying will be the same in both with the same Object-Lens df : For since EB or EC is equal to CH , by supposition; the Angle AEC , or AEB , will be equal to the Angle AHB . But since the optic Angles are equal from either Glass, the Power of magnifying must be so too.

8. Since the Pencils of Rays bF and aD , tend towards the same Parts, where the Extremities of the Object lie from whence they proceed, after they are refracted by the Concave DF , the Eye will perceive the Position of the Image, to be the same with that of the Object, and not *inverted*, as by a Convex Eye-Glass, wherein the Image is seen by Rays which cross or intersect each other before they enter the Eye.

9. Since the Rays, after they are refracted through the Concave DF , pass on more diverging, it is easy to understand, that an Eye placed any where over the said Glass, can perceive only that Part of the Object, whence
those

those Rays which enter the Pupil do proceed ; thus, if the Eye receive the Rays FL , it will see the Part b ; if the Rays DK , it will see the other extreme a , but if the Eye take in the middle Rays EC , the middle Part C of the Object will only be visible.

10. Hence it appears that the visible Area, or Field of View in this Microscope, is in Proportion to the Magnitude of the Pupil of the Eye and the Density : For as the Pupil is greater or lesser, so a greater or lesser Quantity of Rays will enter it ; and consequently the visible Area, or Part of the Object, will be proportionably greater or less.

11. Again, in Regard of the Density of Rays, it is evident the Pupil, placed where they are most dense, will collect most Rays, and therefore see the largest Part of the Object possible, and that is at the Concave Surface of the Lens itself. For as the Rays diverge from thence, they grow more rare, and so the higher the Eye is raised above the Glass DF , the lesser Quantity of Rays it will collect, and consequently the more will the visible Area of the Object be diminished.

12. In order to compute the visible Area of the Object by this Microscope, let PP be the Diameter of the Pupil of the Eye, (Fig. 2.) applied as near the Concave DF as possible, df the Object-Lens, and AB the Object. From P, P , thro' the Center e , of the Lens df , draw the Lines Pa, Pb , these will

cut the Object in the Points a and b; and so a b will be the Diameter of the visible Area of the Object A B. Suppose the Object, Lenfes, and Pupil, all parallel to one another; it will be, *As the Distance of the Concave DF, from the Lens df, is to the Distance of the Object A B; so is the Diameter of the Pupil PP, to the Diameter of the visible Area ab.*

13. And therefore, for Example, If the Distance of the Concave be 3 Inches, and that of the Object $\frac{1}{2}$ an Inch, and the Diameter of the Pupil $\frac{3}{20}$ of an Inch; the Analogy will be, As $3 : \frac{1}{2} :: \frac{3}{20} : \frac{3}{120} =$ the Diameter a b, of the visible Area of the Object. Now, as in this Case, this Diameter is 6 times less than that of the Pupil, the whole circular visible Area, will be but one 36th Part so big as the Pupil of the Eye. If the Power of magnifying were also 6 times, then the apparent or visible Area in the Image, would be just equal to the Pupil of the Eye.

14. Since these Microscopes are capable of exhibiting but so very small a Part of the Object, they are never used; in as much as the Convex Glasses magnify as much, and take in a vastly larger Area, which gives a Pleasure always in Proportion; for the more we can view, at once, the more we are delighted with the View. Indeed there is nothing to be said for the Use of Concaves in Microscopes, but the shortening of the Instrument: For supposing the Eye-Glasses of equal focal Lengths, that

that is, $EC = CH$, it is plain from the Figure, and what has been taught, that the Eye in view with the Concave, is nearer to the Object-Lens $d f$, than it is in viewing with the Convex, by 3 times the focal Length of the said Eye-Glasses.

C H A P. X.

Of Cata-dioptric or Reflecting Microscopes.

1. **T**H E S E Microscopes differ from the common Sort, principally in this; that whereas they consist *wholly of Lenses*, and perform all their Effects by *Refraction*, these are constructed with *one or more Speculums and Lenses conjointly*, and perform their Effects partly by *Reflection*, and partly by *Refraction*. Of these there are two Sorts, *viz.*

2. The first Sort is that represented in Fig. 3. Plate XXV. Where instead of the Lens $d f$, there is placed a small Speculum $d e f$; the Object $a c b$ being placed above it, at a little greater Distance than the Focus g , has it's Image $A C B$ formed by Reflection, as in the other Case it was by Refraction thro' the Lens $d f$. Now if we suppose the focal Distance of the Object-Speculum $d e f$, and
 Lens

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Lens def the same, the Effect of the Microscope will, in other Respects, be the same also.

3. For, (1.) The Distance of the Object ab above the Speculum, will be equal to the Distance of the Object ab below the Lens, in order that the Image may be formed at the same Distance Ce . (2.) *The Position of the Object will be inverted*; for all the Rays flowing from the Point a , will be reflected by the Speculum to the Point A , in the same Manner as if they came by Refraction through the Lens from the Point a ; thus the Part b in the Object, will be reflected to the Focus B in the Image, which therefore is inverted. (3.) *The Power of magnifying will also be the same in both.* For since the Image AB , and the Object ab , are seen under equal Angles, from the Vertex e of the Speculum, the Triangles aeb and AeB will be similar; and therefore $AB : ab :: Ce : ce$; but in the other it is, $AB : ab :: Ce : ce$. But the latter Ratio of these Analogies are the same in both, and consequently the first are so too.

x 4. This Microscope is not so easy to manage as the common Sort; for Vision, by Reflection, as it is much more perfect, so it is far more difficult than that by Refraction. Nature sells her best Commodities at very great Prices generally. Nor is this Microscope so useful, for any but very small or transparent Objects; for the Object being
between

between the Speculum and Image, would, if it were large and opake, prevent a due Reflection.

5. To make this Microscope answer well, the following Things must be observed. (1.) The focal Distance of the Speculum ought to be $1\frac{1}{2}$ or 2 Inches. (2.) The Diameter or Aperture df , $\frac{1}{2}$ Inch. (3.) The small Speculum should be placed in the Center of a larger one, inclined to the Bottom of an open Tube in an Angle of 45 Degrees, that so the Object may be illuminated by the Light reflected therefrom. (4.) In the Place where the Image AB is formed, should be placed a Diaphragm of Wood, as OP , with a Hole QR , of such Size as to exclude all the imperfect Margin of the Object, or Field of View, and exhibit only the most perfect and distinct Part thereof. (5.) The Eye-Glass DF may be $1\frac{1}{2}$ Inch focal Distance, and should be 10 or 12 Inches distant from the Speculum df . (6.) Lastly, the Aperture or Hole in the Eye, Plate, or Piece at Y , should be just as big in Diameter, as the principal Pencil CE , *viz.* equal to mn , which is easy to define; for since the Triangles dCf and mCn are similar; it will be as $Ce : CE :: df : mn$, which therefore is known.

6. The second Sort of Reflecting Microscopes, is that of Fig. 4. Plate XXV; whose Performance is by *two Reflections, and one Refraction*; and is the Invention of Dr Smith, Professor

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Professors of Astronomy, &c. at Cambridge: the Theory and Construction of which follow.

7. AD is a large *Concave Speculum*, and ad a small *Convex one*; each perforated in the middle with the Holes BC, bc: Both these are Segments of the same Sphere, or ground on Tools of an equal Radius, viz. of 2 Inches, that so the focal Distance of each Speculum may be just one Inch.

8. These two Specula are placed at the Distance of about $1\frac{1}{2}$ Inch from each other, that so an Object OPQ being placed a little below the nether Speculum might be between the Focus F, and Center E, of the larger Speculum. Things thus conditioned, the Rays PA, PD, which flow from the Point P in the Object, on the Speculum AD, will be reflected towards a Focus p, where an Image *opq* would be formed, if the Rays were not intercepted by the Convex Speculum ad, and the Point p being nearer than it's Focus f, the Rays Aa, Dd, which tend or converge towards it, will be reflected to a Focus P, where the last Image OPQ will be formed, to be viewed through the Eye-Glass G, by the Eye at I.

9. Such being the Structure of this Microscope, the Effects thereof may be explained as follows. Because the Object and Image is seen from the Vertex V of the Concave Speculum, under the same or equal Angles OVQ and oVq; therefore (supposing the Speculum
ad

ad away) the Object OQ would be to it's Image oq , as VP to Vp ; and so the Object is not much magnified by the large Concave, which is not the Design of it, but to give the Rays a proper Degree of Convergence on the lesser or Convex Speculum ad , for the Purpose of magnifying.

10. For the Image opq , is now to be considered as a virtual Object to the real Image, formed in the Focus P , by Rays reflected from the Convex Speculum ad . And since by the Theory, the Object and Image appear under equal Angles at the Center of any Speculum, if from e , the Center of the Speculum ad , be drawn the Rays eqQ and eoO ; then shall OPQ be the last magnified Image of the Object OPQ . Which seen by the Eye I , through the Eye-Glass G , is to the Object OPQ , seen by the Eye at the Distance of six Inches, in the compound Ratio of

$$\frac{6 \times Vp \times eP}{VP \times ep \times PG}.$$

11. For, (1.) The Object OPQ is seen by the naked Eye, at the Distance of 6 Inches, under an Angle, which is to the Angle OVQ , under which it is seen from the Vertex of the Speculum AD , as VP to 6; and so the first

Ratio of magnifying is $\frac{6}{VP}$. (2.) The Image

opq is to be considered now as an Object, and the Angle oVq , under which it is seen from

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from the Vertex of the Mirrour AD , is to the Angle oeq , under which it appears from the Center of the Mirrour ad , as ep to pV , and so the second Ratio of magnifying is $\frac{Vp}{ep}$. (3.) The Image OPQ appears under

an Angle, at the Eye-Glass G , which is to the former Angle QeO , as eQ to PG , and therefore the third Ratio of magnifying is $\frac{eP}{PG}$. And so the whole Power of magnifying is expressed by a Composition of these

Ratios, *viz.* $\frac{6}{VP} \times \frac{Vp}{ep} \times \frac{eP}{PG}$.

12. To exemplify this, suppose the Distance of the Mirrours $Vv = 1.6$, and that $vP = 0.1143$, so that $VP = 1.7143$; then will

the first Ratio be $\frac{6}{1.7143}$. Then having the

Distance VP of the Object, that of the Image Vp will be found $= 2.4$; by Theor. 14, of the Theory. And therefore we have $ep = 1.2$; and so the second Ratio will be

$\frac{Vp}{ep} = \frac{2.4}{1.2} = 2$. Lastly, having the Distance

of the Object vP from the Convex Mirrour $= 0.8$; we shall find the Distance of the Image $vP = 4$, by Theor. 10, of *Catoptrics*; and

Fig. 1.

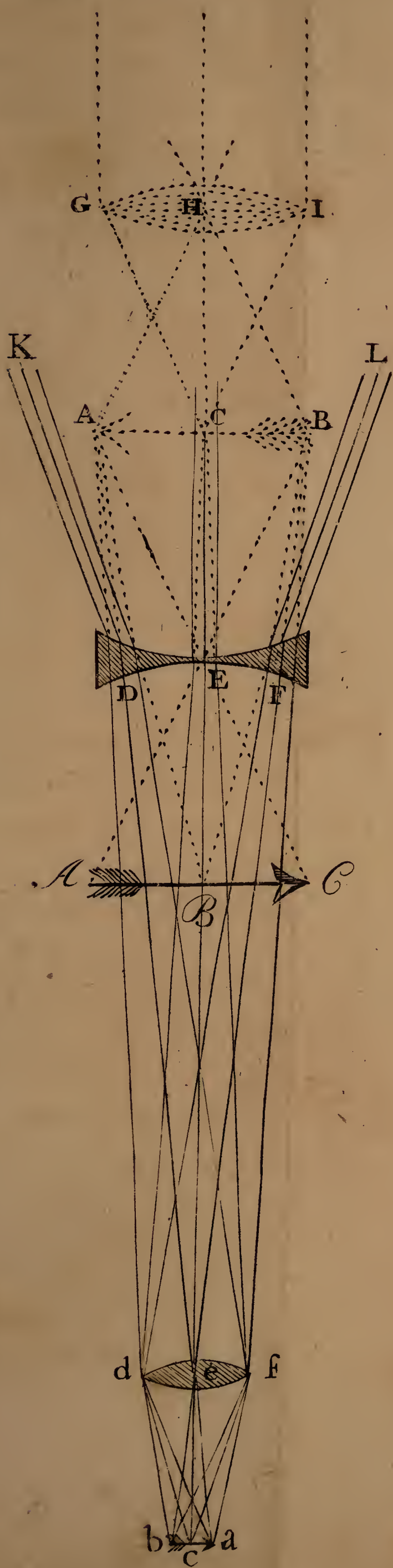


Fig. 2.

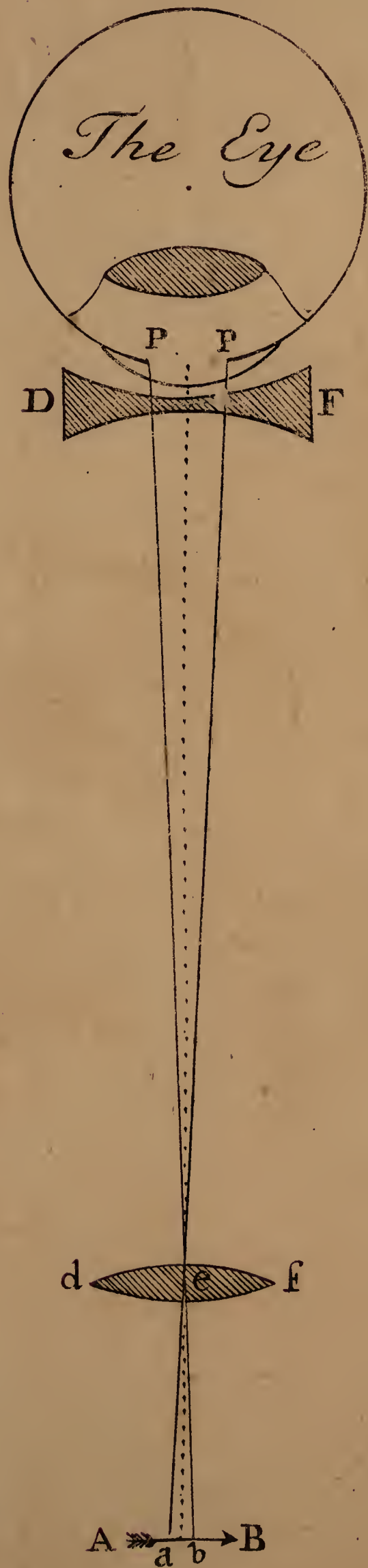
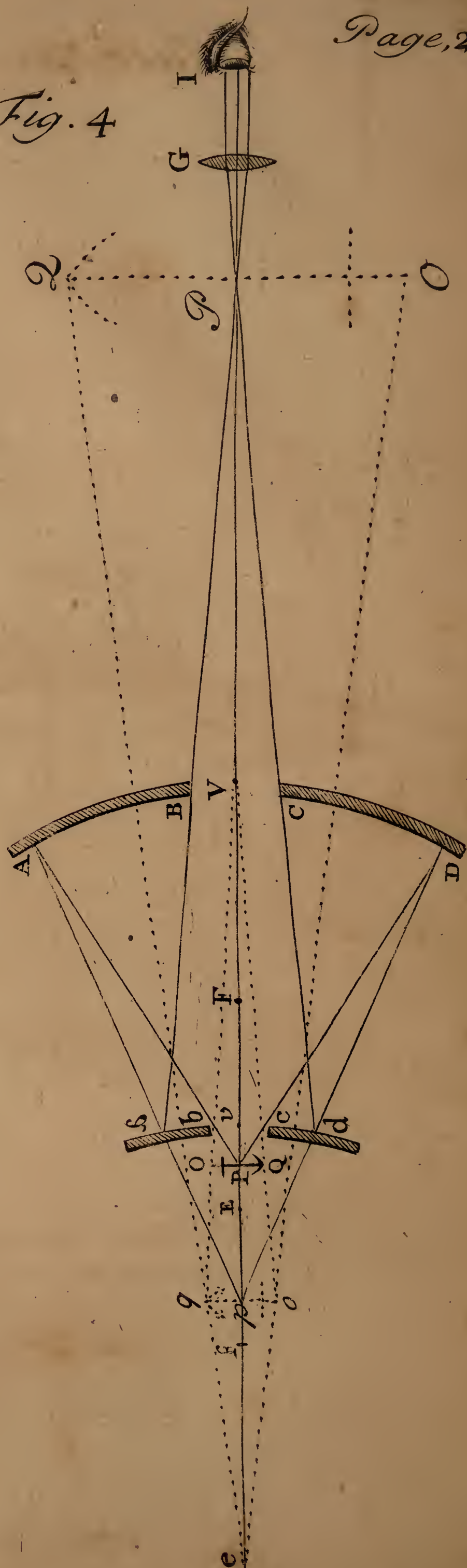
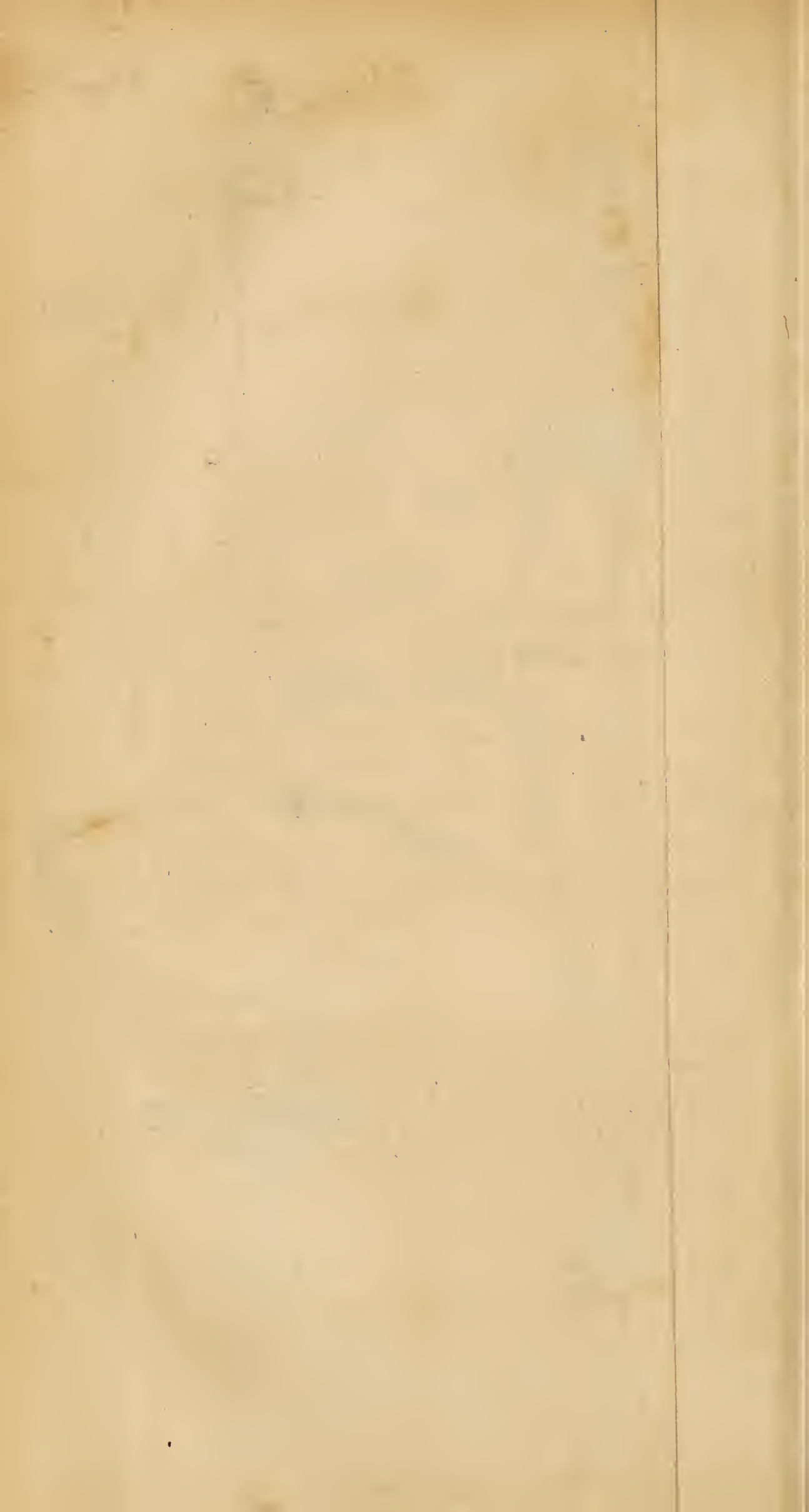


Fig. 3.



Fig. 4.





and therefore the third Ratio is $\frac{eP}{PG} = \frac{4}{0,5}$

if we suppose the focal Distance PG , of the Eye-Glass G , to be $\frac{1}{2}$ an Inch. So that the

whole Power of magnifying will be $\frac{6}{1,7143}$

$\times 2 \times \frac{4}{0,5} = 56$; that is, the Object will

be magnified 56 times by this Microscope.

13. But the Author of this Microscope supposes the Object to be at 8 Inches from the naked Eye, for distinct Vision; and also that by such a Combination of Speculums, the Confusion arising from an Aberration of Rays is so far avoided, that an Eye-Glass G may be used, whose focal Distance PG is not greater than 0,18, or 0,15 of an Inch, and that then the Microscope will magnify 300 times. They that will, may see a great deal concerning the Structure and Dimension of Parts, with all the necessary Cautions for Workmen in fitting up such a Microscope, in the Author's Remarks on his Treatise of Optics, from Page 87 to 97.

14. This Microscope being so near allied to the Reflecting Telescope in it's Nature, that what might be here further said with Respect to the Aberration of Rays, &c. will be referred to those Chapters, wherein the Theory of these Telescopes is more perfectly considered.

C H A P. XI.

Of Telescopes in general; and of the common Dioptric Telescope in particular.

1. **T**HE *Telescope* is the next principal and most useful optical Instrument. The Word is of *Greek* Derivation, and signifies, *The Perfection of the Sight or View*. For as our Sight, at best, extends not far with Distinctness, so most Bodies situated on the Earth, and all those in the Heavens, are so remote from the Eye, that unless it were assisted with a proper Instrument, it would be uncapable of a nice, distinct, and compleat View, and so we could never be able to form any just, regular, or adequate Ideas of those Objects, or their Parts, Forms, Colours, Magnitudes, &c.

2. But, by means of the Telescope, remote Objects are made to seem near, small apparent Magnitudes are enlarged, confused Objects are rendered distinct, and the invisible and obscure Parts of very distant things, are brought out to Sight, and rendered clear to the View; which therefore it greatly *perfects*, and merits the Name it bears with the greatest Propriety.

3. Of

3. Of Telescopes there are two Kinds, according to the two different Sorts of Vision, viz. by *Refraction* through Lenses, and *Reflection* from Mirrours. The Telescope therefore which is constructed with Lenses, and performs it's Effects wholly by refracted Light, is called a *Dioptric Telescope*; and is that in common Use. But the other Kind of Telescope performs it's Effects partly by *Reflection*, and partly by *Refraction*, and therefore is composed of *Mirrours* and *Lenses* jointly; on which Account it is called a *Cata-dioptric Telescope*, or, most commonly, a *Reflecting Telescope*.

4. The essential Parts of a *Dioptric Telescope* are two Lenses, *AB* and *EY*; (See Fig. 1. Plate XXVI.) of which *AB* is called the Object-Glass, and *EY* the Eye-Glass, for the Reasons those were, in the Microscope, so called. These two Glasses are so combined in a Tube, that the Focus of each is coincident in the same Point *F* between them.

5. This being understood, let *OB* be a vastly distant Object; the Rays then which come from it to the Object-Glass will be nearly parallel. Suppose two of these Rays be *OA*, and *BB*, these Rays in passing through the Glass *AB*, will be made to converge to it's focal Point *F*, where they intersect each other, and pass on to the Eye-Glass *EY*; but *F* being also the Focus of the Glass *EY*, these

P

in

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in passing it will be again made parallel, and so entering the Eye, *will produce distinct Vision of the distant Object* OB, according to the *Theory*.

6. The Eye sees it also *magnified*; for OE being vastly distant, the Length of the Telescope is inconsiderable in Regard of it; and therefore supposing the Eye viewed from the Center of the Object-Glass C, it would see it under the Angle OCB; let OC and BC be produced to the Focus of the Glass, they will there limit the Image IM, of the Object formed in the said Focus. From the Extremities of the Image IM, let the two parallel Rays proceed to the ocular Lens EY, these will be converged in it's Focus D, and the Eye will there see the Image under the Angle EDY; all which is evident from the Principles before laid down.

7. Therefore the apparent Magnitude of the Object, seen by the naked Eye, is to that which is viewed in the Telescope, as the Magnitude of the Angle OCB, or ICM, to that of the Angle EDY, or IGM: (for since $GD = GF$, and $IM = EY$, the Angle $EDY = IGM$) But the Angle IGM is to the Angle ICM, as CF to FG. (by Art. 9. Chap. II. of this Part.) Therefore the Magnitude of the Object by the naked Eye is to that by the Telescope, as CF to FG, that is, *as the focal Length of the Object-Glass to the focal Length of the Eye-Glass.*

8. TH

8. The Object will also appear inverted by this Telescope, in the Focus IM, the Reason of which has been explained in the Construction of the Microscope with a single Eye-Glass, and is evident from the Scheme itself. And, in short, they who understand the Doctrine or Theory of the Microscope, must needs know that of the Telescope, because the Analogy of Construction in both is so very much the same.

9. As to the Power of magnifying by these Telescopes, it appears from the foregoing Proportion, (Art. 7.) to be capable of vast Augmentation; because the focal Length of the Object-Glass CF may be vastly increased, in Regard of the focal Length of the Eye-Glass FG, in the lengthening of the Instrument. Thus, if $CF = 36$ Inches, and $FG = 1$ Inch, this Instrument will magnify the Diameter of Objects 36 times, their Surfaces 1296 times, and their Solidity 46656 times.

10. If now you would enlarge the Telescope, and chuse an Object-Glass 10 Feet Focus, or 120 Inches, the focal Length of an Eye-Glass of about 2 Inches, will be sufficient; and then will the Diameter of Objects be magnified 60 times, their Surfaces 3600 times, &c. which is vastly more than the former, and yet the Object nearly as distinct.

11. Again, if you would use an Object-Glass of 40 Feet focal Length, you fit there-
to an ocular Lens of $3 \frac{1}{2}$ Inches; and then

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will such a Tube magnify the Diameters of Bodies (with the same Distinctness) near 1400 times. If, lastly, the Length of the Focus in the Object-Glass be 100 Feet, or 1200 Inches, then an Eye-Glass of $5\frac{1}{2}$ or 6 Inches Focus will be sufficient; and such an one will magnify the Diameter of Objects 200 times; the Surfaces 40000 times; and the Solidity 8000000 times; which, though it is so prodigious an Effect, has been exceeded by Telescopes of this Sort made by some Artists; particularly that of Mr *Huygens*, which was 120 Feet long.

12. But then these very long Telescopes magnifying so much, are not fit for viewing Objects on the Earth, because they have not Light enough when thus magnified, to render their Images visible. For suppose of two Telescopes, one magnifies 10 times as much as the other, then in the focal Space or Area, there will be 10 times less in Diameter (and consequently 100 times less in Surface) of the Object represented by the greater Magnifier, than by the lesser; but the Light on the Surface of the Object being the same, it is plain the Quantity of Light in the *focal Area*, (of the same Dimensions in each Tube) will be 100 times less in the great Telescope, than in the small one; and therefore a Telescope above 6 or 8 Foot long, is not fit for terrestrial Uses.

13. What

13. What has been said about the Illumination of the Image in the Telescope, supposes the Aperture of the Object-Lens to be the same in both the Tubes; but when the Tube is to be enlarged, it will bear a greater Aperture also, and will therefore admit of more Light; thus if a Tube of 4 Feet may have an Aperture of 1 Inch, a Tube of 30 Feet will admit of an Aperture of 3 Inches. But the Light entering by these Apertures, will be as the Squares of their Diameters; that is, as 1 to 9; or 9 times more Light will enter the Aperture of the greater Lens, than that of the lesser; but then, since the Image of Objects, by the greater Lens, is to that formed by the lesser, as 30 to 4 in *Diameter*, they will be in Surface, as 900 to 16, or as $55\frac{1}{4}$ to 1; and therefore the Light on the same Area of the Image, will be 55 times less in the greater Tube, than in the lesser one; therefore the Light in these two Telescopes, will be as 9 to 55 nearly; that is, above 6 times greater upon the Image in the lesser, than in the larger one.

14. Since the *Power of magnifying*, is the Proportion of the focal Length of the Object and Eye-Glasses, and this Proportion being to be varied in any Degree, in any Length of a Telescope, it may seem strange that a short Telescope will not answer that Purpose, as well as a long one. But they who thus think, must consider, that if the Power of magnifying be

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augmented, the Length of the Telescope being the same, it is necessary to diminish the focal Length of the Eye-Glass in the same Proportion; but this can't be done by reason of the great Confusion of Colouring and Distortion of the Image, on Account of the too great Convexity of the ocular Lens in such a Case.

15. For suppose in a Telescope of 3 Feet, the Power of magnifying be as $\frac{1}{36}$, (See Art. 9.) if you would augment this Degree of magnifying 5 times, (*viz.* makes it as $\frac{1}{180}$) then must you have an Eye-Glass not above $\frac{1}{2}$ of an Inch focal Length, and this would be so very small as to admit of an Aperture not bigger than the Pupil of the Eye, nor would the Aperture of the Object-Glass admit of Light enough to illuminate the Object. On both which Accounts, the Design and Use of a Convex ocular Lens would be frustrated.

16. But suppose you require it to magnify but *twice* as much as before, *viz.* that the Power be as $7\frac{1}{2}$; even this can't be done conveniently for the same Length of the Tube. For, (1.) The Lens being but $\frac{1}{2}$ an Inch focal Length, will not admit an Aperture of above $\frac{2}{10}$ of an Inch conveniently, to view the Object distinctly, which is still too small. (2.) The Aperture of the Object-Glass will then be too small, to admit a requisite Degree of Light. (3.) This Aperture of the Object-Lens can't be enlarged, because there would be

be then too much Light for the Power magnifying, which would produce Confusedness in the Image. (4.) Neither must we attempt to enlarge the Aperture of the ocular Lens; for then the Rays would fall too far from the Axis of the Lens, to be sent parallel to the Eye, and therefore would prevent *distinct Vision*.

17. On these, and some other Accounts, it is evident, that if we would increase the Power of magnifying, we must necessarily increase the Length of the Telescope, by using an Object-Lens of a greater focal Length; for it principally depends on that, the focal Length of the ocular Lens being very little augmented. As for Example, if the Power of magnifying in any Telescope be known, or made a Standard, that Power, in any other Telescope, may be augmented to any Degree you please, in the following Method.

18. The Eye-Glass must have the focal Distance such a Number of Inches, as is expressed by the proposed Degree of magnifying; and the same Number squared, and multiplied by the focal Distance of the Object-Lens of the lesser Telescope, will give the focal Length of that required.

19. Thus if the Telescope, in Art. 9, be the given one, or Standard, whose Power is $\frac{1}{36}$; and you would increase this Power *once and an half*, or make it $\frac{1}{54}$, then must the Eye-Glass of the enlarged Telescope be $1\frac{1}{2}$
P 4 Inches;

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Inches; the Square of which is $2\frac{1}{4}$, which multiplied by 3, is $6\frac{3}{4}$ Feet for the focal Length of the Object-Glass in the Telescope required. Again, if it be required the Telescope should magnify *twice* as much, then must the focal Length of the ocular Lens be 2 Inches; the Squares whereof, 4 multiplied by 3, gives 12 Feet for the Distance of the Focus in the Object-Lens. And thus a Telescope that shall magnify 3 times as much, shall have an Eye-Glass of 3 Inches focal Length, and an Object-Glass of 27 Feet. And universally in Telescopes of this Sort, *The focal Distances of the Object-Glasses, should be always nearly in Proportion to the Squares of the focal Distances of the Eye-Glasses.*

20. But (as I have before observed) when Telescopes of this Sort are made of a great Length, they are not fit for viewing terrestrial Objects, but are appropriated to celestial Observations: But for such Purposes the Glasses are not managable in Tubes, which are either too slight and apt to bend, or too bulky, heavy, and unwieldy, if made strong in Wood: And therefore the Object-Glass is generally fixed on the Top of an high Pole, &c. and an Eye-Glass so adjusted thereto, that their Axis exactly coincide, and thus they view the heavenly Bodies which pass along the Ecliptic. This Form is called an *Aërial Telescope*.

21. For

21. For Tubes in Telescopes, as they answer the Purpose of a *Dark Chamber* to represent the Images of Objects in, have here no Use in that Respect; for the Night being the Time for Astronomical Observations with the Telescope, it is generally dark enough of itself, without a Tube, and the celestial Objects being very bright, make very clear, distinct, and bright Images, let the Telescopes magnify as much as they will.

22. Tho' the brighter the Object, the less the Aperture of the Object-Glass should be; thus in viewing the Sun, or Venus, we are obliged to use a smaller Aperture than for the Moon, Mars, Jupiter, or Saturn, and their Satellites. And tho' (as was shewn Art. 8.) in this Telescope with one Eye-Glass, the Object is inverted, yet this is not heeded by Astronomers, it is all one to their Purposes which Part of the Sun, Moon, or Planets, appears East or West, North or South.

23. But in viewing terrestrial Bodies, this Inversion of the Object can't be admitted; it would be unnatural and displeasing to set the Earth above, the Sky beneath; Houses, Trees, &c. turned Topsy-turvy, &c. To remedy this Inconvenience therefore, there are added two other Eye-Glasses, LN and RS, (See Plate XXVI. Fig. 2.) which erect the Object, and so make all things appear in their natural Positions.

24. In

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24. In order to effect this, the first of these two Glasses, *viz.* LN , is placed at twice it's focal Distance from the former Eye-Glass EY , (for they are all three supposed to be of the same Sphere) and the other RS , at the same Distance from it. Now the Pencils of Rays being made to intersect each other, by the first Glass EY , in it's Focus D , and at the same Time, the Rays of each Pencil made parallel; these Pencils will be rendered parallel by the second Glass LN , and their Rays made to converge in it's Focus F ; where they now represent the Image of the Object erect, in the Points I, F, M . The Pencils proceeding parallel from hence to the third Eye-Glass RS , are by it collected together in it's Focus P , where their Rays again made parallel, render this erect Image distinctly visible. All which is evident from the Scheme, and the foregoing Theory.

25. Notwithstanding it is usual to use three Eye-Glasses, as before explained; yet two will erect the Object, and keep the same Magnitude of the Image. For suppose the middle Lens LN , taken quite away, if the first Lens EY , be placed at D , which is double it's focal Distance from the Image IM ; it will, at the same Distance DF on the other Side, form a secondary Image IM , equal to the primary Image IM , and also in a contrary Position, (by Theor. 17. Chap. II. of

of *Dioptrics*) that is, it will be just the same in Magnitude, Position, and Place, as it was by two Lenses.

C H A P. XII.

Of Galileo's Telescope, or Prospective Glass.

1. **T**HIS is a Telescope composed of a Convex Object-Lens *AB*, and a Concave ocular Lens *EY*. (see Fig. 3.) It is the first Form we have of a Telescope, and was invented, and applied to Use, by that celebrated *Italian* Philosopher *Galileo*, who lived in the Beginning of the last Century.

2. The Theory of this Telescope, is entirely the same with that of a Microscope, with a Concave ocular Lens, in Chap. IX, already explained. For this Telescope represents Objects *distinct and erect*, and magnifies them in Proportion of the *focal Length of the Object-Glass*, to the *focal Length of the Eye-Glass*; and the Eye the nearer it is applied to the Lens *EY*, the larger Area it takes in; and still greater there in Proportion to the *Largeness of the Pupil of the Eye*; the Reason of all which has been there explained, and is evident

evident in comparing the Figure of this Telescope and that Microscope together.

3. Now though the *visible Area*, or *Field of View*, in this Telescope, depends on the Pupil of the Eye, yet it is of much greater Service here than in the Microscope; because in that the small invisible Parts of near Bodies were to be magnified in a very large Image, of which the Pupil can take in but a small unsatisfactory Part; whereas in the Telescope, the Parts of large distant Bodies are to be rendered distinct in a small Image, in the Focus of the Object-Lens, which Image the Pupil of the Eye can generally take entirely in, and thereby view the whole Object, or Picture of Objects, at once, which is sufficient Satisfaction.

4. I shall make this easy by an Example. Fig. 4. Let AB be the Object-Lens of this Telescope, 4 Feet focal Length, and EY the ocular concave Lens. Let OB be an Object to be viewed, whose Height is 40 Feet, and Distance AO 3 Miles, or 15840 Feet. Now since it was shewn in the Theory, that the Object and it's Image are as their Distances from the Lens AB , say, As the Distance AO 15840, is to BO 40, so is the focal Distance of the Lens AB 4, to $\frac{160}{15840}$ of a Foot for the Length of the Image IM , which is about $\frac{1}{10}$ of an Inch, the Pupil of the Eye being larger than that, will take in the whole Image IM , and so be able to view the whole Object

Object O B, at once, with great Pleasure and Satisfaction.

5. If the focal Length of the Concave be 1 Inch, then will the Object be magnified in the Proportion of 48 to 1, that is, it will appear 48 times higher, and therefore 48 times nigher also than to the naked Eye; that is, a Person, by the Help of the Telescope, will be as well able to view the Object O B at 3 Miles Distance, as he would by the naked Eye, at the Distance of 330 Feet; which is but the 48th Part of 3 Miles.

6. On this Account they are of great Use to People for viewing of terrestrial Objects; but still of much greater Use for viewing of the heavenly Bodies. For an Object-Lens of 5 Feet Focus, will bear an Eye-Glass of but 1 Inch; and such a Telescope will magnify the Diameters of the Planets 60 times, and their Surfaces 3600 times, which is sufficiently enough to render the Spots in the Sun, the Horns of Venus, the Satellites of Jupiter, and Ring of Saturn visible, if well managed; the Truth of this I know very well by Experience.

7. Again, the *Field of View* being so small in these Tubes, is not so much to be insisted on here, where the Images of the Planets are vastly less than the *visible Area*, except of the Sun and Moon; Jupiter, and all his Satellites, are many Times to be seen at one View, in these Telescopes; though it must be acknowledged, that there is some Difficulty in finding
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ing Objects thus with Telescopes, more than in the other Sort to Persons not used to them.

8. The principal things which recommend this Telescope, are, (1.) That it is cheaper than the other, having but one Eye-Glass instead of three. (2.) It is shorter than the other with Convexes, by 8 or 12, or 16 Inches, sometimes, though the Power of magnifying be the same in both. (3.) The Vision is more distinct and perfect, the Rays being not so much lost and refracted thro' one very thin Lens, as through three very thick ones. In short, were People aware of the Usefulness of this Telescope for Astronomical Purposes, they would be much more frequent than they are.



C H A P. XIII.

Of the manifold Uses of a Dioptric Telescope, viz. in shewing the Spots of the Sun, Eclipses, &c. in a dark Chamber; in Surveying and Leveling; in Measuring the Distances of Objects at one Station; and divers Astronomical Purposes.

1. **T**HE Uses of the Dioptric Telescope are many, and of vast Importance to the Sciences and civil Uses of Life, some of the principal of which I shall concisely point out and describe in this Chapter.

2. And, first, *to view the Spots in the Sun through this Telescope.* Fit the Telescope to a Focus for viewing the celestial Bodies, and then take a circular Piece of plain and clear Glass, and hold it a little above the Point of the steady Flame of a Candle, till it be uniformly blackened over with the Smoke; then applying this smoked Glass before the Eye-Glass next the Eye, and you will be able to view the splendid Face of the Sun without Offence to the Eyes, and with Pleasure see
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the various Spots, &c. dispersed here and there upon it.

3. But the best Way of viewing the Spots in the Sun, is by fixing the End of a Telescope of about 4 or 5 Feet long, into a Scioptric Ball, fixed to the Hole PQ of a Window-Shut HK, in a darkened Chamber. (See Fig. 5.) In this Case one Eye-Glass EY is sufficient, whose Focus let be F, and let IM be the Image of the Sun, formed in the Tube by the Object-Glass AB, in the moveable Ball. Now it has been shewn, that in viewing Objects with the Eye through the Tubes, the Image IM must always be exactly in the Focus F of the Eye-Glass, to produce distinct Vision.

4. For then the Rays EO, YO, proceeding from any Point O in the Image, would have passed parallel after Refraction through the Eye-Glass EY. But now, for this Purpose, the Eye-Glass is to be drawn a little farther from the Image than is it's focal Distance, and then a Pencil of Rays EOY, proceeding from the Point O, will be made by the Eye-Glass to converge to a Focus R, according to Theor. 16, of Chap. I, of *Dioptrics*. In this Focus R therefore, the Image IOM will be formed anew, and in an *erect Position*, in the Image IRM.

5. The Diameter IM of this secondary Image, is to the Diameter of the former IM, as the Distance GR, to the Distance GO, by the
the

the Theory. Thus if GO be $1\frac{1}{4}$ of an Inch, GR will be 5 Inches, and so the Diameter of the Image IM , to the Diameter IM , as 5 to $1\frac{1}{4}$. If the Focus GF be 1 Inch, and it's Distance from the Image FO be $\frac{1}{10}$ of an Inch; then will GR be 11 Inches, *viz.* 10 times as large as GO , and therefore IM will be 10 times as large as IM , and consequently the Surface of the secondary Image will be 100 times as large as that in the Tube IM . And therefore a Sheet of Paper held in that Focus R , will receive the vast Image IM of the Sun's Face, and the Spots therein will be very large and distinct. I have sometimes exhibited the larger Spots this Way, near an Inch in Diameter. The Moon may likewise be shewn this Way; but her Light is too faint to make the dark Parts appear well. But an Eclipse of the Sun is this Way, the best of any, shewn to an Advantage. This is a most agreeable Sight.

5. Suppose then ELC and YMC , two Rays proceeding from the extreme Limbs of the Sun's Disk, and passing through the Center C of the Object-Lens; they will define the Extremities of the Image IM , and falling on the Glass EY , will be converged to the Point D , where crossing each other, they proceed to limit the Extremities of the Image IM . Now IDM is the Angle under which this Image is seen at the Distance DR , by the Eye, and is equal to the Angle EDY : But

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the Angle EDY is a little larger than it would be, if the Image IM were in the Focus F : (because GC is then somewhat shorter, and consequently GD somewhat longer) Therefore the Image IM , at the Distance DR , is seen somewhat larger than the Sun appears in the Telescope adapted for distinct Vision.

6. In *Surveying*, the Use of the Telescope is admirable; the Business here is to take the angular Distances between distant Objects on a Plane truly horizontal; or else the angular Elevation, or Depression of Objects, above or below the Plane of the Horizon; in order to obtain either of these Sorts of Angles to a requisite Degree of Exactness, it is necessary the Surveyor should have as clear and distinct a View, as possible, of the Objects, or *Stations* *Staves*, which he fixes up for his Purpose, that he may more certainly determine the Point or Part of the Object viewed, which exactly corresponds to the Line or Edge of the movable Arm (or *Alidade*) on the Theodolite, which cuts off the Degrees and Minutes contained in the required Angle.

7. Now as these Objects are generally at too great a Distance, for the Surveyor to be able to discern so well with the naked Eye, he takes the Assistance of the Telescope, and thus obtains his Ends compleatly on two Accounts, *viz.* (1.) His having a more perfect Sight of the Object, by the magnifying Power of the Glass; and, (2.) His being able to determine

determine the aforefaid Point in the Object by means of the Cross-Hairs in the Focus of the Telescope.

8. That he is affifted greatly, in the Sight of his Objects, by means of the Telescope, is evident; for fuppofe the Telescope were but 13 Inches long, it will admit of an Object-Lens of 12 Inches, and an ocular Lens of 1 Inch, and will therefore magnify the Diameter of Objects 12 times; that is, it will make the Objects appear 12 times nearer to the Surveyor than they are, and confequently he can view them 12 times more diftinctly than by the ~~naked~~ Eye.

9. Again, in the fecond Place, by means of the *Cross-Hairs* in the Focus of the Telescope, he is able to determine exactly that *Point* or *Line* in the Object, which is in the Plane that paffes through the Center of the Theodolite, and the Degree and Minute in the Limb of the Circle, which is the Quantity of the Angle required. For thefe Cross-Hairs, or Wire, being fo placed in the Focus, as that one of them is parallel to the Horizon, and the other perpendicular thereto, and their Point of Interfection being exactly in the Axis of the Tube; it follows, that the Point in the Object, which answers the Point of the Interfection of the Wires, or the Line in the Object, which is covered by the vertical Wire in the Tube, is the Point or Line required; and which alone can give the Angle

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with Certainty, and this is the very Grounds and Foundation of all *Surveying* and *Levelling*.

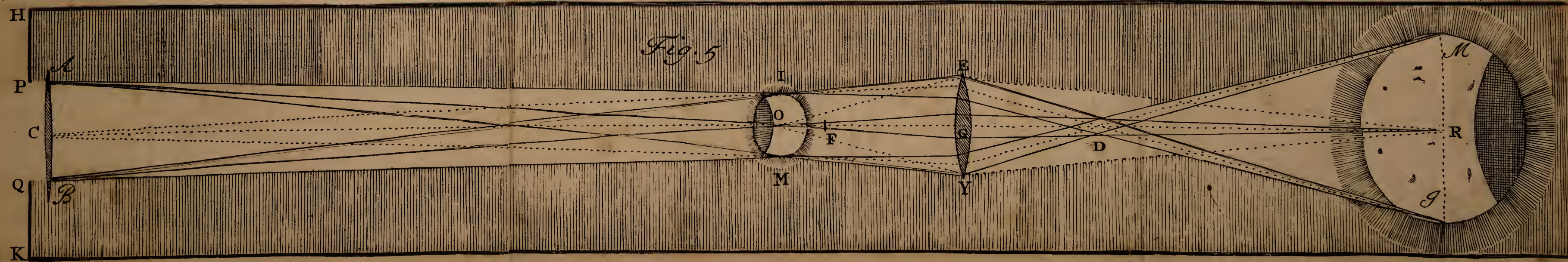
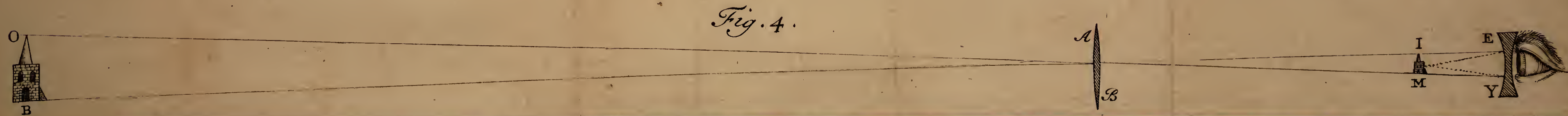
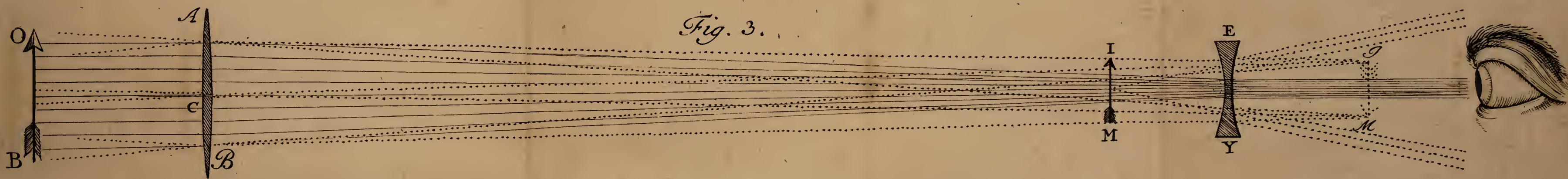
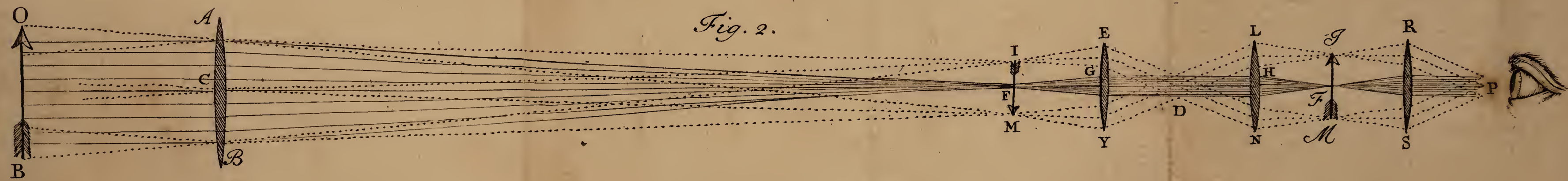
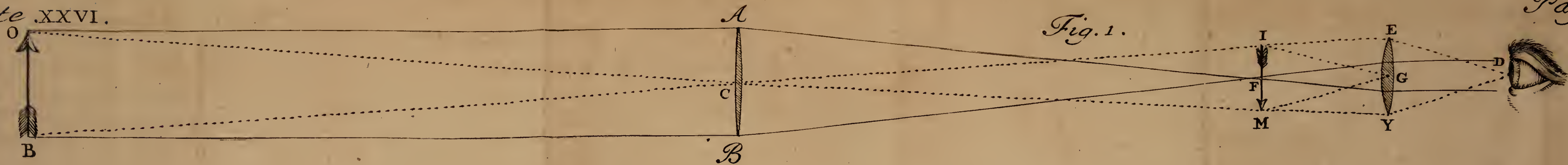
10. There is another Use to be made of the Dioptric Telescope, very singular in it's Nature; and that is, *to measure the Distance of an Object at one Station*. This is the great *Desideratum* of Geometry, which furnishes Methods enough to do it at *two or more Stations*, but none at *one*. Yet this, in Theory, is perfected by the Telescope, and in a good Degree to be ascertained in Practice.

11. The Theorem for this Purpose is obtained from Theor. 16, Chap. I, of *Dioptrics*,

viz. $\frac{dr}{d-r} = y$. For hence we have $dr =$

$dy - yr$, and then $dy - dr = yr$, whence we have this Analogy, $y - : r :: y : d$; that is, in Words, *As the Difference between the focal Distance of the Object, and Radius of the Object-Lens is to the said Radius; so is the focal Distance of the Object, to it's true Distance from the Object-Lens*. But the three first Terms of this Analogy are known, or may be known very easily; and therefore the fourth Term or Distance of the Object is given.

12. For the Radius of the Object-Glass is known by holding it in the Sun, and measuring the Distance of the Sun's Image therefrom, when brightest and most distinct, for that will be the Radius of the Lens, as was shewn





shewn in the Theory. Then for the focal Distance of the Object in the Tube, you are first to get a pretty good Sight of the Object, and then slide in the Drawer till the Object begins to appear obscure, and mark that Place on the Tube precisely; again, draw out the Tube till the Object begins again to be obscured, and then make another mark on the Tube at the End of the Drawer, as before. Lastly, take the middle Point between these two Marks, and that will be the Point where the Image of the Object will be formed most distinct, which you are to measure nicely from the Object-Lens, and you have the Data required. An Example will render this Matter easy.

13. Let AB be the Object-Lens, EY the ocular Lens, EC the Radius of the Lens AB , and CF the focal Distance of the Object OB , whose Distance QC , from the Glass AB , is to be measured. (See Fig. 1. Plate XXVII.) Now suppose $CF = 48$ Inches, or 4 Feet; and you find, by the Method above, that CF is 50 Inches; then FF is 2 Inches, and the Analogy is, As $FF = 2$, is to $CF = 48$; so is $CF = 50$, to $CQ = 1200$ Inches, or 100 Feet; and so far is the Object OB , from the Lens AB . Again, suppose you find $CF = 49$ Inches, then will $FF = 1$ Inch; and the Analogy is $1 : 48 :: 49 : 2352 = QC$, or 196 Feet. If CF be found $48\frac{1}{4}$ Inches; then $\frac{1}{4} : 48 :: 48\frac{1}{4} : 9264 = QC$, or 772 Feet.

Q_3

Feet. So that this Telescope will measure only small Distances.

14. But suppose AB a Lens, whose Radius, or solar Focus $CF = 12$ Feet, or 144 Inches; and you find by the above Method, (in Art. 12.) that CF is 146 Inches; then will $FF = 2$ Inches; and then by the Analogy, As $2 : 144 :: 146 : 21024$ Feet $= QC$, or 1752 Feet, the Distance of the Object. If with this large Telescope you look at an Object OB , just 100 Feet distant from the Lens AB , it will give $CF = 163 \frac{6}{10}$ Inches, and therefore FF will be $19 \frac{1}{2}$ Inches. On the contrary, suppose in viewing an Object OB , you find FF but $\frac{1}{10}$ of an Inch, this will give the Distance of the Object $CQ = 17292$ Feet, or $3 \frac{1}{5}$ Miles nearly.

15. Since then the Difference FF , between the Radius and focal Distance of the Object, is so considerable as 2 Inches in a Tube of 4 Feet, and 18 in one of 12 Feet; it is easy to contrive Methods for shewing the Distance of nigh Objects by the former, and of very distant ones by the latter, by Inspection only. For it is but well adjusting or drawing a spiral Line round the Drawer, or Tube, thro' the *two Inch Space* in the small Telescope, and by Calculation graduate it for every 100 Feet, and the intermediate Inches; and you will no sooner view an Object, but at the same Time see it's Distance upon the Tube.

16. But

16. But in regard of the larger Object-Lens, it might be a better Way to fix it in a Scioptric Ball, in a darkened Chamber; and at the focal Distance of 12 Feet, on a Table or Stand M N, fix up an Instrument consisting of two Planes A B C D, E H K L, and a Screw O F, which is fixed into the Plane A C, and moves the Plane E K over the Space F G, which here represents the Difference between the Radius and focal Distance of the Object, *viz.* 18 Inches. If now on the Plane A C, be described Concentric Circles, or (which would be better) a Spiral from the Center to the Circumference, and graduated by a Calculation made for every 100 Feet, beginning from the Center O, for the Measures next to G, and proceeding towards the Circumference for the Parts towards F, which, because they will run very near together, will have the larger Space to be defined in.

17. These Things being done, it will be easy by the Screw to move the Plane E K backwards and forwards, on the Space G F, till the Image *F* appears most plain and distinct, and then the Index O P (fixed to the Screw) will shew the Distance of the Object on the graduated Spiral. The larger the Plane A C is, the more exactly will the Distance of remote Objects be shewn thereon; if it be a Foot square, it may do pretty well, but not less. Note, the best Way to get the true Place of the Plane E K, will be that in Art. 12.

Having just hinted these things, I shall leave it to the Invention of the Artist to vary Circumstances.

18. In the Business of *Astronomy*, scarce any thing is done but by the Aid of the Telescope; for which Purposes it is applied in various Shapes, and compounded with such a Number of Instruments, that to give an Account of them all, would fill a Volume; I shall therefore content myself with giving of one Instrument, which is of a general Nature, and by which the Construction of others will be easily understood. See Fig. 3.

19. This Instrument consists of a Telescope A B, whose Object-Glass is B, and Eye-Glass A, in whose Focus is a Ring of Cross-Hairs C; this Tube is fixed to the Side of a Quadrant E F G, so that the Axis of the Tube, and Side of the Quadrant E F, are exactly as possible at Right Angles, or perpendicular to each other. The Quadrant is suspended by, and movable about a Center-Pin D, passing thro' it's Center, and fixed into the square Pilaster D H, which supports it very steadily. On this Pilaster at O, is fixed a small Plate N O, called a *Nonius*, from the Name of the Inventor; whose Use I shall next shew.

20. Fig. 4. Let A B be a Portion of the graduated Limb of the Quadrant, and E D the *Nonius* Plate by it; whose Index is D C. The Artifice of this Contrivance consists in this; the whole Length of the *Nonius* is divided

divided into 12 equal Parts, which are made equal to 13 of the equal Divisions on the Limb of the Quadrant A B ; and therefore as $12 : 13 :: 1 : 1 \frac{1}{12}$; that is, one of the equal Parts of the *Nonius* exceeds one of equal Parts (*viz.* a *Degree*) on the Limb, by a 12th Part of the latter, that is, by 5 Minutes.

21. Suppose now you have taken a Sight of an Object, and then observe the Index D C of the *Nonius*, at some Point between the 17th and 18th Degree, as in the Figure ; the *Nonius* will tell you the Minutes above 17 Degrees thus : Look to see where any Division of the *Nonius* stands exactly against a Division on the Limb, and you will perceive in the present Case, that it is at the 6th Division of the *Nonius* ; then, since 1 Division gives 5 Minutes, (Art. 20.) 6 Divisions will give 30 Minutes, which shews that the Index points to $17^{\circ} : 30''$ on the Limb. And hence you see the Reason also, why they are numbered from the Right-Hand to the Left in the *Nonius*.

22. If the Quadrant were sufficiently large, not only the *Minutes* of a Degree, but the *Seconds* of a Minute might be shewn by this excellent Contrivance ; as in that called the *Mural Arch*, in the Royal Observatory at *Greenwich*, which is an Iron Quadrant with a Brass Limb, most exactly graduated, and fixed into the Side of a Free-Stone-Wall, which is in the Plane of the Meridian ; the
Length

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Length of the Radius is 8 Feet. On the Center, and by the Limb of this Quadrant, moves a large Telescope, which carries a *Nonius* upon the Edge of the Limb; which shews the Angles in Degrees, Minutes, and Seconds.

23. By the Telescope applied in such a Manner, divers Astronomical Problems and Positions are determined and defined to a great Degree of Nicety and Exactness. Thus suppose you would know the Altitude of a Star, at any Instance, direct the Telescope to it, and bring the Star on the Intersection of the Cross-Hairs, and the *Nonius* will shew the Degrees and Minutes of it's Altitude: Thus the Altitude of the Sun is to be found for many Purposes, as of *Dialing*, &c. And thus the Altitude of the *North Star*, and consequently the *Latitude of the Place* will be very truly known.

24. Again, the Axis of the Telescope being in the Plane of the Meridian, and the vertical Hair; you may observe the Difference of Time, in which any of the heavenly Bodies pass over the Hair or Meridian, and turning into Degrees you will thereby have the *Difference of the Right Ascension* of those Bodies. Note, 15 Degrees of the Equator, answers to one Hour of Time, and 15 Minutes of a Degree to 1 Minute of Time; and therefore 1 Degree to every 4 Minutes of Time.

25. But

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Fig. 1.



Fig. 2. 23047

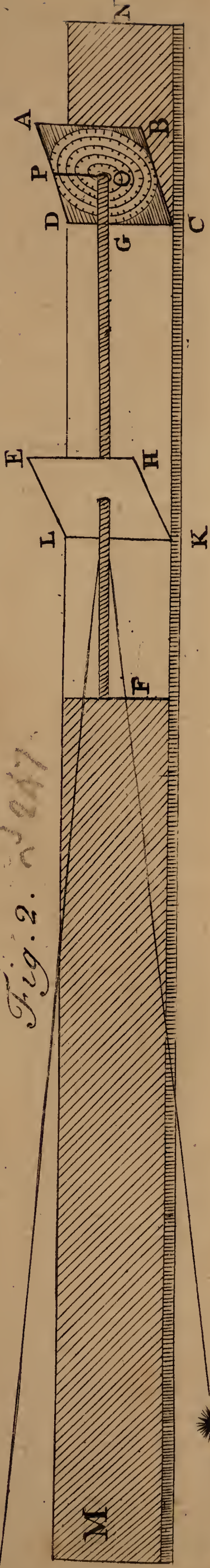


Fig. 3.

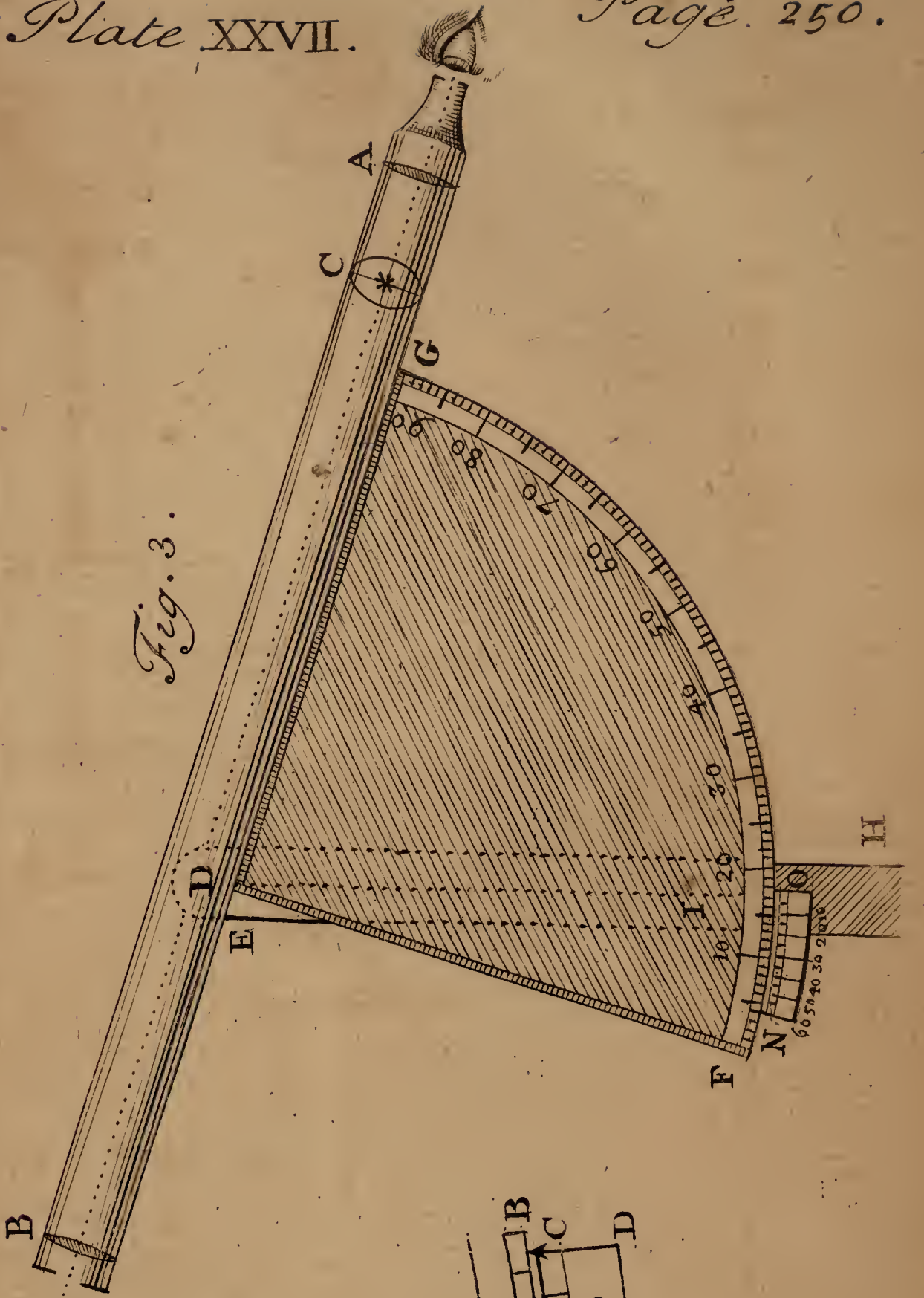


Fig. 4.

A	10				10			
	10				10			
E	60	50	40	30	20	10		
	60	50	40	30	20	10		



25. By this Means also, the Quantity of a Siderial Day is to be determined; for fixing the Telescope precisely in the Plane of the Meridian, direct it to any fixed Star, so that it may come upon the Intersection of the Cross-Hairs, and thus let it remain till the next Night; and then again observe when the Star comes to the *vertical Hair*; the Time between these two Moments is the Length of the Siderial Day; and is easy to be measured by a good Pendulum Clock.

26. When the vertical Hair in the Telescope is nicely in the Plane of the Meridian, it is easy then to find the *apparent Time*, and from thence the *true Time*. For by a good Pendulum Clock observe the Moments of Time, when the vertical Hair touches the two extreme Edges of the Sun's Disk. The middle Moment between these, is the *apparent Time* of *Noon*, and if the Hand be then at XII, the Clock is true for the apparent Time; if not, it's Distance from it, on this Side or that, shew how much the Clock is too slow or too fast; and having the *apparent Time*, the *true Time* is found by a common Equation Table.

27. There are great Numbers of other Uses to be made of a Telescope thus mounted, furnished, and adjusted to a larger Quadrant, especially when a *Micrometer* is fitted in it, which the Reader may see in Treatises of Astronomy, &c. but cannot expect in this Compendium.

C H A P. XIV.

29101
Of the Colours of the Sun's Light;
of the different Refrangibility and
Reflexibility thereof; and the Im-
perfection of Dioptric Telescopes a-
rising from thence.

1. **I**N the Theory we considered a Pencil of the Sun's Rays reflected from Mirrors, or refracted thro' Lenses to one *single Point* in the Axis thereof; but that, strictly speaking, is not the Case in Nature; though nothing to the contrary was understood or taught in Optics, till Sir *Isaac Newton's* Time. That great Man was the first, who discovered that a Beam of the Sun's Light, when reflected or refracted so as to be made converging, did yet not converge or tend all of them to one sole Point or Part of the Axis; but were so severed, and differently inclined by the Medium, that some Rays tended towards one Point, and some towards another.

2. He not only discovered, that the Sun's Light was differently *refrangible* and *reflexible*, but also at the same Time, that it was within certain Limitations, and in a certain Order
and

and Proportion; and that Rays of each several Degree of Refrangibility, were of a different Colour from each other when separated; and accordingly that Bodies were tinged with Colours thereby.

3. So much of this new Doctrine, as is necessary to be known in Optics, or the Construction of Instruments, I shall here relate from his excellent Book on that Subject; and give the Experiments by which he made these Discoveries. The first whereof is as follows. Let $A B C$ represent the transverse Section of a Prism, $G F$ a Beam or Cylinder of the Sun's Light coming thro' the Hole H , in the Window-Shut of a darkened Chamber, and falling on the said Prism. (Fig. 1. Plate XXVIII.)

4. Now this Beam $G F$, will be refracted out of it's natural Course $E T$, in passing thro' the Prism, into some other $D Q$; nor will it now appear in a round cylindric Form $D Q S E$, as it would were it in every Part equally refrangible; but in a broad, dilated, diverging Form $D X Y E$. The Beam thus new modified, being received on a Sheet of Paper $L M$, at a proper Distance, will not now, as before, be white, but will paint a various coloured *Spectrum* or Image $X Y$, of an oblong Form. If the refracting Angle of the Prism $A C B$, be 64 Degrees, and the Distance of the Paper from the Prism about 18 Feet,
the

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the Length of the Image XY will be about 10 Inches, and the Breadth 2 Inches.

5. Now it is evident, that since some Part of the Beam DX , is refracted much farther out of it's natural Course ET , than some other Part of the Beam, as EY , the Rays towards DX have a much greater Disposition to be refracted, than those towards EY ; and that this Disposition does arise from the naturally different Qualities of those Rays is manifest, since the refracting Angle, or Power of the Prism, is the same in Regard of the superior Part of the Beam, as the inferior. Thus the Inequality of Refractions, or the *different Refrangibility* of the Rays of Light, in equal Incidences, is plainly proved by this Experiment.

6. The Rays of Light are variously coloured, as they differ in Refrangibility; thus the most refrangible Rays DV , are *Violet*; the least refrangible ER , are *Red*; the Order of Colours, thro' the whole Image, being as follows, *Violet, Indigo, Blue, Green, Yellow, Orange, Red*, as represented in the Figure. This is the Reason why Rays, falling near the Edges of a Lens, are differently refracted, and consequently paint or tinge the Object with Colours; which makes it necessary to exclude such Rays as much as possible.

7. For that a Lens, as AB , Fig. 2, has the same Property as the Prism acb , in refracting Rays, is evident from this Experiment,

ment, *viz.* Let O be an Object of a *Violet* Colour, the Ray DA proceeding thence to the Lens, and falling on the Surface AaB, at A, will be refracted from the perpendicular CD, to the Point E; but were the Object O red, it's Focus would be found at F, much farther from the Lens than E; and therefore the red Ray AF, will not be so much refracted from the perpendicular by the Lens, as the Violet coloured Ray AE.

8. But the best Experiment of this Sort, is to take a Slip of Pastboard, Card, &c. as ABEF, (Fig. 3.) and paint one half ABCD Red, the other half CF Violet or Indigo, and tying black Threads a-cross it, set it near the Flame of the Candle G; then take a Lens HI, and holding a Sheet of white Paper behind it, move it backwards and forward upon the Edge of a graduated Ruler, till you see the black Threads most distinctly in the Image; and you will find the Focus of the Violet Part *fc*, much nearer than that of the Red *ac*; which plainly shews, that Bodies of different Colours can never be shewn without some Degree of Confusion, by Refraction thro' Lenses.

9. The same Author also found that the Rays of the Sun's Light were *differently reflexible*, or that some Rays of a Beam of Light were reflected farther from the perpendicular to the reflecting Surface than others; and also that those Rays, which are most refrangible,

refrangible, are also most reflexible. All which he confirmed by the following Experiment.

10. In Plate XXIX. Fig. 1. F M is a Beam of Light propagated into the dark Chamber, through the Hole F, of the Window-Shut E G. A B C and B C D are two Prisms tied together in the Form of a Paralleloepid, their Sides B C and C B being contiguous, and A C and B D parallel. H I K is a third Prism by which the said Beam is refracted at O, to the white Paper at V G; for since the two Prisms A C B and B C D, are placed in such a Manner, it is easy to conceive, that the Effects of the first Prism are reversed or destroyed by the second, and so the Beam will pass through both to the third Prism, without any Alteration in the Nature of it's Light.

11. The Beam therefore is refracted at O, into the different coloured Rays O V, O I, O B, O G, &c. Now upon turning the Paralleloepid A C D B about it's Axis, in the Order of the Letters A, C, D, B, when the contiguous Planes B C and C B become sufficiently oblique to the incident Beam F M, the said Beam will begin to be reflected at the Point M; and there will vanish totally out of the refracted Light V G, first of all the most refracted Rays O V, or Violet, (the other remaining as before) then the Rays I O, B O, G O, &c. successively to the least refracted or red. Hence it is very manifest, that the
Plane

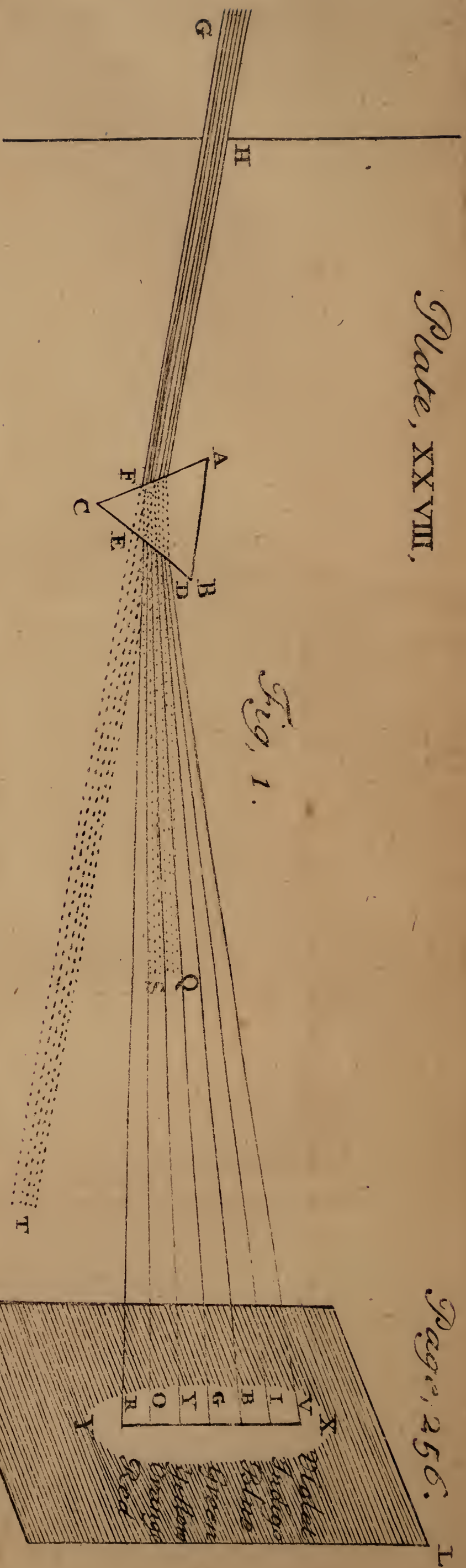


Fig. 1.

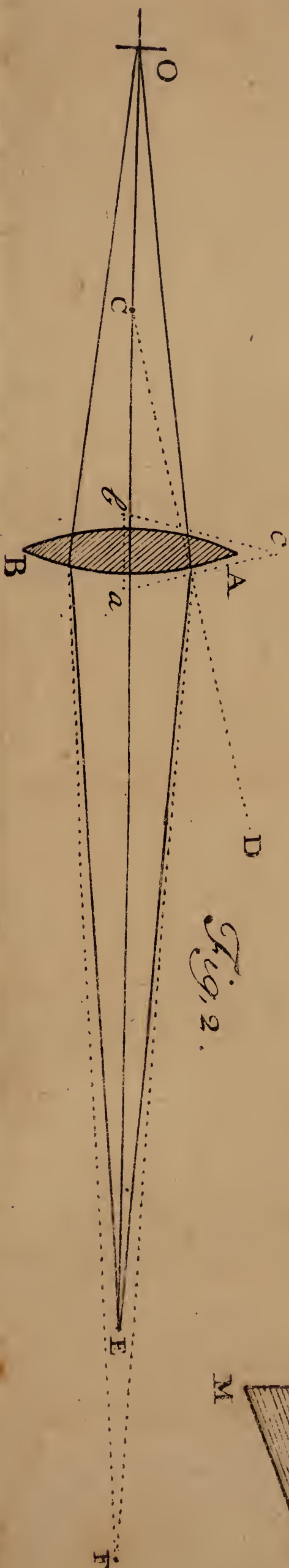


Fig. 2.

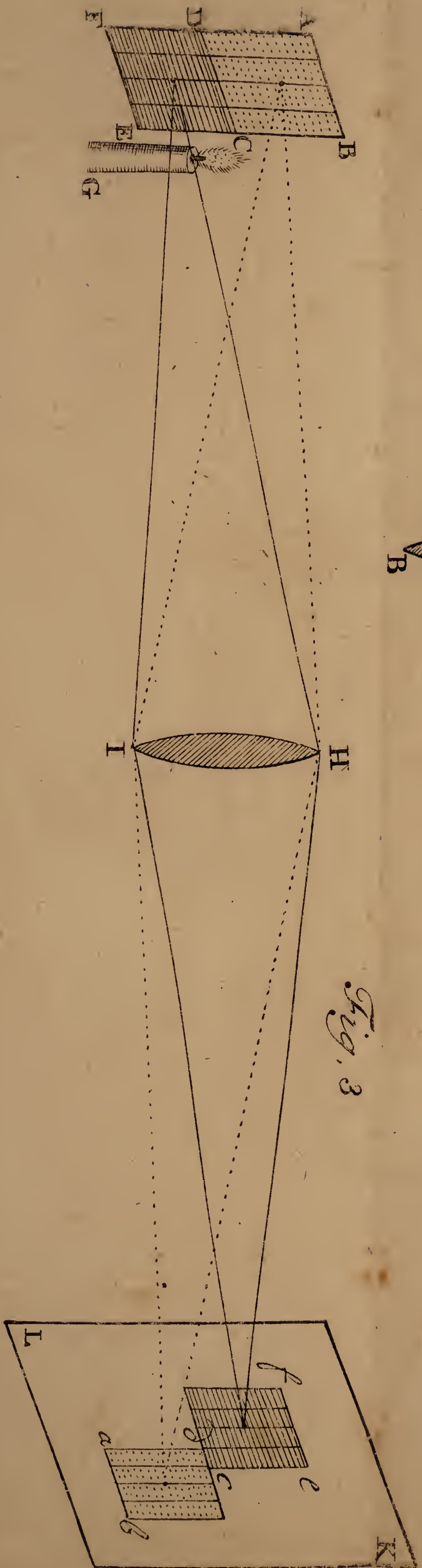
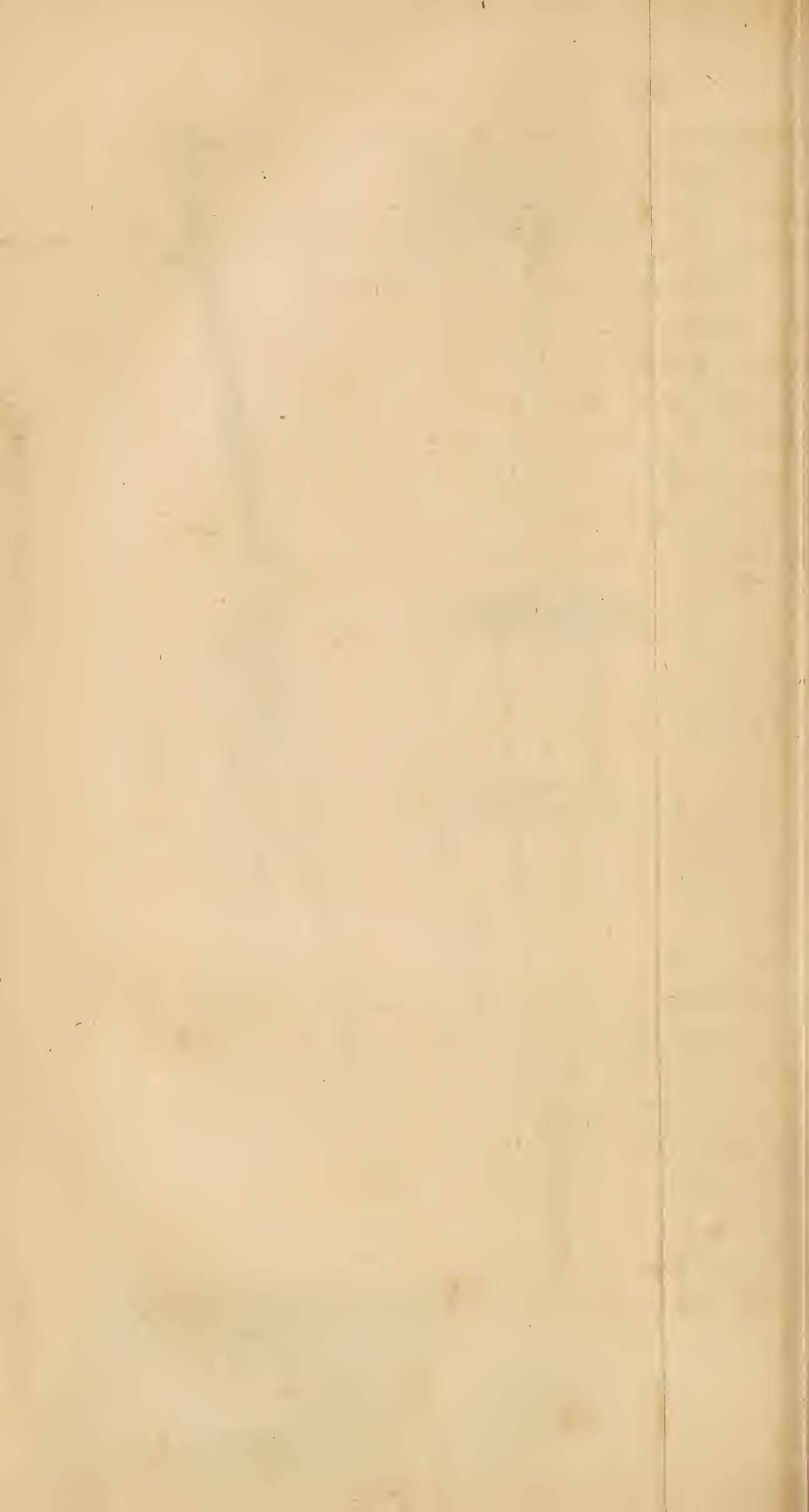


Fig. 3.



Plane B C, did first of all reflect the most refrangible Rays, and the rest in Order to the least refrangible ones, which were reflected last.

12. This he farther illustrated by adding a fourth Prism X Y Z, to receive the reflected Beam M N, and to refract it upon the Paper at *vg*. For then the Light N O, which in the fourth Prism is more refracted, will become fuller and stronger when the Light O V vanishes at V; and so as all the rest vanishes at I, B, G, &c. the coloured Light at *i*, *b*, *g*, &c. becomes increased successively, that is, the Colours at *i*, *b*, *g*, are increased, while those at *v*, *i*, *b*, receive no farther Increase. And as the trajected Beam M O, is always of such a Colour, as ought to result from the Mixture of Colours at V G; so the reflected Beam is always of that Colour as results from the Colours at *vg*.

13. Those Rays which have a peculiar Degree of Refrangibility, and are tinged with a proper, simple Colour, are called *Homogeneous Light*, and all others *Heterogeneous Light*. Let A C be a Ray of heterogeneous Light, falling upon a Medium of Glass O P Q, in the Point C. This Ray will be separated by the Medium into all it's homogeneous Rays, of which C D is that of the greatest Refraction, and C E of the least. (See Fig. 2.) The Sines of these Refractions are G D and E F, and of the incident Ray A B. Sir Isaac Newton
R has

has shewn that AB is to GD , as 50 to 78, and to FE , as 50 to 77. (See his *Optics*, page 73.) Make GI and FH equal to AB ; then will ID and HE express very nearly the greatest and least Degree of Refrangibility, and are to each other, as 28 to 27. So that ID exceeds HE by a $\frac{1}{28}$ of ID ; and therefore we may take this Difference of the greatest and least Refraction to be a $27\frac{1}{2}$ Parts of the mean Refraction.

14. Let AB be a Plano-convex Lens, (Fig. 3.) Q it's Center of Convexity; QG it's Axis produced; DA an heterogeneous Ray parallel to the Axis, which by the Lens at the Point A , is refracted into all it's homogeneous Rays, of which AL is that of the least Refraction, AK that of the greatest, and AO that of a mean Refraction. Suppose in like manner another Ray EB , refracted into BN , BM , and BO . These Rays intersect the Axis in the Points F , O , G . And those of greatest and least Refraction intersect each other in H and I . And therefore HI is the Diameter of the Circle, in which the Rays of every Degree of Refraction will be found.

15. Draw DAP parallel to the Axis QG and HP perpendicular thereto. Then will PH , PO , PI , be the Sines of the *least*, *mean*, and *greatest Refractions*, or of the Angles HAP , OAP ; and IAP , or of the Angles equal to them, AGC , AOC , AFC . But the Sines of the Angles AGC and AOC are

are as the Sides AO and AG , or as CO and CG nearly (by Princip. II). Thus the Sines of the Angles AOC and AFC , are as CF and CO ; consequently PH , PO , and PI , are as CF , CO , and CG , which therefore may represent the least, mean, and greatest Refractions.

16. Now since FG is a 28th Part of CG , (by Art. 13.) OG will be a 56th Part of CG : But since the Triangles HOG and ACG are similar, $OG : CG :: OH : AC :: HI : AB :: 1 : 56$; that is, the Diameter of the Circle HI , into which all the Rays which come parallel from any Point of an Object, are collected by a Plano-convex Object-Lens, is the 56th Part of the Diameter of the Aperture of the said Object-Lens.

17. What this Diameter HI is, in the Focus of reflected Rays, may be thus shewn. Let AB be a Concave Speculum, whose Center is Q ; DA a Ray parallel to the Axis QY , and reflected most of all towards F , suppose O the Focus of Rays of a mean Reflection, and draw OI perpendicular to the Axis, which will be the Semidiameter of the circular Space into which Rays of every Sort will be reflected from the Point A .

18. Since F will always fall between O and C , the Point A may be taken at such a Distance from the Vertex C , that FA may be equal to FQ : In which Case, on F describe the Semicircle QAY , cutting the Axis in Y ;

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then shall OF be equal to $\frac{1}{2} CY$. For $OF = QF - QO$, and so $2 OF = 2 QF - 2 QO = QY - OC = CY$; and therefore $OF = \frac{1}{2} CY$. But CY is the Excess of the Secant QY , above the Radius QA . Consequently when the Angle AQC is given, CY is known, and therefore it is half OF ; and then since $OF : FC :: OI : AC$, the Ratio of IO to AC , the Semi-aperture of the Glass will be known.

19. For Example. Suppose the focal Length OC of the Speculum AB be 12 Inches, and the Angle $AQC = 2^{\circ} 3''$; whence the Arch AC will be very near $\frac{1}{2}$ an Inch. Then if the Radius QA or $QC = 100000$, the Secant QY will be 109095; therefore $CY = \frac{95}{100000}$, the half whereof $\frac{475}{1000000} = FO$. Since then FO is the $\frac{475}{1000000}$ Part of QC , it will be the $\frac{95}{1000000}$ Part of OC ; and therefore IO will be the same Part of AC , which is near 20 times less than a $\frac{1}{56}$ Part, as in the Convex Lens.

20. If each individual Point of an Object were to affect but single or individual Points on the *Retina*, the Picture there made would be absolutely *perfect*, and the Idea conveyed to the Mind, or *Vision*, would also be perfectly *distinct*; but since it has been shewn, that Rays coming from any single Point in an Object, and collected by a Lens, Mirrour, or the Eye, do not represent that Point in a Point, but circular Space of the Picture; and
since

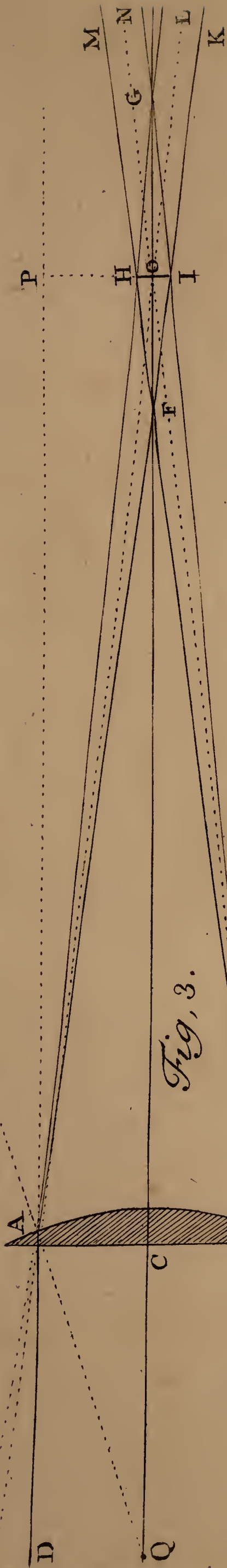


Fig. 3.

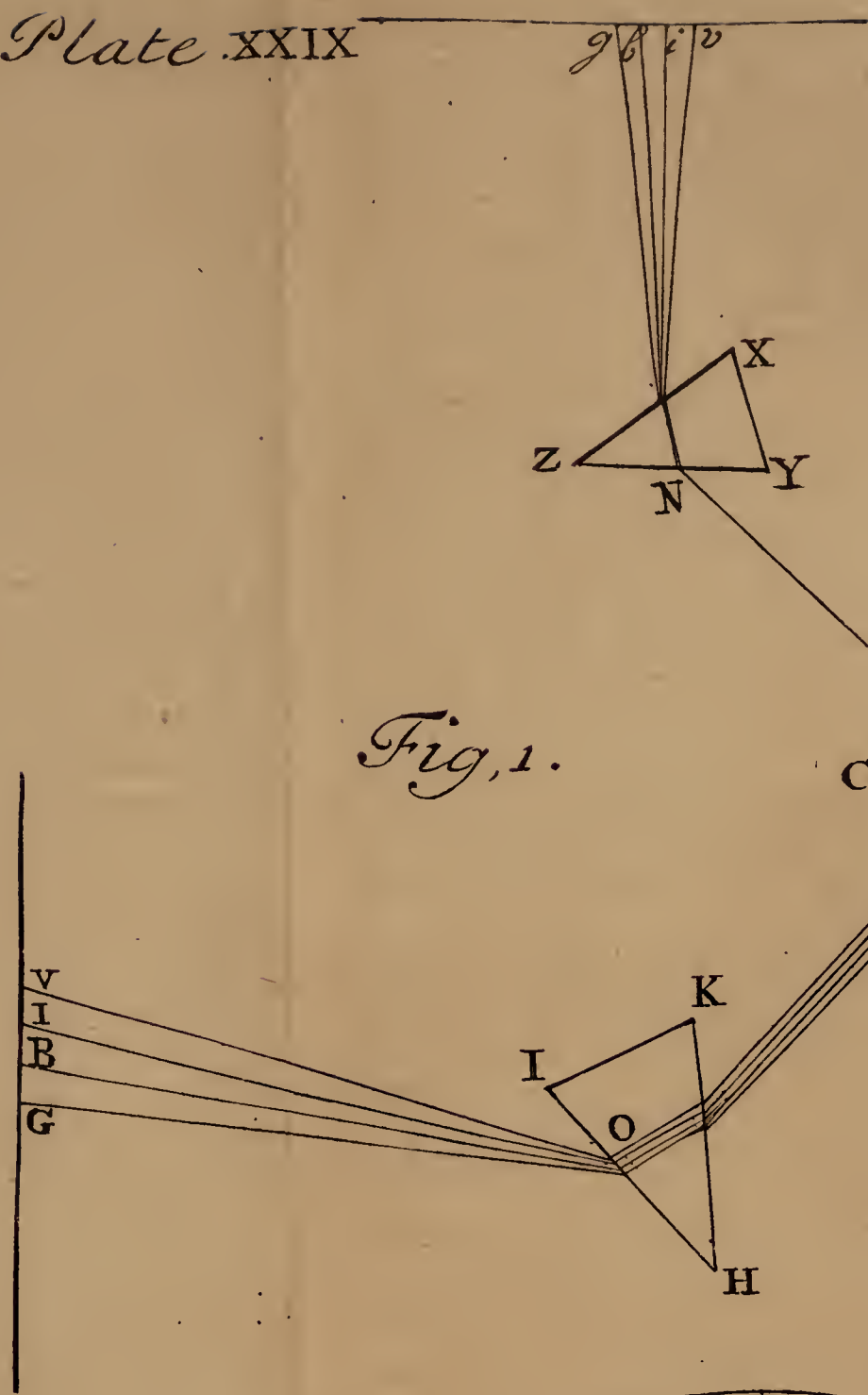


Fig. 1.

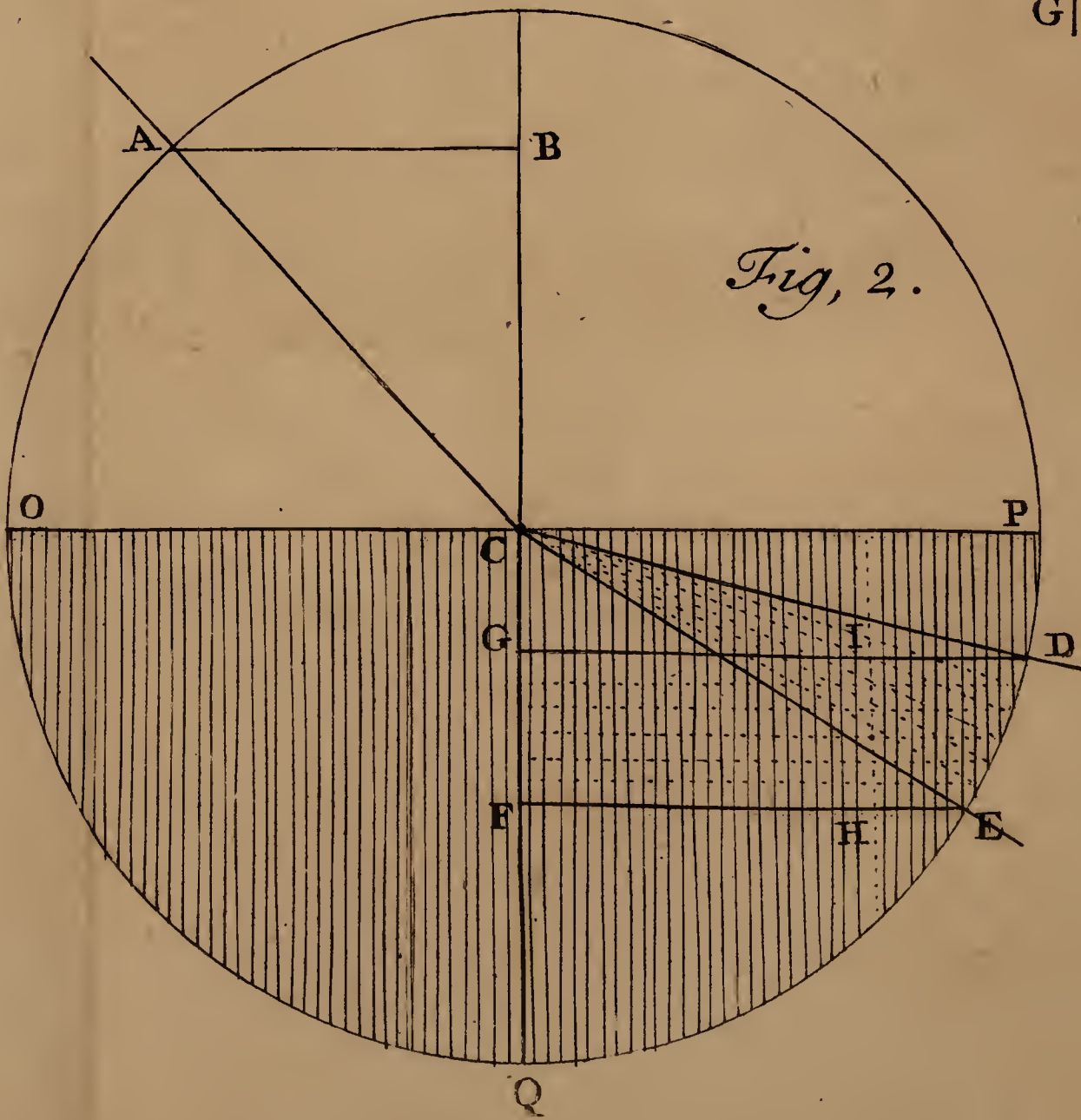


Fig. 2.

since every Point of the Picture may be the Center of such a circular Space, and this circular Space will admit of a Mixture of as many others, as there are Points within the Distance of a Semidiameter ; and, lastly, since the very central Point of this Space will be affected or covered with all those Circles, whose Centers fall within it's own Circumference: I say, from all these Considerations it is very manifest, the Picture must be extremely confused and indistinct, and consequently the Vision will be so too ; and that in Proportion to the Area of the Circle, Confusion is painted in the Picture on the *Retina* of the Eye.

21. Now since the Areas of these Circles, formed by reflected Rays, are incomparably less than those formed by refracted Rays, it is evident, the Vision by reflecting Instruments must far exceed that by refracting ones, and consequently that reflecting Telescopes are vastly preferable to refracting ones in their Effects, as well as Conveniency of Form ; to the Theory of which I now proceed.

C H A P. XV.

Of the Cata-dioptric or Reflecting Telescope ; it's Theory, Construction, and Use.

1. **T**H E imperfect Vision, by Dioptric Telescopes, of a great magnifying Power, unless very long, which then were very cumbersome and unmanageable, gave the Opticians Occasion to meditate and contrive some Method of shortening, and making this excellent Instrument of a more commodious Form, and of more general Use.

2. This, from the Nature of the Science, they were soon convinced, was to be effected by *Reflection* ; and therefore several Persons, in several Parts, began to contrive and make *Reflecting Telescopes*, some one Way and some another. The first Hint and Figure of any thing in this Kind was by Dr *James Gregory*, a *Scotchman*, in his Book called *Optica promota*, pag. 93, 94.

3. The Form and Manner of the *Gregorian Telescope* was as follows. In Fig. 2. let A B E F be a Tube open at the End A F, towards the Object ; at the other End he placed

placed a parabolic Concave Speculum B E, with an Hole C D perforated in it's Vertex; a little beyond the Focus *e* of this, he placed another small Concave G, (on the Foot G H) of an Elliptic Form; at the End of the great Tube B E, he screwed in a small Tube C D K I, containing an ocular Lens of a conoid Form on one Side, and plain on the other.

4. Now suppose *a, b*, two parallel Rays, falling on the Speculum B E, in *c, d*, from thence they are reflected to, and collected in, it's Focus *e*, where the Image is formed and inverted; this Point *e* is also one Focus of the Elliptic Speculum G, and therefore the Rays coming thence, and falling on G, are by it reflected to the other Focus thereof, in the little Tube at *f*. Here again the Image is reformed, very large, and erect, which being also in the Focus of the Plano-conoid Lens I K, is seen by parallel Rays very clear and distinct.

5. Such was the Form and Theory of the original *Gregorian* Tube, but the Doctor, being deficient in Mechanics, never brought it to Perfection, but proposed it for others to execute. He had also spherical Speculums of Metal, but could not use them for want of a good Polish. And thus nothing was done in the Telescope of this new Invention, till about the Year 1666, when Sir *Isaac Newton* thought proper to alter a little the Construc-

tion of the *Gregorian* Tube; and instead of placing the Eye-Glass at the End, he put it into the Side of the Tube, as in Fig. 3.

6. Where ABEF is the Tube, BE the Object spherical Concave, which reflect the parallel Rays *a*, *b*, to a small plain Speculum G, placed a little within the Focus of BE, and so as to throw the converging Rays directly to the Side of the Tube, upon a small Eye-Glass I, placed in a Hole therein; and the Situation of the Speculum G was such, that the Focus of Parallel Rays from BE, was made to fall exactly on the Focus of the Eye-Glass in *e*; and consequently the Eye must then have distinct Vision by Parallel Rays. This was the Structure of the *Newtonian* Tube.

7. A few Years after one Monsieur *Cassegrain* published a Description of a Reflecting Telescope, as his own Invention; but since it was entirely of the Form of that in Fig. 4, where the only Difference between it and the *Gregorian* Telescope is, that the small Speculum G is in this a *Convex*, and in Dr *Gregory*'s a *Concave*. And therefore this of Monsieur *Cassegrain*'s seems to be only that of Dr *Gregory*'s disguised.

8. About this Time also Dr *Hook* contrived a Telescope of this Kind; the Form thereof was that of Fig. 5, wherein you see it differs nothing from the *Gregorian* Telescope, but the placing of the Eye-Glass I, in the Hole
of

of the great Concave B E. But whether Dr *Hook* insisted on this as his own Invention I cannot say, though he seems to do so in a Letter he wrote about it to some Lord.

9. To these several Forms, I shall add one that I contrived for my own Use, which is somewhat of the *Newtonian* Structure, but of a perpendicular Position, as represented in Fig. 6. A B E F is the Tube, in which there is an Opening or Aperture O P, in the upper Part; against this Hole, within the Tube, is placed a large plane Speculum G H, at half a Right Angle, with the Axis or Sides of the Tubes, with an Hole C D perforated thro' the middle thereof. The Parallel Rays *ab*, falling on the inclined Plane G H, are reflected perpendicularly and parallel on the great Concave B E, in the Bottom of the Tube; from thence they are reflected converging to a Focus *e*, through the Hole of the Plane C D, which being also the Focus of the Eye-Glass I K, the Eye will perceive the Object very much magnified and distinct.

10. Now though the Forms and Structure of these several Tubes are different, yet their Effects all flow from one Principle, *viz.* *The superior Perfection of reflex Vision.* For they all perform their Effects either by one Reflection, as Fig. 3 and 6; or else by two, as Fig. 2, 4, and 5. And an Image formed by reflected Rays is so very perfect and distinct, that it will bear viewing with an Eye-Glass
of

of a much shorter focal Distance, than when it is painted by refracted Rays; for in that Case, if it be magnified too much, the Confusion of the Picture will be rendered sensible, and the distinct Vision of the Object entirely impeded.

11. I shall first consider the Effects of the *Gregorian Telescope* by two Reflections. In Plate XXXI. Fig. 1. A B E F is the Tube containing the great Concave B E, and little one G; let a, b , be two Parallel Rays coming from a vastly distant Object O B, and falling on the Concave in the Points c, d ; from hence they will be reflected to the Focus e , where they form an Image I M inverted. Let f be the Focus of the small Concave G, and since the Image I M is a little farther distant from the Concave G than it's Center, the Rays come from it with such a Degree of Divergency, as to be reflected by the little Concave to another Focus k , and there crossing again form another Image I M, which will be now in an erect Position; and being supposed in the Focus also of the Eye-Glass K L, the Eye will see a Part of it very distinct, and greatly magnified. This has been in every Part so particularly demonstrated, in the Theory and Practice of *Catoptrics*, that I need not here again repeat it.

12. Now as to the *Power of magnifying*, let bQ and gQ be two Rays coming from the Extremities of a distant Object, and meeting
ing

Fig. 1.

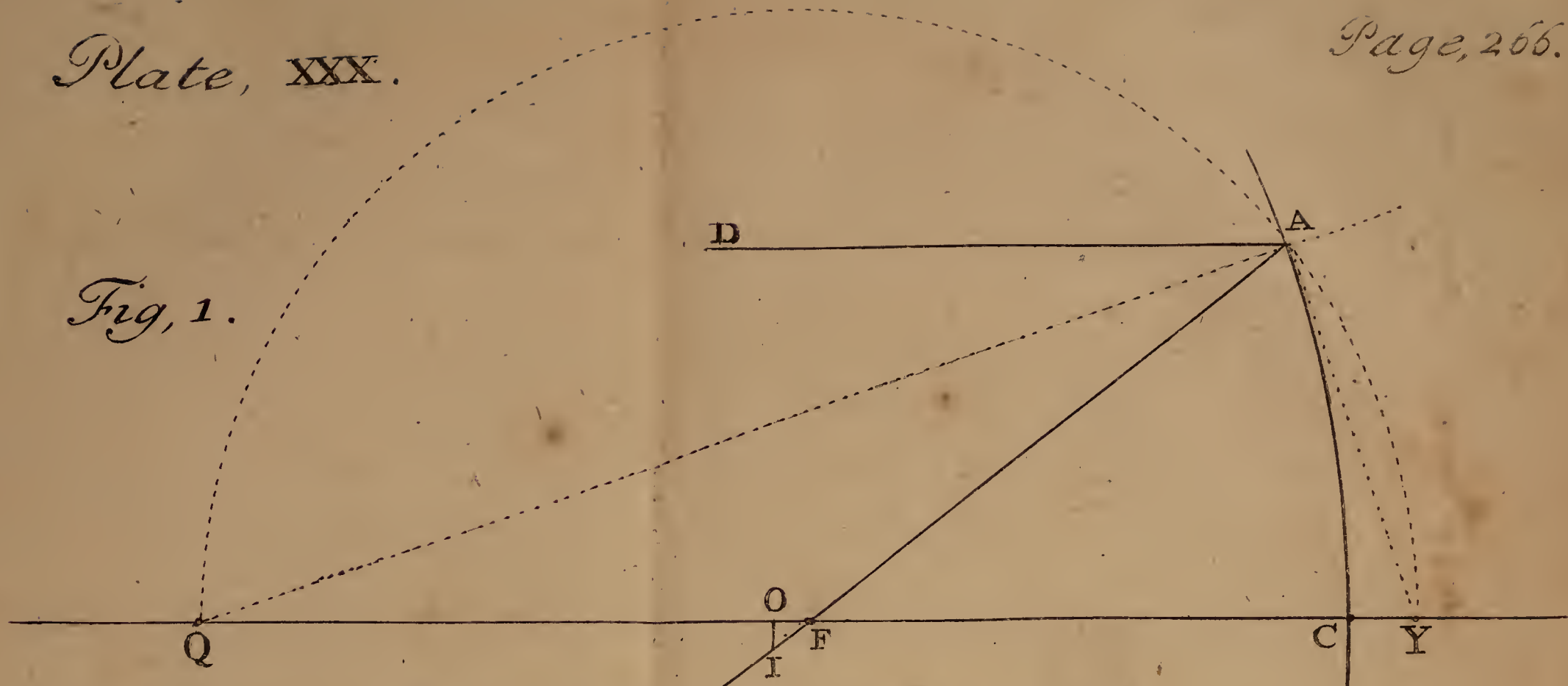


Fig. 2.

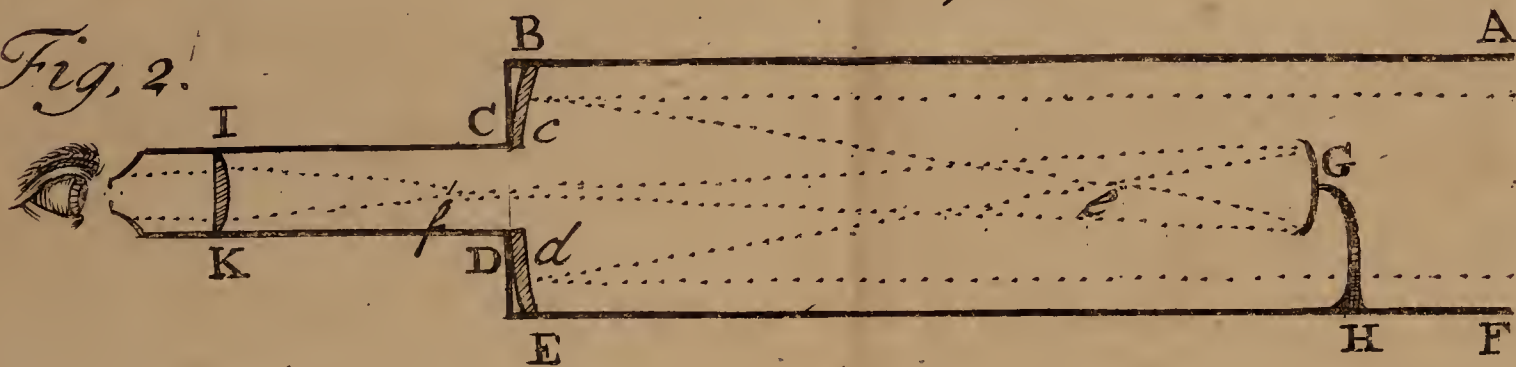


Fig. 3.

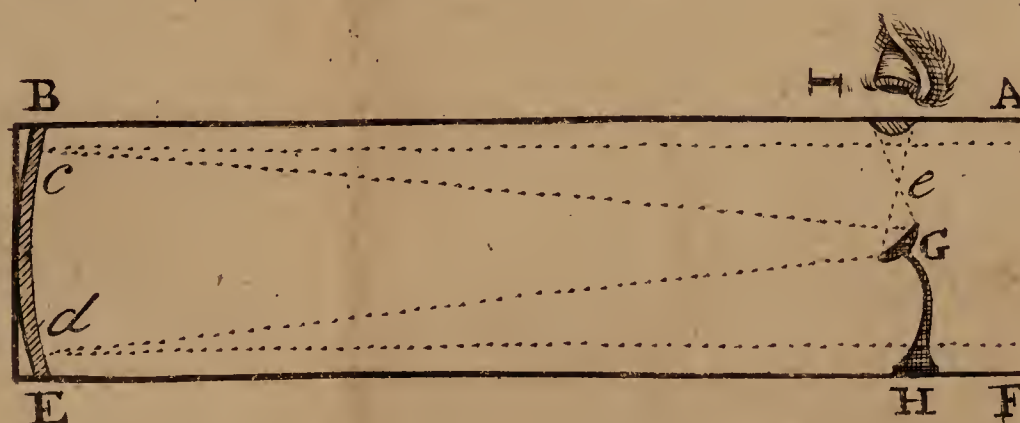


Fig. 4.

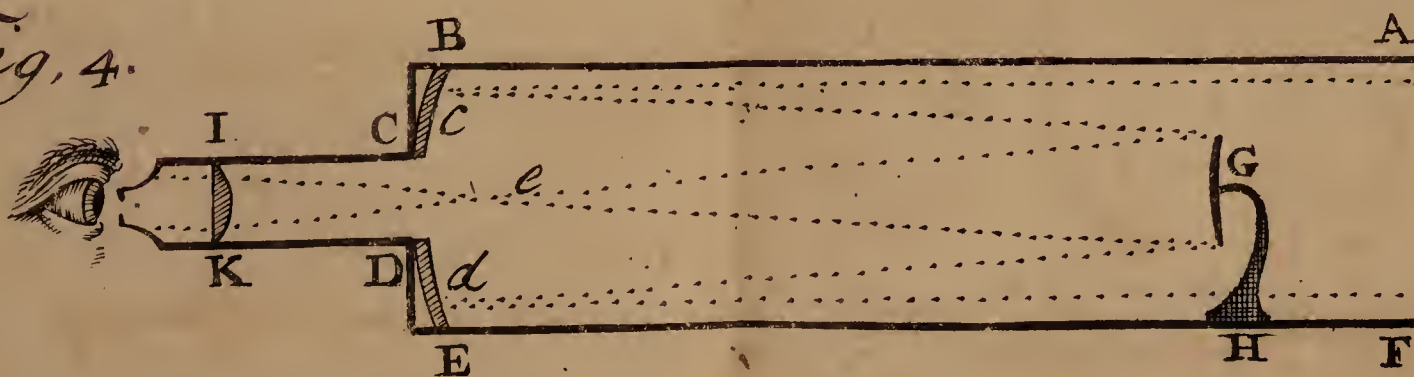


Fig. 5.

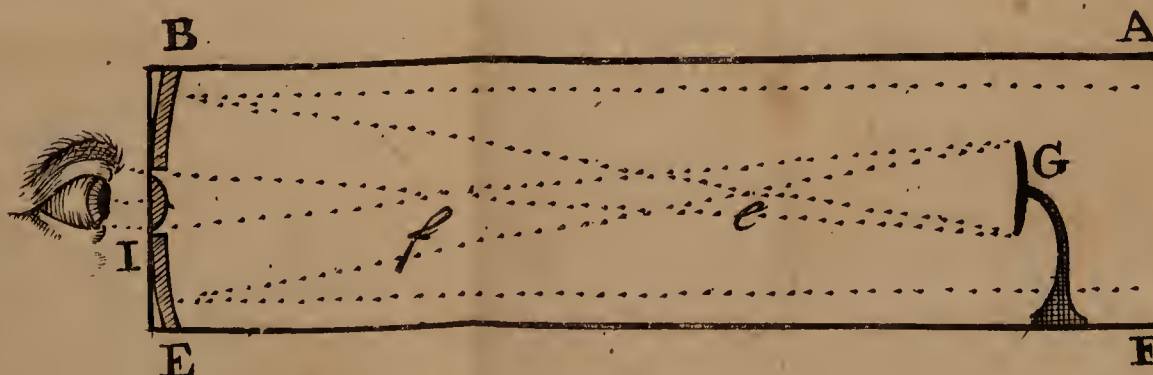
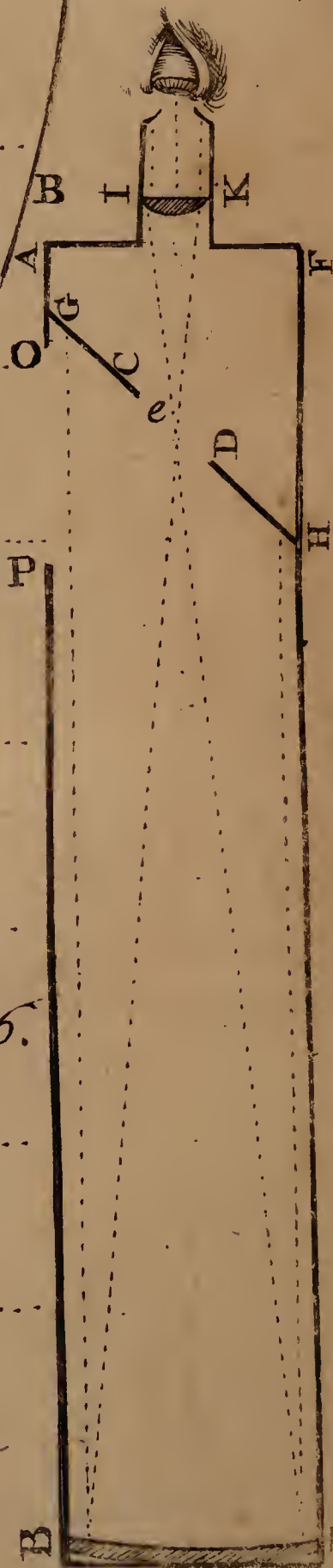


Fig. 6.



ing in the Center Q of the Speculum BE ; these Rays will terminate the Image IM , because the Object and it's Image are both seen under the same Angle gQb , to an Eye at the Center or Vertex of a Speculum, by Art. 5. Chap. IV, of *Catoptrics*. Now this first Image IM may be considered as an Object, with Respect to the small Speculum G , whose Image IM is formed in the Focus k , and terminated by the Rays IG and MG , drawn through the Extremities of the Object IM . This secondary Image IM , is that which is viewed by the Eye-Glass KL , under the Angle INM , supposing it could be all seen at once. And therefore the Object is to the Image in this Telescope, as the Angle gQb , to the Angle INM .

13. But these Angles are in the compound Ratio of Qe to eG , and of kG to kN , for the Angle IQM is to IGM , as eG to eQ ; and IGM ($= IGM$) is INM , as kN to

$$kG; \text{ and therefore } \frac{IGM}{IQM} \times \frac{INM}{IGM} \left(= \frac{INM}{IQM} \right) = \frac{Qe}{eG} \times \frac{kG}{kN}. \text{ That is, the Angle}$$

INM , is to the Angle IQM or gQb , as $Qe \times kG$ is to $eG \times kN$. All which is demonstrated in Chap. II, of this Part.

14. Hence as Qe , the focal Distance of the Object Speculum BE , is increased, or kN ,
the

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the focal Distance of the Eye-Glass is diminished, the magnifying Power of the Telescope will be increased. And it is evident, that the same things are at the same Time shewn for *Cassegrain's* Telescope, Fig. 2. where the small Speculum *G* is a Convex, and the Image *IM* only *virtual*, or behind it, and the Image *IM* *inverted*, other things being the same.

15. If the focal Distance Qe , of the Object-Speculum *BE*, and Gf of the small one *G* be given, and you have determined the Point *k*, where the second Image is to be formed, the Distance Ge , of the first Image from the Speculum *G*, may then be found, and a Theorem raised for calculating the magnifying Power of the Instrument. For having the focal Distance Qe , and the Distance Qk , there is known ke ; then put $ke = X$, $Ge = d$, and $Gf = \frac{1}{2}r$; and we shall have

$$X + d = Gk = \frac{dr}{2d - r}; \text{ by Theor. 14.}$$

Chap. III, of *Catoptrics*.

16. Then — —	1	$X + d = \frac{dr}{2d - r}$
Therefore — — —	2	$\left\{ \begin{array}{l} 2dX + 2dd - Xr \\ - dr = dr. \end{array} \right.$
Add dr , — — —	3	$\left\{ \begin{array}{l} 2dX + 2dd - Xr \\ = 2dr. \end{array} \right.$
And by Transpos.—	4	$\left\{ \begin{array}{l} 2dX + 2dd - 2dr \\ = Xr. \end{array} \right.$

Divide

Divide by 2 — —	5	$\left\{ \begin{array}{l} dX + dd - dr \\ = \frac{1}{2} Xr. \end{array} \right.$
Put — — — —	6	$X - r = 2Z.$
Then — — — —	7	$dd + 2Zd = \frac{1}{2} Xr.$
Compare the Squ.—	8	$\left\{ \begin{array}{l} dd + 2Zd + ZZ \\ = \frac{1}{2} Xr + ZZ. \end{array} \right.$
Extract the Root —	9	$d + Z = \sqrt{\frac{1}{2} Xr + ZZ}.$
Therefore — — —	10	$\left\{ \begin{array}{l} d = \sqrt{\frac{1}{2} Xr + ZZ} \\ - Z, \text{ Theorem.} \end{array} \right.$

17. Having thus found d = Distance of the first Image from the small Mirrour G; the magnifying Power of the Tube may easily be computed as follows. Let the focal Distance of the great Mirrour BE be 6 Inches, that of the lesser 1 Inch; and let it be proposed to have the second Image, or Point k , at 1 Inch, before the Mirrour BE; then will $X = ke = 5$, $r = 2$, and $X - r = 2Z = 3$, and so $Z = 1.5$. Whence by the Theorem, (Art. 16.) we shall find $d = 1.192$. See the Operation.

$$\frac{1}{2} Xr = 5$$

$$ZZ = 2.25$$

$$\frac{\frac{1}{2} Xr + ZZ}{4} = \frac{7.25}{4} \quad \left(\begin{array}{l} 2.692 = \sqrt{\frac{1}{2} Xr + ZZ} \\ 1.5 = Z. \end{array} \right.$$

$$46 \begin{array}{r} 325 \\ 276 \\ \hline \end{array} \quad 1.192 = d = Ge.$$

$$529 \begin{array}{r} .4900 \\ 4761 \\ \hline .13900 \end{array}$$

18. Having

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18. Having $G e = 1.192$, and $Q e = 6$, we have $\frac{Q e}{G e} = \frac{6}{1.192}$, which is the first Part

of the magnifying Power. Then if the focal Distance $N k$ of the Eye-Glass be $1 \frac{1}{4}$ of

an Inch, we have $\frac{G k}{N k} = \frac{6.192}{1.25}$. And there-

fore $\frac{Q e}{G e} \times \frac{G k}{N k} = \frac{6}{1.192} \times \frac{6.192}{1.25} = \frac{37.152}{1.49}$

$= 25$ very nearly. Therefore an Eye-Glass $K L$, whose focal Distance is $N k = 1 \frac{1}{4}$ of an Inch, will magnify 25 times, and if $N k$ be but $\frac{1}{2}$ Inch, it will magnify the Diameters of Objects above 50 times.

19. There is one Case when the Power of magnifying will be, as the *Square of the focal Distance of the great Mirrour, to the Rectangle under the focal Distances of the smaller Concave, and the Eye-Lens.* And that is, when $Q k = f e$; for then the said Power will be as

$\frac{Q e \times Q e}{G f \times N k}$. Thus suppose $Q e = 6$ Inches, and

$G f = N k = 1$ Inch, such a Telescope will

magnify 36 times; for $\frac{Q e \times Q e}{G f \times N k} = \frac{6 \times 6}{1 \times 1}$

$= 36$.

20. If $Q e$, the focal Distance of the greater Concave $B E$, be 9 Inches, and that of the lesser

leffer G be $1\frac{1}{2}$, and of the Eye-Glass 1 Inch;

then if $Qk = fe$, we shall have $\frac{9 \times 9}{1.5 \times 1} = 54$,

the Number of Times such a Telescope will magnify. If the focal Length of the Eye-Glass be but 1 Inch, the Power will be

$\frac{9 \times 9}{1 \times 1} = 81$; for so many times will it mag-

nify the Diameters of Objects.

21. If $Qe = 12$ Inches, $Gf = 2$, and $Nk = 1.5$; then if $Qk = fe$, we have

$\frac{12 \times 12}{2 \times 1.5} = \frac{144}{3} = 48$, the Number of Times

the Diameters of Objects will be magnified; but if $Gf = 1.5$, and $Nk = 1$ Inch, then

the Power will be $\frac{12 \times 12}{1.5 \times 1} = 96$ times. If

the Image IM be required at any other Distance before or behind the great Concave Q , the Power of magnifying may be found as in Art. 16 and 17.

22. If the Telescope be larger, *viz.* $1\frac{1}{2}$ Foot, or 2, 3, 4, 5, 6, &c. Feet long, there are two Eye-Glasses applied, as in Fig. 3. Where the Image IM is projected to some Distance Qk , behind the great Mirrour BE , and by the Interposition of the first Eye-Glass WX , it is contracted into RS , by Refraction of the Rays WM , XI into WN , XO , at
the

the second Eye-Glass $Y'Z$, which again refracts them to the Point P , where the Eye views the Object in it's last Image, under the Angle NPO . The Effects of a Combination of two Eye-Glasses have been already explained in Chap. VII, of the Microscope, where the Power of magnifying, the Amplification and Quantity of the optic Angle NPO , are considered in Comparison with the same in a single Eye-Glass.

23. For if the Nature and Construction of this Telescope be well observed, we shall find it of a *Telescope and Microscope conjoined in one, or compounded in one Machine*. For, (1.) We are to consider the Object Mirrour BE , forming the Image IM of distant Objects, in it's Focus e , and the small Speculum G , placed at it's focal Distance from the Image, will give distinct Vision thereof by Parallel Rays to any Eye at Q , which is the Function of an Eye-Glass, and magnifies the Object in Proportion of their focal Lengths Qe and eG ; which is all that is performed in a common Telescope. But, (2.) If the Image IM be now looked upon as a Microscopic Object, it is removed a little from the Focus f of the small Speculum, so far as to cause a large Image thereof IM , to be formed near the Eye, which is the Effect of an Object-Glass in a Microscope; this Image IM is distinctly viewed in the Focus of an Eye-Glass or two, which

which is all that is performed in a common Microscope.

24. Therefore since the Reflecting Telescope is nothing but a common Telescope and Microscope combined together, and it's Effect compounded of the Effects of those two Instruments; it is evident the Nature or Theory of this admirable Instrument will be easy to understand, when those of the simple Telescope and Microscope, before explained, are well considered, and understood.

25. I shall conclude this Account of the Reflecting Telescope with an Observation or two, necessary for those that shall undertake to make them; and, first, with Regard to the Hole CD , in the great Speculum BE , it's Diameter should be equal to that of the smaller Speculum G ; for if it be less, no more parallel Rays (which make the principal Pencil $c y x d$) can be reflected, than if it were equal to xy , and so it can answer no Purpose, but may do Harm, in contracting the visible Area within too narrow Limits.

26. Again, it must not be bigger than the Mirrour G , because some parallel Rays will then be lost, and those of most Consequence too, as being nearest the Center; if any Part of the great Mirrour can be spared, it must be on the Extremity. Now the Breadth of the little Mirrour xy , is easy to be determined by the focal Distances Qe and eG , and Aperture of the great Mirrour cd ; for

$$\frac{S}{Qe}$$

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$Qe : eG :: dc \propto y$. Suppose $QE = 6$, $eG = 1.2$, and $cd = 1.5$; then $\frac{1.2 \times 1.5}{6} = 0.3$

of an Inch, and therefore $\propto y$ may be a little more than $\frac{3}{10}$ of an Inch. If $Qe = 9$, eG

$= 1.5$, and $cd = 2$; then $\frac{1.5 \times 2}{9} = 0.34$

$= \propto y$. If $Qe = 12$, $eG = 2$, and $cd = 2$;

then $\frac{2 \times 2}{12} = 0.14 = \propto y$, which may be

then made about $\frac{1}{4}$ an Inch. For the Diameter of the Mirrour should be a little bigger than the Pencil of Rays, which it receives.

27. The next thing to be regarded is the small Hole at P, in the End of the Eye-Piece, which must be nicely adjusted to the Size of the Cylinder of Rays, emerging from the principal Pencil in the nearest Lens YZ. For if it be bigger, it will permit the foreign Light of the Sky, &c. to enter the Eye, which cannot be suffered; for the Eye must receive nothing but what comes from the Surface of the small Mirrour G. If the Hole be smaller than the Diameter of the Cylinder Prs, then some of the necessary Light of the principal Pencil will be excluded, and the Object rendered more obscure.

28. If the Eye-Glass YZ were alone, and the Focus thereof, the Diameter of the Hole would

would be found by this Analogy; As $G l : l r :: x y : r s$ = the Diameter of the Cylinder or Hole. Suppose $G l = 8$, $l r = 1$, and

$x y = 0.3$; then $\frac{1 \times 0.3}{8} = 0.04 = r s$, near-

ly; therefore the Hole must not exceed $\frac{1}{25}$ of an Inch in Diameter. Again, suppose $G l =$

16, $l r = 1.4$, and $x y = 0.5$; then $\frac{1.4 \times 0.5}{16}$

$= 0.044 = r s$; and therefore in a Telescope of this Length, the Hole at P must not exceed $\frac{1}{250}$ or $\frac{1}{25}$ of an Inch in Diameter. And this will be the Size of the said Hole, if the Image R S be formed in the middle between two Eye-Glasses W X, and Y Z. For since $l r = l n$, $n o$ will be equal to $r s$; and it is $G l : l m :: x y : n o = r s$, as before.

29. It is usual to place a Plate T V, in the Focus l of the Eye-Glass Y Z, with a Hole in the middle of such a Diameter R O, as will circumscribe the Image, and limit the Angle of Vision, in such a Manner, as to exhibit only so much of the visible Area, as appears distinct, and exclude the confused, coloured, and contorted Part thereof; and all the collateral and superfluous Rays, which enter by the Sides of the little Mirrour, or are reflected from it's Margin. The Diameter of this Hole R S is various, according to the various Dimensions of the Telescope, and

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Apertures of the Eye-Glasses W X and N O. And had better be adjusted by Trials, than Rules which might be given.

30. This famous Instrument owes it's Original to the Invention of Dr Gregory, (as before said) but it's Perfection to the Sagacity, Skill, and Industry, of *John Hadley, Esq; Vice-President of the Royal Society*, about 18 or 20 Years ago. Since which Time it has been attempted by divers Persons, and executed so well, that when once they can be made as common as they are useful, and be sold cheap, the long Dioptric Telescopes will be heard of no more. And that this will be the Case e'er long, I can almost venture to predict, being well assured, from what I have done in it myself, that the Construction is not so difficult, and the Workmanship and Materials so expensive, as is generally understood.



Fig. 1.

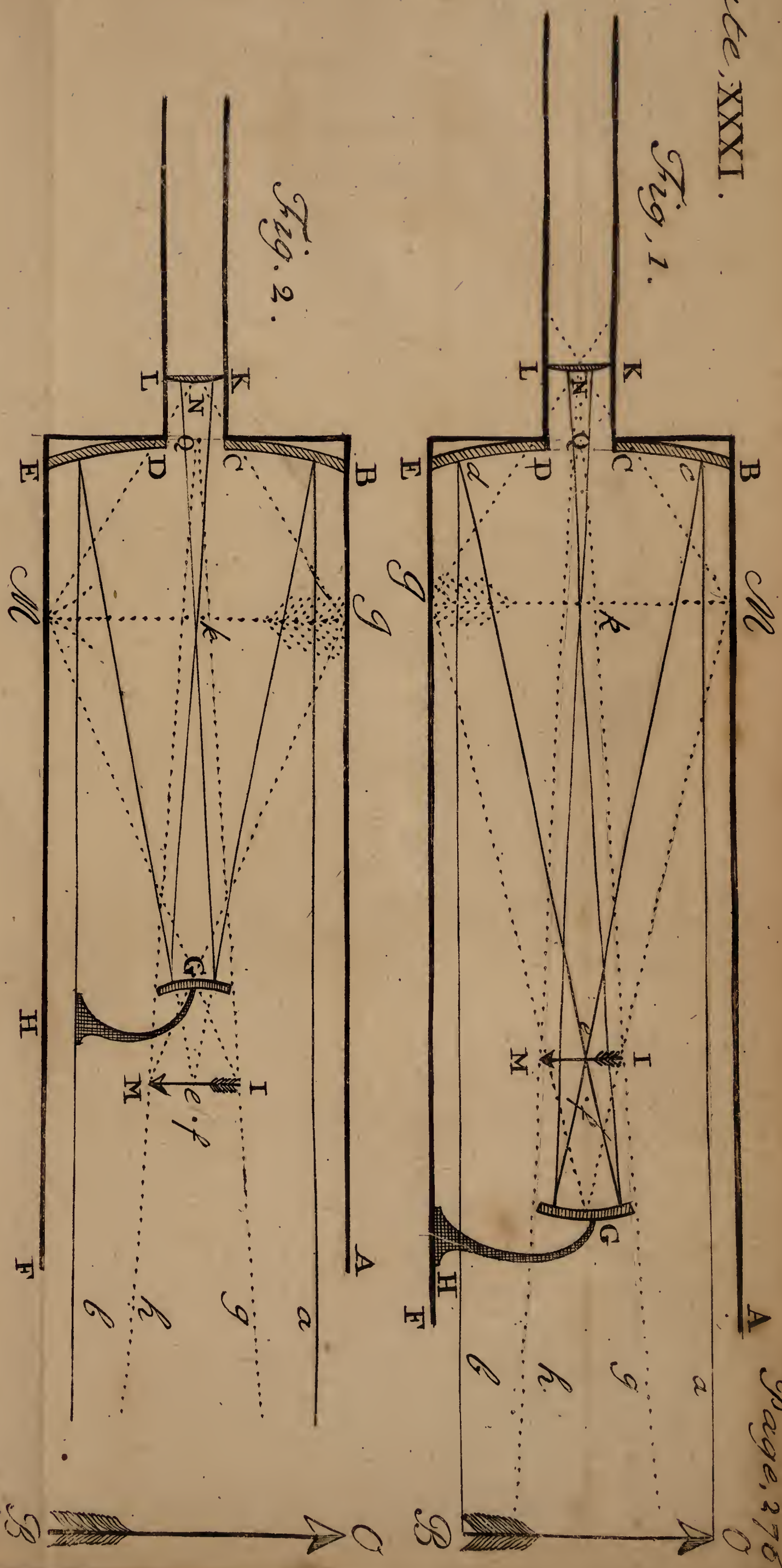


Fig. 2.

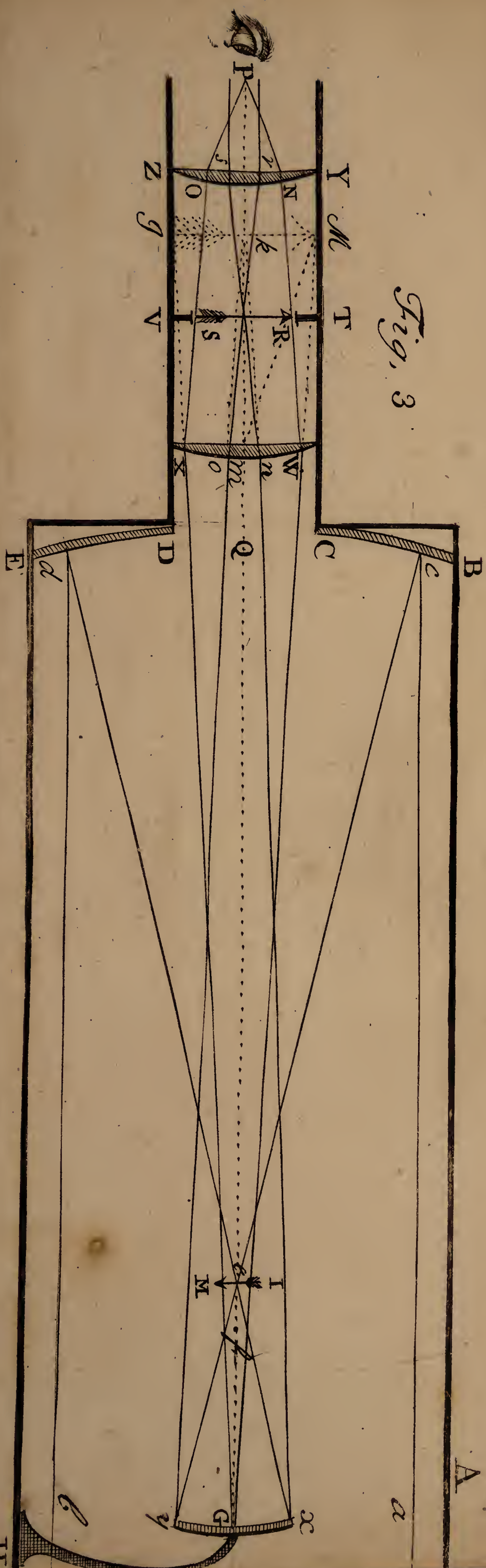
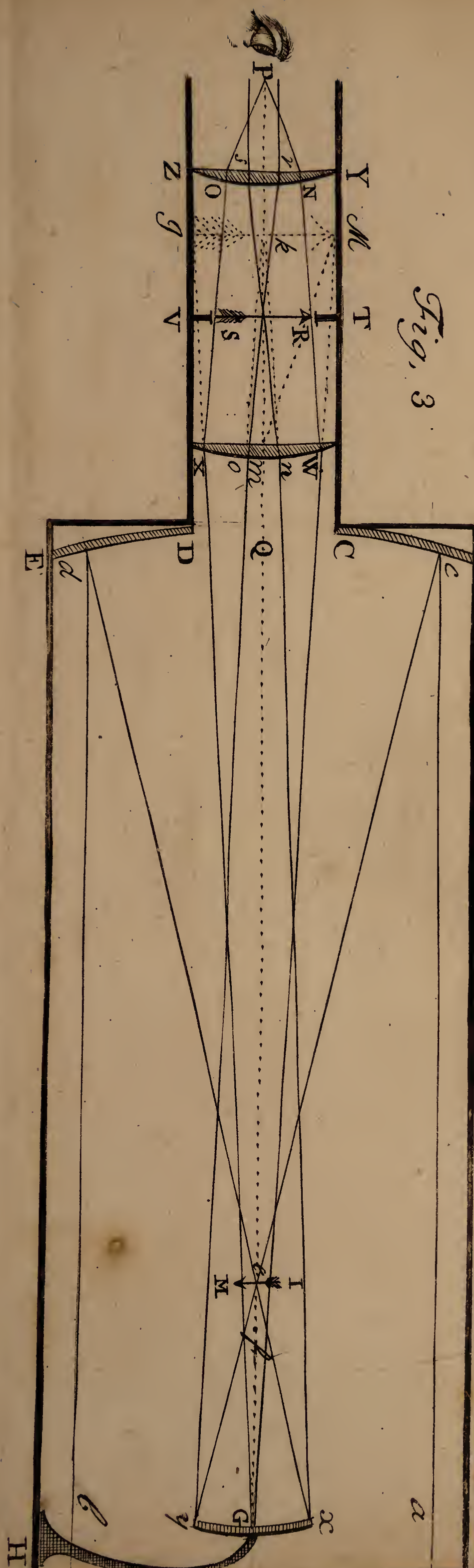


Fig. 3.





C H A P. XVI.

Of Micrometers, and the Method of fitting them to Microscopes and Telescopes.

1. **T**HE Word *Micrometer* is Greek, and signifies an Instrument *to measure small Objects*, as those which are the Subjects of our View through the Microscope or Telescope; and therefore any Contrivance in either of these, or any other Ways applied, by which we can measure such small Objects, whose Dimensions cannot be taken by a common Rule, is called a Micrometer. Of these there are several Kinds, some of which have been applied to the Telescope, but none fixed in a Microscope before that which I make and sell under the Title of the *Pocket reflecting Microscope with a Micrometer*.

2. Those which I make and use in the Microscope are of two Sorts, which I shall now describe. The first consisteth of a circular Piece of Glass *A B*, on the middle Part of which are strait parallel Lines *c d e f*, drawn with the fine Point of a Diamond, (in an Instrument made for that Purpose) of which

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40 are contained in an Inch, or the Interval between two Lines is precisely the 40th Part of an Inch. See Fig. 1. Plate XXXII.

3. The Intervals of these Lines, tho' scarce discernable to the *naked* Eye, are very distinct, and appear very large through a Lens, whose focal Distance is but an Inch, larger than the *Tenth of an Inch* to the *naked* Eye. And therefore, if it be placed in the Focus of the Eye-Glass of the Microscope, the Image of the Object will be painted upon those Lines, and the Parts thereof may be compared with the Intervals, and their true Magnitude or Dimensions thereby very nearly known.

4. Let AB be the Focus of the Eye-Glass DF, (Fig. 2.) in this Focus I either place another Lens for amplifying the Optic Angle, or else the Glass Micrometer for measuring small Objects; let this Micrometer be represented by the Circle GHIK, and the parallel Lines by LMNO, all magnified by the Eye-Glass DF. Let the Edge of a Ruler *gh*, graduated into Inches and Tenths, be laid under the Object-Glass *df*, and *ab*, two of those Tenths which will be represented by AC, CB, in the Focus. Again, let each Tenth *ac*, and *cb*, be divided into four equal Parts, these Divisions will be shewn by the dotted Lines between AC and CB in the Image, and will appear very large.

5. Now

5. Now you are to consider, that so many Times as the Distance Ce , is greater than CE , so many Times AB is bigger than ab ; and consequently, so many Intervals of 40th Parts in the Glass Micrometer, will be equal to one such Interval in the Image AB . If $Ce : CE :: 4 : 1$, then four such Intervals in the Micrometer are equal to one in the Image; if $Ce : CE :: 5 : 1$, or $6 : 1$; then will five or six of those in the Micrometer correspond to one of those in the Image; and therefore by an Observation of this Kind it will be easy to know, at any Time, what the Ratio of Ce to CE is, and consequently how much the Microscope magnifies an Object.

6. With this Micrometer it will be very easy to measure any minute Object very exactly; for the Image being cast upon it, it will be easy to judge what Proportion the Diameter of the Object, or Part to be measured, bears to that of an Interval between two Lines of the Micrometer, and from thence to determine it in Parts of an Inch. Thus, if I observe the Width of an Object to be just the same with that of an Interval, I know it is the 40th Part of an Inch; if half the Width, one 80th Part; if $\frac{1}{4}$ of the Width, it will be one 160th Part of an Inch.

7. Thus if an Object cover a 5th Part of an Interval, it will be but the 200th Part of an Inch in Diameter; if I find it is in Length equal to one Interval, or a 40th Part of an

Inch, the Superficies of that Object will be $\frac{1}{200} \times \frac{1}{40} = \frac{1}{8000}$, or one 8000th Part of a square Inch. And thus the Length, Superficies, and Solidity of any minute Object, or Part thereof, may be known, and measured to a sufficient Degree of Exactness by this Glass Micrometer, which is so easy to be understood, that I need say no more of it.

8. The second Sort of Micrometer is more artificial, and, if well understood, would be of very great Use where the utmost Exactness is required. It consists of a Screw and Nut; (see Fig. 3.) the Nut is fixed into the Side of the Microscope at G, at the Focus of the Eye-Glass D F; on the external Part or Face of the Nut is a graduated Circle, represented in the Fig. N T, in the Center of which the Screw H O moves, the Hand H Q pointing to the Divisions of the Circle.

9. The Screw H O, that I use in my Pocket Microscope, has 50 Threads in an Inch precisely; and the Circle N T is divided into 20 equal Parts, and numbered as in the Figure. Now since one Turn or Revolution of the Hand moves the End of the Screw O, over a Space equal to the $\frac{1}{50}$ of an Inch, the Motion of the Hand over one of the Divisions of the Circle, will cause the Point O to move over the $\frac{1}{20}$ of $\frac{1}{50}$ of an Inch, that is, over the $\frac{1}{1000}$ Part of an Inch.

10. Now when the lower Eye-Glass G K is taken away, the Image A B, of any Object
a b,

a b, is projected into the Place of the Screw, or Focus of the Glass *D F*, and so the Screw *H O* will appear to lie or move upon the Surface of the Image; and as the Screw will appear very large, so it's Motion will be very visible over the least Part of the Image. Suppose now the Image *A B*, be 4. times larger than the Object *a b*; then, since the whole Revolution of the Screw moves the Point *O* over one 50th of an Inch in the Image, it will be but one 4th of one 50th in the Object *a b*; that is, it will measure a 200th Part of an Inch in the Object. Again, since the Motion of the Hand over one Division in the graduated Circle, measures the 1000th Part of an Inch in the Image, it will measure but a 4000th Part of an Inch in the Object.

11. And thus if the Image *A B* be 5, 6, 8, or 10 times greater than the Object *a b*, one Revolution of the Hand will accordingly measure the 250th, 300th, 400, or 500th Part of an Inch in the Object; and the Motion over one Division of the Circle will be over one 5000th, 6000, 8000th, or 10000th Part of an Inch in the Object. And thus knowing the Measure of one Revolution, or of one Division, the Measure for any Number of Revolutions, or Divisions in the Circle, is also known. For Instance;

12. Let *A B* be to *a b*, as 8 to 1; and suppose in measuring the Length or Breadth of an Object, your Index makes four Revolutions,

tions, and stands at the 13th Division of the Circle. Then since in such a Microscope, one Revolution is the 400th Part of an Inch, four Revolutions will be the 100th Part of an Inch in an Object *ab*, and, again, since one Division is the 8000th Part, 13 Divisions will be $\frac{13}{8000} = 0.0016$ of an Inch; but $0.01 + 0.0016 = 0.0116$ of an Inch; that is, the Length or Breadth of such a Part of the Object was $\frac{116}{10000}$ Part of an Inch.

13. Or thus; 4 Revolutions make 80 Divisions on the Circle, which, with the odd 13, make 93 Divisions in all; then $\frac{93}{8000} = 0.0116$, the Decimal Part of an Inch, as before, for the Length of the Object, or Part measured. In Practice, the best Way is to determine the Proportion of *Ce* to *CE*, or of *AB* to *ab*; and then to form a Table, shewing, at Sight, the Measure in Parts of an Inch, answering to all the Revolutions and Divisions thereof, as you judge there will be Occasion for; which is easy to be done by those who have any Skill in such Matters.

14. Note, whenever you go to measure an Object, set the Index precisely at 10, or the Beginning of the Divisions; also let the Screw stand out from the Plate somewhat farther than you judge to be the Length to be measured; and, lastly, with one Hand turn the inner Tube of the Microscope, and with the other move the Object *ab*, so that the Part to be measured may be brought very nicely to touch

touch the sharp Point O of the Micrometer ; then turning the Index about, you will easily perceive when the said Point is just on the other Extremity of the Part, then counting the Revolutions and Degrees, you turn them into Parts of an Inch, as before taught.

15. In Telescopes of the refracting Sort, Micrometers have had a long standing. In those of great Lengths, a very curious Machine of this Kind has been used to measure small Angles ; for in the Telescope, it is the Angles themselves, not their Subtenses, as in the Microscope, that are measured by the Micrometer. And to know the Angle which any Body or Object subtends, being of little Use, except in the heavenly Bodies ; the Micrometer in the Telescope is only applied to measure the Angles subtended by the Diameters of the Sun, Moon, and Planets, &c.

16. But since long Telescopes are going out of Use, and since the Micrometer abovementioned is a very compound and expensive thing, and it's Description would be very tedious, I chuse to pass it by, and give an Account of a more simple one, which contains in it the true Nature and Essentials of a Micrometer ; consisting only of two Screws in a Piece of Wood, Ring of Brass, &c. fixed to that Part of the Tube of the Telescope, which is the common Focus of the Object and Eye-Glass. See Plate XXXIII. Fig. 1.

17. Let

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17. Let AB and DE be the Object and Eye-Glass of a Telescope; and KG, HL, the two Screws in their common Focus. Now if the focal Length of the Object-Glass CF be known, and also the Number of Threads in the Screws which are equal to an Inch, then will it be easy to compute the Quantity of an Angle measured by one Revolution, or Part thereof, of the Screw. Thus suppose $CF = 10$ Feet, or 120 Inches, and that 40 Threads of the Screw make an Inch precisely; again, let GH be the greatest Opening of the Screws, and therefore GCH, the greatest Angle that can be measured in this Telescope.

18. Then if $GH = 1$ Inch, HF will be $\frac{1}{2}$ an Inch, and therefore in the Right Angle Triangle FCH, there being given the Side CF and FH, we shall find the Angle FCH by this Analogy.

As the focal Length $CF = 120 = 2.0791812$
 Is to the Side - - - $FH = 0.5 = 9.6989700$
 So is the Radius - - $45^\circ : 00' = 10.0000000$
 To the Tangent of the An- } 7.6197988
 gle FCH $= 14' 20'' =$ }

19. Now the Screw in passing over FH will make 20 Turns or Revolutions, and since $14' : 20'' = 860'$; therefore say, As $20^R : 860'' :: 1^R : 43''$; that is, an Angle
 of

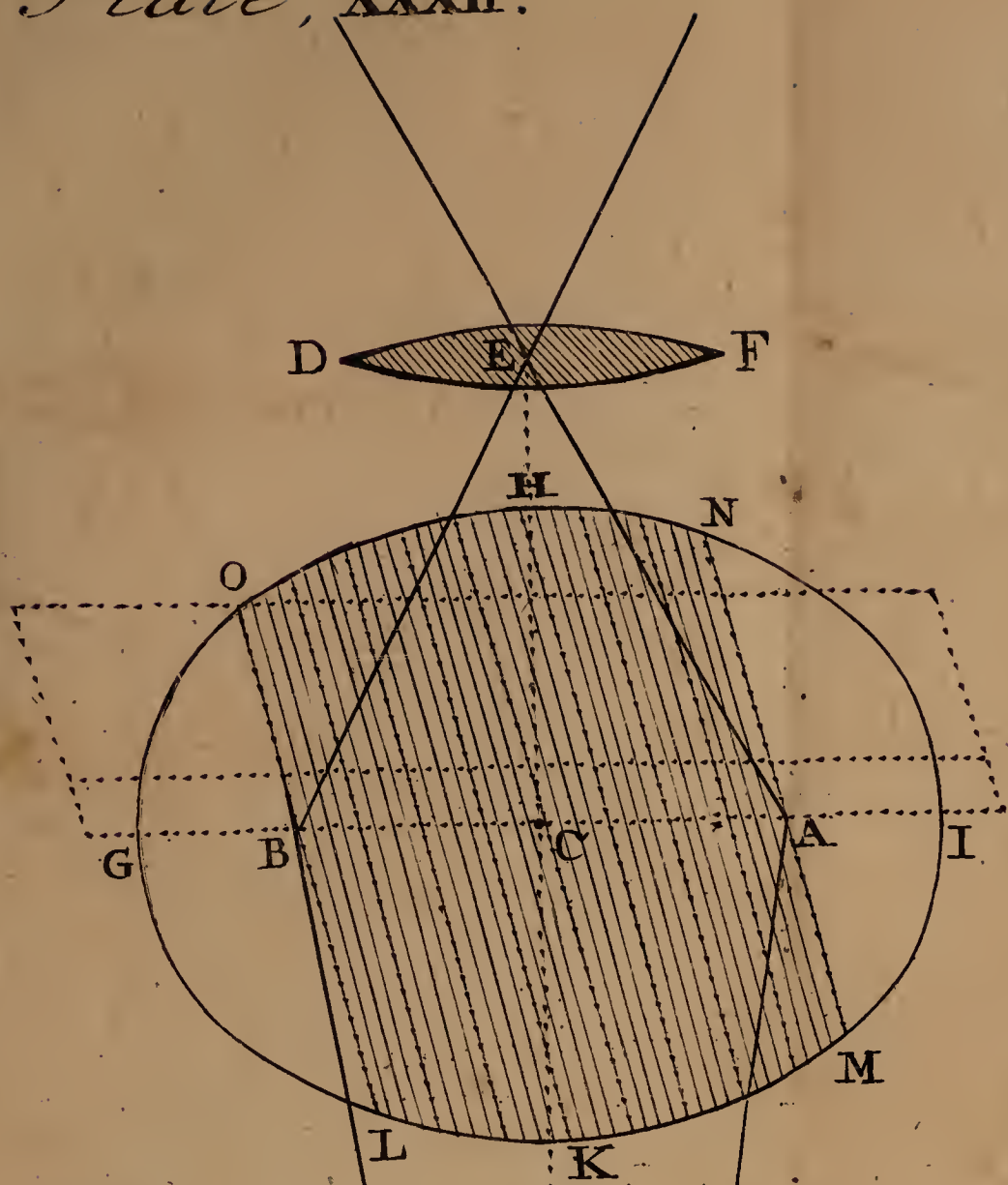


Fig. 2.

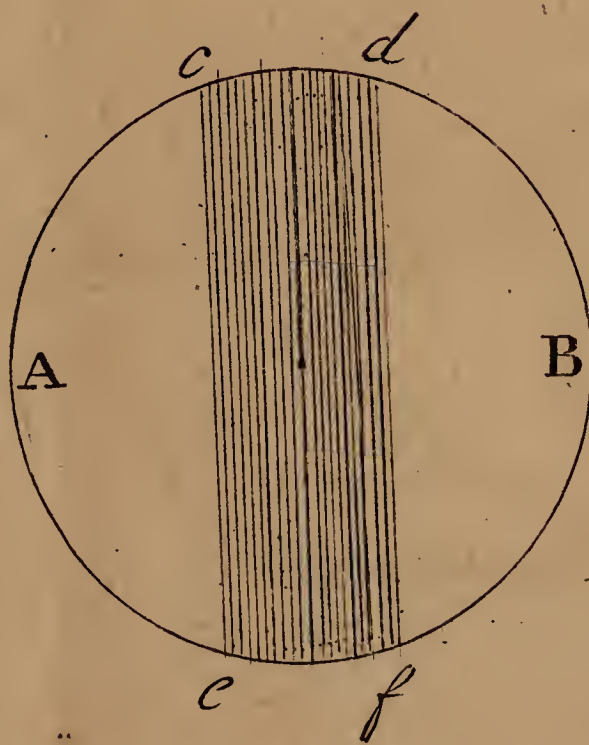


Fig. 1.

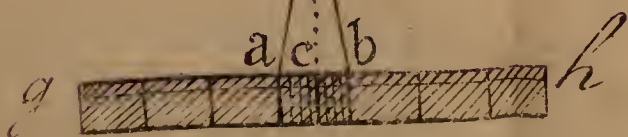


Fig. 3.

of $43''$ is measured by one Revolution of the Screw. If the Index, or Hand of this Screw, moves over a Circle on the Face of the Micrometer, divided into 10 equal Parts, and these subdivided into 10 others each; then every tenth Division will measure $4'' : 18'''$; and every hundredth Part $25''' \frac{8}{10}$. And if the focal Distance CF be greater, the Angles measured will be smaller in Proportion.

20. If it happens that no Number of Threads are exactly contained in an Inch, or any known Measure, there are several Ways whereby the Angle measured by a Revolution, or Part thereof, may easily be computed. Thus suppose AB , CD , were two Lines drawn parallel to each other, on a Wall at a sufficient Distance, and EF their Distance; then if the Telescope be directed to the Line EF , so that the Axis thereof KL , be perpendicular thereto; and the Distances EF , KL , precisely measured, the Angle EKF will be found by Trigonometry, as above, and therefore it is equal to fKe . Then through the Telescope view the Image fe , and adjust the Ends of the Screws to the Extremities thereof very nicely, and count how many Revolutions and Parts of a Revolution are made before the Screws meet. Then say, As the Number of Revolutions is to the whole Angle fKe , so is one Revolution to the Angle it measures. Fig. 2.

21. Or thus, suppose EF represents a Portion of the Equinoctial in the Heavens, and let E be a Star in or near it; then having directed the Telescope to the Star, and set the Ends of the Screws at the Distance of a certain Number of Revolutions, bring the Star to touch one of them, and observe, by a good Pendulum Clock, the Interval of Time which the Star takes up in passing to the other, then turn that Time into the *Minutes*, *Seconds*, and *Thirds*, of an Arch, and they will be the Measure of the Angle of that Aperture of the Screws.

22. Or, if the Star be at any considerable Distance from the Equinoctial, the Interval of Time observed in the Star's Transit over the Opening of the Screws, must be lessened in the Ratio of the following Analogy, *viz.* *As the Radius to the Sine of the Star's Distance from the Pole, so is the Interval of Time observed, to the Time required;* which turned into Minutes and Seconds, gives the Angle of the Aperture of the Screws.

23. In the same Manner you find the Angle for any Aperture of the Screws in a Reflecting Telescope; and consequently a Micrometer may be fitted in these, as well as in the other Sort. I shall conclude with an easy, practical Method of finding the Power of magnifying in a Reflecting Telescope, which is as follows.

24. In

24. In a Tube of 6 Inches Length, fix at one End a Piece of Glas, on which are drawn some parallel and equi-distant Lines, and at the other End a Piece of Pastboard or Wood, with a very small Hole in the middle; with this view some distant Object, whose apparent Diameter will just equal the Interval of two of those parallel Lines. Then put the same Piece of Glas into the Focus of the Eye-Glas of the Telescope, and viewing the same Object again through the Telescope, observe very nicely, how many of those magnified Intervals the magnified Diameter of the Object now equals; then may the Power of magnifying be easily known.

25. For suppose the focal Distance of the Eye-Glas be 1 Inch, the Intervals of the Lines will be magnified 6 times; again, suppose the Diameter of the Object through the Telescope appears equal to 8 of those Intervals, it is evident the Telescope magnifies 6 times 8, or 48 times.

C H A P. XVII.

An Optical Instrument for measuring the Angle of Vision, or estimating the apparent Magnitude of Bodies; also for viewing Perspective Prints, Pictures, &c.

1. **T**H E Instrument for these Purposes is that of Fig. 3. Plate XXXIII. consisting of a Tube $ACBD$, containing two Convex Lenses AB and CD , of an equal focal Distance ib or bn , and placed at twice that Distance from each other; exactly in the midst between them, and therefore in the Focus of each, is placed the Glass Micrometer (before described) EF ; upon which the Image gb , of any distant Object GH is formed by the Object-Glass CD .

2. This Image being also in the Focus of the Eye-Glass AB , is seen distinctly by the Eye placed in the other Focus m ; the Rays gl and bk being parallel, gb is equal to kl ; and therefore the Angle $kml = big = GiH$; and therefore the Image, formed in this Instrument, is seen under the same or an equal Angle, as the Object subtends to the naked Eye;

Eye; And consequently the Object is neither magnified nor diminished thereby.

3. Now suppose the Lines on the Micrometer EF, are 40 to an Inch; (as in that I use) and the focal Distance of the Glasses be one Inch; then the Angle which the Interval between two of those Lines on the Micrometer subtends to the Eye, is of $1^{\circ} : 26'$; so two will subtend an Angle of $2^{\circ} : 52'$; and three an Angle of $4^{\circ} : 18'$; and so on: the Angles and their Subtenses being very nearly proportional, when so small. *Hence the optic Angle, or Angle of Vision, under which any Object at any Distance is seen by the Eye, is immediately known by this Instrument.*

4. *The apparent Magnitudes of all Objects are hereby easily estimated and compared*; for as these are always as the Images formed on the Micrometer, so the Lines or Spaces on that will readily shew the comparative Magnitude of them, and consequently of the Objects themselves in Appearance; for the Object whose Image measures two Lines is apparently twice as big as one that measures in it's Image but one Line, and but $\frac{2}{3}$ as big as another, whose Image measures three Lines.

5. By this Instrument *you also may measure very nearly the Distance of Objects*; thus, suppose you observe an Object, whose Image measures two Lines or Intervals, and you go back till it measures but one, or forward till it measures four, in either Case, the Space or

T

Length

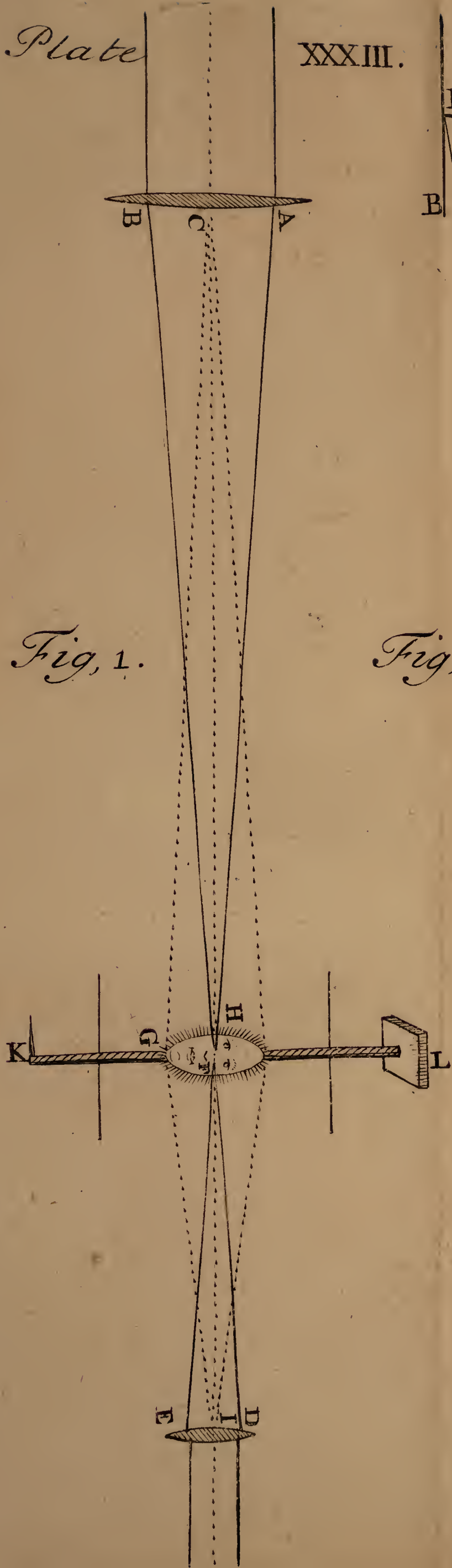
Length you went, is equal to the Distance of the Object from the Place where you first observed it.

6. Again, *the Distances of Objects being known, their Heights are measured pretty exactly by this Instrument.* For the Proportion is As the focal Distance of the Eye-Glass to the Length of the Image, so is the Distance of an Object to it's Height; that is, $ig : gh :: iG : GH$.

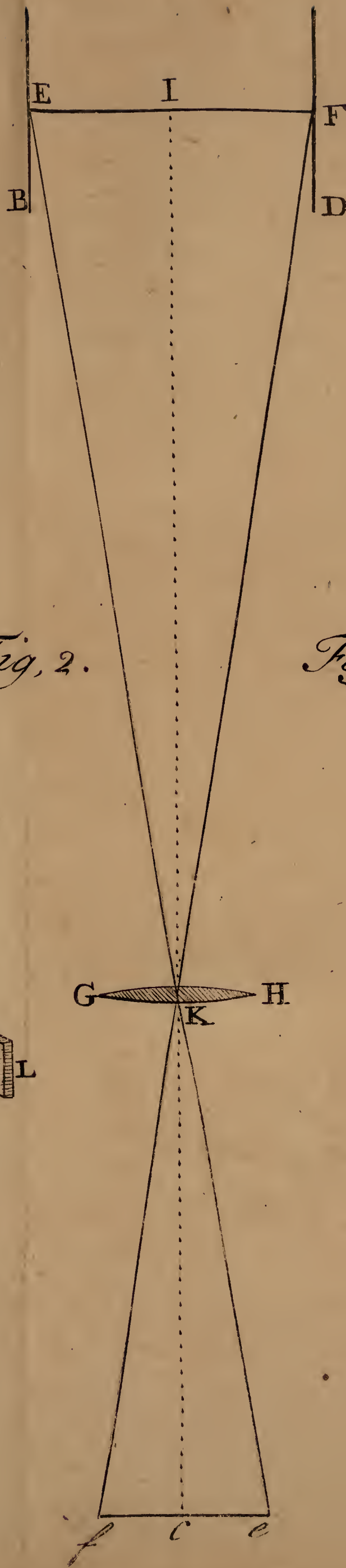
7. *This Instrument is very useful in drawing the Out-Lines of any Landscape or Object you would delineate.* For by means of a circular Piece of Glass, with small Squares drawn thereon, and put in the Place of the Micrometer EF, you may represent the Object or Picture in the same just and natural Proportion and Disposition on any larger Squares, as you see in the Image on the small ones; so will the Perspective of the Piece be perfect.

8. *This small Machine is extremely useful for viewing perspective Prints, Views, and Pictures, &c.* For as when you view Nature, it gives you a beautiful Projection thereof on the perspective Plane; so, on the contrary, when you view a Print, Picture, or any Piece in perspective, it resolves it all into Nature, and gives you the same Ideas of the Positions and Distances of Objects in the Print, as you would have by viewing the things themselves in Nature. It gives to Pictures such a natural and surprizing *Relievo*, as make the Life itself be there. If it be a Face, the Cheeks are protuberant,

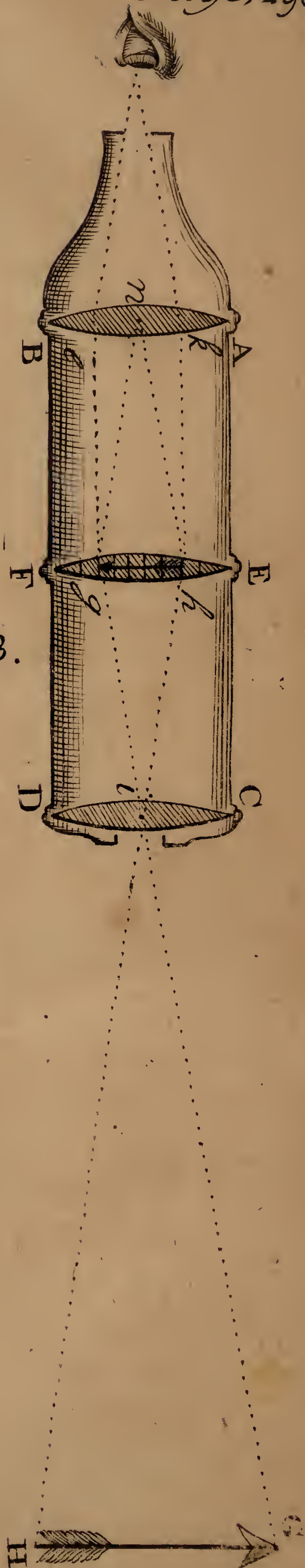
Fig, 1.



Fig, 2.



Fig, 3.





protuberant, the Nose projects, the Eyebrows over-hang and darken the Cavities beneath, in which the Eyes and Eye-lids seem to put themselves in a Posture to view you, and the Lips are about to speak. In short, the Picture or Landscape being well done, and in it's proper Colours, it is almost impossible to distinguish it, seen thro' such a Glass, from Nature, or the Life itself.

9. But most especially those Pictures of Nature's own drawing, (*viz.* those by a *dark Chamber*) viewed by such a Glass, are most delightfully fine and beautiful. For as they are painted always inverted, so this Instrument erects the whole, and gives that Pleasure and Perfection which only was wanting to make the Pictures of a darkened Room absolutely compleat the Representation of Nature.



C H A P. XVIII.

*Of the Magic Lanthorn, and Opera
Glas.*

1. **T**HE *Magic Lanthorn* is an optical Machine, said to be the Invention of Mr — Kircher, in order to magnify small Objects in a dark Room; and has been since used rather to surprize and amuse ignorant People, and for the Sake of Lucre, than for any other Purpose, and thence it had it's common Name: It has been also called *Lanterna Megalographica*, from it's Property of *magnifying small Objects*.

2. The Construction and Theory of this Instrument are very easy to be understood, and are as follows. In a darkened Room ABCD, is placed a dark Lanthorn EFGH, in the Side of which FH is fixed a round Tube K L N M, within which slides another Tube O P, so that the whole may be lengthened or shortened as Occasion requires. In the inmost End of the first Tube is placed a large Plano-convex Lens K L, and towards the external Part of the other another double Convex Lens S T. In the first Tube there is a Contrivance for passing through it a small Frame or Plane of Glas, on which are painted
divers

divers small Objects in transparent Colours, as at Q R. See Plate XXXIV. Fig. 1.

3. In this Lanthorn is a Lamp or Candle I, which by means of the great Convex K L before, and a large Concave Speculum X Y behind, does very strongly illuminate the Object Q R. If now the Lens S T be moved a little farther from the Object Q R than is it's focal Distance, it will form a Representation of the same at a great Distance on the opposite Wall, in a large Image as V W ; which will be as much larger than the Object Q R, as the Distance Z V is greater than Z R. All which is evident from the Theory of a Convex Lens.

4. As the Tube O P is moved farther out of, or into the Tube M N, the Image V W will be smaller or larger, according to the Distance of the opposite Wall. And tho' those Objects are generally some humerous, ridiculous, or frightful Figures, to divert or scare the Spectators ; yet, I believe, this Machine might be applied to more useful Purposes, in magnifying the transparent Parts of Animal and Vegetable Substances, as Wings of Flies, Membranes, &c. especially if enlightened by the Sun-Beams in a darkened Chamber, as I have many Times experimented.

5. But enough of this Instrument. That which I shall next describe is called an *Opera Glass*, from it's being used by Gentlemen in Play-houses, and sometimes a *Diagonal Perspective*, from it's Construction, which is as follows.

follows. $ABCD$ is the wooden Tube about 4 Inches long; EF , GH , are two Holes on each Side, exactly against the middle of IK a Plane Mirrour, which reflects the Rays falling upon it to the Convex LM , thro' which they are refracted to the Concave Eye-Glass NO , whence they emerge parallel to the Eye at the Hole rs , in the End of the Tube. Fig. 2.

6. Let $P a Q$ be an Object to be viewed, from whence proceed the Rays Pc , $a b$, and Qd , these Rays being reflected by the plane Mirrour IK , will shew the Object in the Direction cp , ba , dq , in the Image pq , equal to the Object PQ , and as far behind the Mirrour IK , as the Object is before it; the said Mirrour being placed so as to make an Angle of 45 Degrees, or half a Right Angle with the Sides of the Tube, all which is evident from the Theory of a Plain Mirrour, heretofore explained.

7. Therefore considering pq as an Object, the Case of this perspective Glass is reduced to that of a common refracting Telescope of *Galileo's* Form, whose Object-Glass is LM , and Eye-Glass NO . And since in viewing Objects near at hand, no magnifying of them is necessary, the focal Distances of both the Glasses may be nearly equal; or if that of LM be three Inches, and that of NO be one Inch, the Distance between them will be but two Inches, and the Object will be mag-

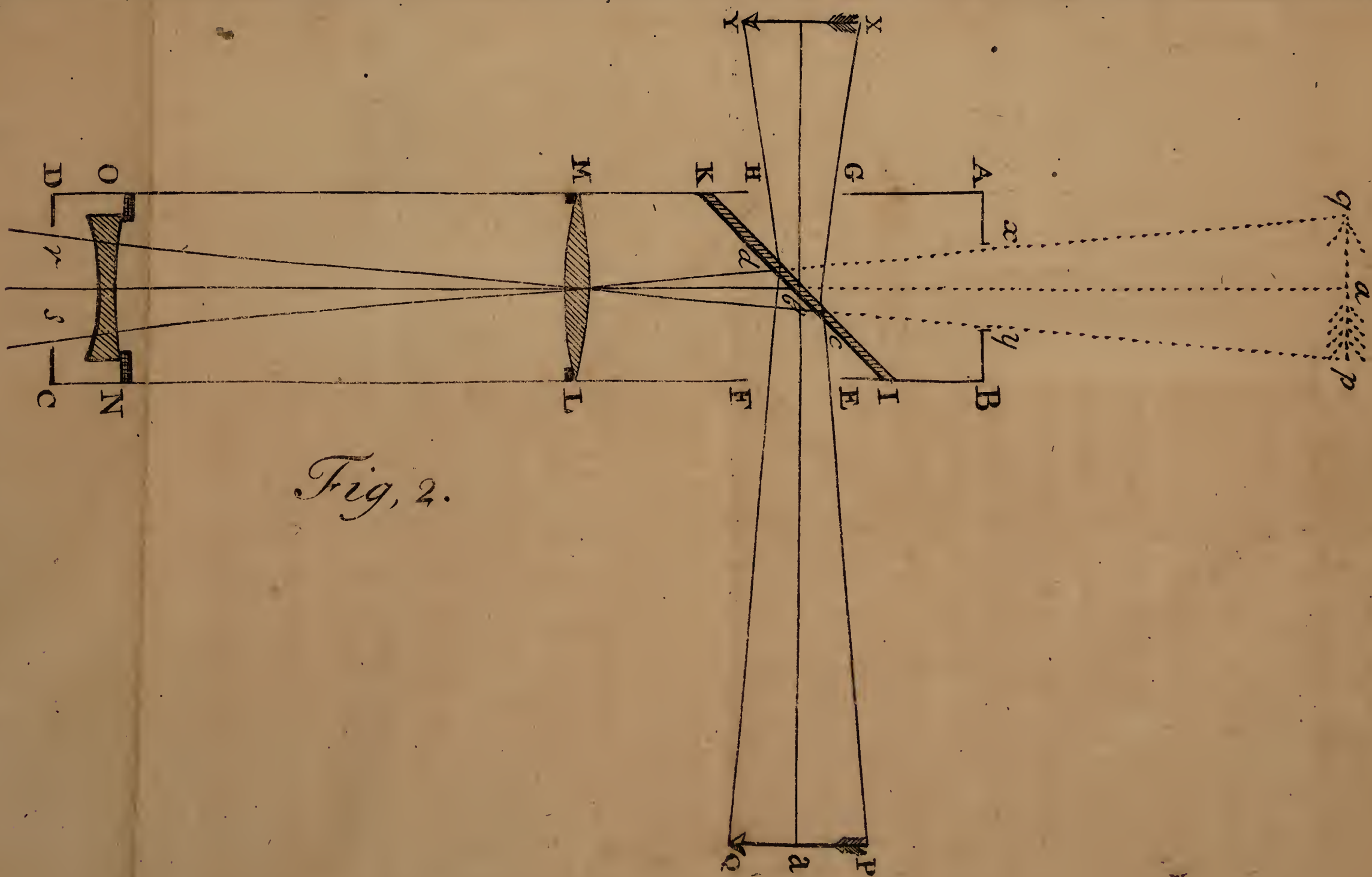
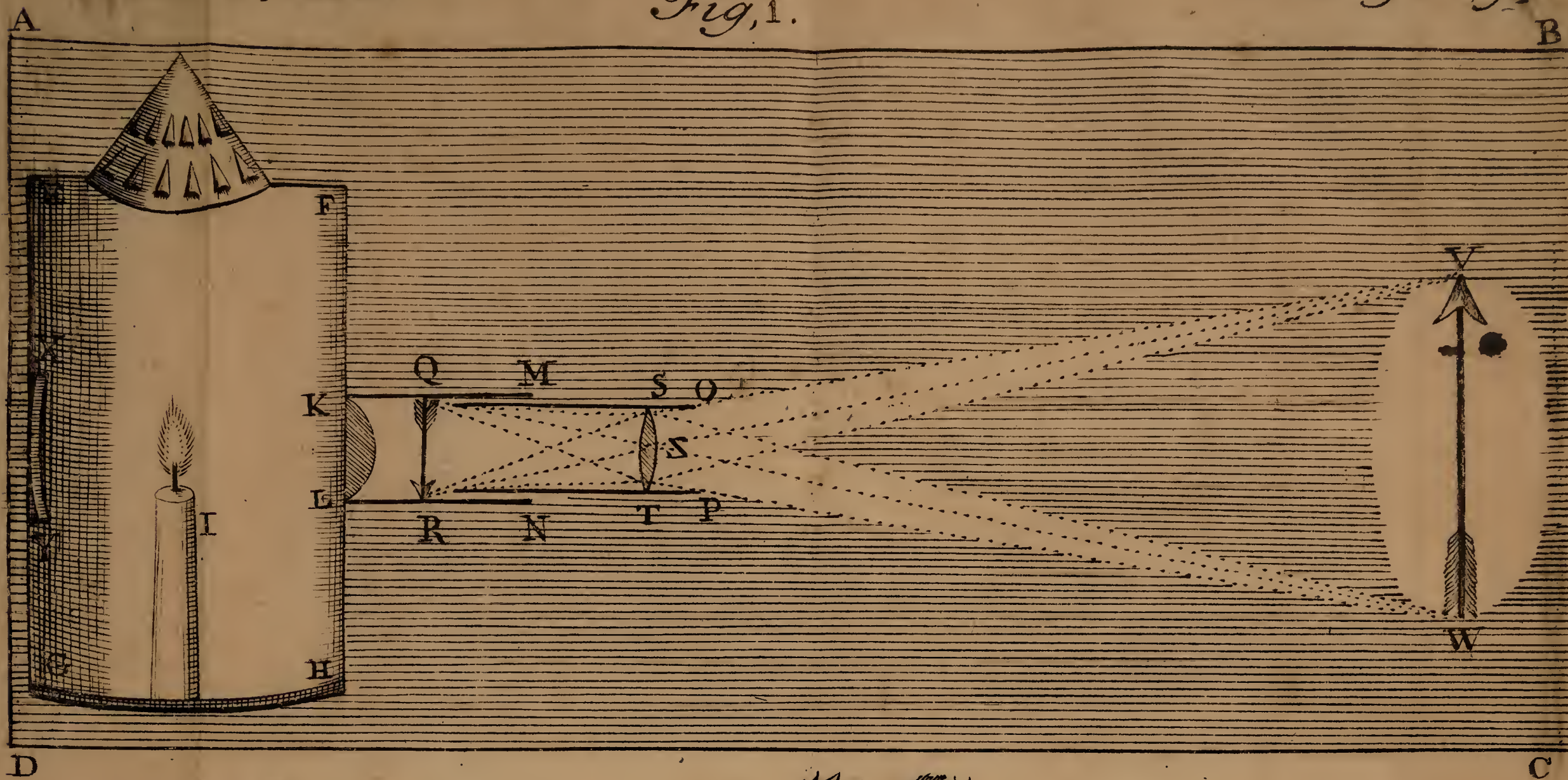


Fig. 2.



nified three times, which is enough to answer the Design of this Glafs.

8. If the Object be very near, as XY , it is viewed through a Hole xy , at the other End of the Tube AB , without an Eye-Glass, the upper Part of the Mirrour being polished for that Purpose, as well as the under. This Tube unscrews near the Object-Glass LM , for taking out and cleansing the Glasses and Mirrour. I presume enough is said to explain the Nature and Construction of this common and well known Instrument.

9. The peculiar Artifice of which is, to view a Person at a small Distance in such a Manner, that no one shall know who it is that is ken'd at, though they know your Design; and that on a double Account, *viz.* because the Instrument points towards another Quarter than that in which the Person is; and because there being a Hole on each Side, it is impossible to know on which Hand the Object is situated which you are viewing. The Position of the Object will be erect thro' a Concave Eye-Glass. And Objects situated high or low will easily be found, by turning the Instrument round one Way and the other about it's Axis.

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